ELEMENTARY MANUAL

ON

APPLIED MECHANICS.

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APPLIED MECHANICS

(AN ELEMENTARY MANUAL ON)

SPECIALLY ARRANGED FOR THE USE OF FIRST-YEAR SCIENCE AND ART, CITY AND GUILDS OF LONDON INSTITUTE, AND OTHER ELEMENTARY ENGINEERING STUDENTS.

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WITH NUMEROUS ILLUSTRATED EXPERIMENTS AND EXAMINATION PAPERS.

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E.W.

PREFACE.

THIS Manual has been written expressly for First-year Students of Applied Mechanics. It therefore forms a suitable companion to the Author's Elementary Manuals on "Steam and the Steam Engine," "Magnetism and Electricity," for it covers the Elementary Stage of the Science and Art Department's Examination in Applied Mechanics. At the same time, the treatment of the subject is sufficiently general to satisfy the wants of other young Engineers and Mechanics, who do not happen to have these Examinations specially in view.

The book has been divided into four stages :—

(1) Forces in Equilibrium and the Principle of Moments and of Work as applied to simple Machines, such as levers, pulleys, cranes, inclined planes, belt and wheel gearing, screws and screw gearing, with and without friction.

(2) Hydraulics and Hydraulic Machines.

(3) Laws of Motion, velocity, acceleration, centrifugal force, balancing fast-speed machinery, and accumulated work.

(4) Properties and Strengths of Materials, chains, shafts, and beams.

These sections have been treated systematically and practically. They are contained within twenty-four Lectures, but both teachers and students will find ample material to occupy their attention for *fully* twenty-eight lessons of one hour each.

In some recent books on Mechanics an attempt has been made to deal with the "Laws of Motion," and the effects of motion on

solid bodies, *before* illustrating and explaining simple machines and hydraulics. The Author has found, from an experience of over twelve years' continuous teaching, that this new order is quite a mistake, for students get confused and disheartened at the very outset by a mass of formulæ and problems which are far more difficult for them to understand than those relating to forces in equilibrium as applied to simple machines.

Great stress has been laid on principles, definitions, uniformity of notation, and symbols. The explanations, illustrations, and examples are such as will enable students to apply leading principles to practical work, and teach them to think out and investigate results for themselves, rather than depend blindly on rules and formulæ.

In every Lecture a number of examples have been fully worked out, and, wherever possible, illustrated experiments have been described, so as to encourage students to carry out similar or more elaborate experiments in actual practice, rather than rely on rule-of-thumb proportions.

At the *end* of *each* Lecture a series of carefully selected questions has been arranged in the *precise order of*, and relating *solely* to, *each* Lecture, so that teachers and students may have a minimum of trouble in finding suitable examples. Full advantage has been taken of the questions set annually by the Science and Art Department's examiner on this subject; in fact, all the more important questions which have appeared in the Elementary papers for the last six years have been incorporated, with many others.

The book, as a whole, will form an easy introduction to the Author's more Advanced Text book upon the same subject, now in preparation. Consequently, all points requiring mathematical knowledge higher than has been herein employed, as well as descriptions of more complicated mechanisms, have been left over for treatment to the Advanced Course.

In the preparation of the drawings the Author is indebted

for help to his students, Messrs. George W. Shearer, John A. Sloan, David A. Ramsay, and John F. Neilson, as well as to the following firms for working-drawings and illustrations of special machines and tools: Messrs. John Lang & Sons, toolmakers, Johnstone; Messrs. P. & W. MacLellan, and Messrs. Loudon Brothers, Glasgow; Messrs. Weems & Co., Coatbridge; and Messrs. Holt & Willetts, of Cradley Heath. Finally, his thanks are due to Mr. R. M. Anderson for revising the proofs.

If any errors should be observed by readers, the Author will be glad to have them pointed out, and to receive any suggestions tending to increase the usefulness of the book, his desire being, as far as possible, to keep it abreast of the times and of the wants of Students.

ANDREW JAMIESON.

THE GLASGOW AND WEST OF SCOTLAND
TECHNICAL COLLEGE:

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INSTRUCTIONS TO BE FOLLOWED IN THE WRITING OF HOME EXERCISES.

1. Put the date of handing-in each exercise at the right hand top corner.

2. Leave a margin an inch wide on the left hand side of each page; and in the margin place the number of the question, and nothing more.

3. Leave a space of at *least* four lines between your answers for remarks or corrections.

4. Be sure you understand *exactly* what the question requires you to answer, then give *all* it requires, but *no more*. If unable to answer any question, write down its number and the reason why.

5. Make your answers concise, clear, and exact; and accompany them, whenever practicable, by an illustrative sketch.

6. Make all sketches large, open, and in the centre of page, and do not crowd writing about them.

(NOTE.—The character of sketches will be considered in awarding marks.)

7. Every sketch must be accompanied by an "Index of parts" written immediately beneath it, and must accompany the answer it is designed to illustrate.

(NOTE.—The initial letter or letters of the name of the part must be used, and not A, B, C, or 1, 2, 3, &c.)

8. Unless specially asked by the question, every sketch must be accompanied by a concise written description.

9. Every answer which receives less than five marks must be re-written correctly for next evening, before the usual class work, and headed "Re-written."

REMARKS.—Students are strongly recommended to write out each answer in scroll first, and then to compare it with the question. After committing the answer to their book, they should then read it over a second time, to correct any errors they may discover. Reasonable and easily intelligible contractions are permitted. Students are invited to ask questions and explanations regarding anything they do not understand. Except in special cases, arrears of Home work *will not receive marks.*

N.B.—Students who from any cause have been absent from a lecture should send a post card or note of explanation to their teacher. If they miss any exercise or exercises they must state the reason (in red ink or underlined), in their exercise books en handing them in next night.

ELEMENTARY MANUAL

ON

APPLIED MECHANICS.

LECTURE I.

CONTENTS.—Definition of Applied Mechanics—Force—Matter—Unit of Force—The Elements of a Force—Graphic Representation of Forces —Forces in Equilibrium—Action and Reaction—Resultant and Components—Resultant of Forces acting in a Straight Line.

Applied Mechanics is that branch of applied science which not only explains the principles upon which machines are designed, made and act, but also describes their construction and applications, as well as how to calculate and test their strength and efficiency.

Before a student can successfully master any science, he must thoroughly understand the units of measurement that have been adopted in calculating results, and he should also have a clear conception of the exact meaning of the various terms employed. Consequently, we shall commence the study of Elementary Applied Mechanics with definitions and with units of force, work and power.*

Force *is any cause which produces, or tends to produce, motion or change of motion in the matter upon which it acts.*

Matter *is anything which can be perceived by one or more of the senses, and which can be acted on by force.*

Matter exists under three conditions: (1) Solids, (2) Liquids, (3) Gases. For example, pieces of wood and of iron are solids; water and mercury are liquids; whilst air and oxygen are gases.

* For the units of length, surface and cubic measure, and for the mensuration of areas and solids, the student is referred to Lectures I. II. and III. of Author's "Elementary Manual on Steam and the Steam Engine," issued by the *publishers of this* book.

Bodies are therefore limited portions of matter. When the resistance to motion of a body is equal to or greater than the force applied, so that no motion takes place, the body is said to be subjected to *pressure*.

Solids do not yield readily to pressure, for they tend to retain their original shape and size, whereas liquids and gases yield to a very slight pressure, and consequently possess no definite shape. A gas differs from a liquid since it possesses the property of indefinite expansion. A liquid has therefore a definite size, but not a definite shape, whilst a gas has neither definite shape nor definite size.

Unit of Force.—The British unit of force is the pound avoirdupois, or GRAVITATION UNIT. The magnitude of a force is therefore reckoned by the number of pounds which the force would support against gravity or by the weight in pounds which would produce the same effect. For example, a force of 100 lbs. means that force which would just lift a weight of 100 lbs. if acted on by gravity alone. But the force of gravity varies at different parts of the earth's surface, being slightly greater at the Poles than at the Equator. Consequently, our unit is not an absolute or invariable one, although for all the practical purposes of applied mechanics it is the most convenient unit which could be employed.*

The Elements of a Force.—When a force acts upon a body, then, in order to fully determine its effect we must know the three following elements:—

(1) The point or place of application of the force.
(2) The direction in which the force acts.
(3) The magnitude of the force.

(1) *Place of Application.*—In the case of the force of gravity acting on a body, the place of application may be considered to be the whole mass of the body, or we may estimate the whole weight of the body as concentrated at one point, termed the centre of

* An absolute or invariable *unit of force* is that force which, acting for one second on a *mass* of one pound, gives to it a velocity of one foot per second. It is called the *Poundal*. *Mass* is estimated by the weight in pounds divided by the acceleration of gravity at the place of observation —*i.e.*, $M=\frac{W}{g}$, where M stands for mass, W, for weight in pounds, and, g, for the acceleration of gravity. At London, $g=32 \cdot 2$ feet per second. The mass of a body is therefore a constant quantity.

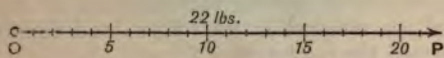
In Electrical Engineering measurements, the unit of force is called the *Dyne*, and is that force which, acting for one second on a *mass* of one gramme, gives to it a velocity of one centimetre per second. In estimating force in dynes, mass is equal to the weight in grammes divided by the [acceler]ation due to gravity, which at London $=981$ centimetres per second.

gravity of the body. When an extended surface is subjected to pressure, as in the case of a tank containing a liquid, or the piston of an engine subjected to the pressure of a gas, the whole area under pressure may be considered as the place of application. When a body is pulled by means of a rope, or pushed by means of a rod, or supported on a small area, then we consider the force as acting at a point.

(2) *Direction.*—The direction of a force is the line or path in which it tends to move the body on which it acts.

(3) *Magnitude.*—The magnitude of the force is the pounds pull or pressure which the force exerts upon the body on which it acts.

Graphic Representation of Forces.—When a force acts on a body at a point, its three elements may be conveniently represented as follows :—



SCALE DIAGRAM OF A FORCE.

Where O represents the *point of application*, the straight line, OP (with the arrow-head), shows the *direction* in which the force acts, and the length of the divided line, OP, indicates to scale the *magnitude* of the force.*

Forces in Equilibrium.—(1) When any number of forces acting upon a body neutralize each other's effects (*i.e.*, leave the body in the same condition as to rest or motion as before the application of the forces), these forces are said to be in equilibrium.

(2) Forces which are in equilibrium may be applied to or removed from a rigid body without altering its condition as to rest or motion.

(3) Two equal and opposite forces destroy each other's effects; and, conversely, no *two* forces can destroy each other's effects unless they are equal and opposite.

(4) A force will have the same effect at whatever point in its own direction it may be supposed to act; and, conversely, if a force have the same effect whether it act at one or other of two given points, then the straight line joining these points will be the direction of the force.

Action and Reaction.—(1) Whenever a fixed rigid body is

* In the case of the above figure the force is represented as equal to 22 lbs. Students will find it convenient to plot down the representation of forces in their exercise books to a scale of $\frac{1}{10}$ of an inch to a pound, or hundredweight, or ton, according to the values of the stresses.

acted on by a force, then naturally there is at once set up in that body a secondary force, or a force of reaction, equal and opposite in direction to the primary force.

(2) Hence action and reaction are equal and opposite, and neutralize each other's effects.

For Example.—Suppose a weight is placed on a rigid horizontal table. In the table there is set up an opposing force or upward reaction which exactly counterbalances the downward force of the weight. If this were not the case, then motion would take place, and either the table would give way, or the weight would sink through the table!

Resultant and Components.—(1) If any number of forces acting upon a body be replaced by a single force which shall have the same effect, then this force is termed the *resultant* of these forces, and the forces are called the *components* of their resultant.

(2) The operation of finding the resultant of any number of forces is called the *composition* of forces; and finding the components is termed the *resolution* of forces.

Resultant of Forces acting in a Straight Line.—(1) The resultant of any number of forces acting in the one direction along one straight line is equal to their sum, and acts in that direction.

For Example.—Let $P_1 P_2 P_3 P_4$ be any four forces acting in one direction along one straight line, then their resultant—

$$R = P_1 + P_2 + P_3 + P_4$$

(2) If the forces do not all act in one direction, then the resultant is equal to the difference between the resultant of those acting in one direction and the resultant of those which act in the opposite direction, and has the direction of the greater of the two resultants.

For Example.—Let $P_1 P_2 P_3 P_4$ be any four forces acting along one straight line to the right hand or in a positive direction; and $Q_1 Q_2 Q_3$ be any three forces acting along the same straight line, but in an opposite or left-hand or negative direction, and let

$$Q_1 + Q_2 + Q_3 \text{ be less than } P_1 + P_2 + P_3 + P_4$$

Then the resultant,

$$R = (P_1 + P_2 + P_3 + P_4) - (Q_1 + Q_2 + Q_3)$$

and acts in the same direction as $P_1 P_2 P_3 P_4$, and along the same straight line.

If equilibrium existed between these two sets of oppositely directed forces, then their algebraical sum would be zero, or the resultant would vanish; *i.e.*,

$$(P_1 + P_2 + P_3 + P_4) - (Q_1 + Q_2 + Q_3) = R = O$$

A familiar illustration of the above reasoning, is the game of "the tug of war," when, say, a batch of sailors are pitted against a corresponding number of soldiers, each batch pulling their utmost at the opposite ends of a rope, and in opposite directions, with the view of obtaining a resultant.

We shall return to the graphic representation of forces, &c., when we come to deal with the parallelogram and triangle of forces and their application to ascertaining the stresses on simple structures.*

* We have intentionally made this Lecture a short one, and have not appended any questions, because at the first meeting of a session the lecturer has to give a series of general instructions to his students, and the class is seldom so completely formed as to make it worth while setting any home work until the second meeting.

LECTURE II.

CONTENTS.—Work—Unit of Work—Examples I. II. III. IV.—Work done against a Variable Resistance—Example V.—Diagrams of Work—With Uniform Resistance—With a Gradually Increasing Resistance—With a Gradually Decreasing Resistance—For Example V.—With a Combination of Uniform and Variable Loads—Power or Activity—Units of Power—The Horse-power Unit—To find the Horse-power of any Working Agent—Example VI.—Questions.

Work.—If a force acts upon a body and causes that body to move through a distance, then the force is said to have done work. It does not matter how long the operation takes, whether a second, a minute, an hour, or a day, or even a year, the same amount of work is done by the force acting through the distance. Time, therefore, does not come into the question of estimating work done, but we must have a force overcoming a resistance through a definite distance. If the force applied be inadequate to overcome the resistance of the body to motion, then no work is done. The amount of work done therefore depends *solely* upon the product of the force applied (or the resistance overcome) and the distance through which it acts.

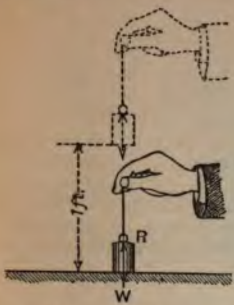
Or, **Work = Force × Distance.**

Unit of Work.*—*The unit of work, is the work done in overcoming unit force through unit distance.* Now, since the British unit of force is the pound, and unit distance the foot, the British *unit of work is called the foot-pound*, and is therefore the work done when a resistance of 1 lb. is overcome through a distance of 1 foot.

EXAMPLE I.—If a weight of 1 lb. be elevated a vertical distance of 1 ft. against the force of gravity, then 1 foot-pound of

* In the case of heavy work the unit *foot-ton* is sometimes used in this country. A foot-ton simply means one ton raised one foot high against gravity, or a force of one ton exerted through a distance of one foot, or a resistance of one ton overcome for a distance of one foot. In Electrical Engineering the unit of work is the work done in overcoming a resistance of one *dyne* through a distance of one *centimetre*. It is called the *Erg*. Since the weight of 1 gramme is = 981 dynes, the work done in raising 1 gramme through a vertical height of 1 centimetre against the force of gravity is 981 ergs or (g) ergs. One foot-pound = $1 \cdot 356 \times 10^7$ ergs.

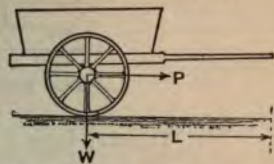
work has been performed. If 10 pounds be elevated vertically through a distance of 10 ft., then result is $(10 \times 10) = 100$ ft.-lbs. of work.



UNIT OF WORK.

W = 1 lb. weight.
R = 1 lb. reaction.

Fig. for Example III.



ILLUSTRATING WORK DONE.

W = Weight in lbs.
P = Pull in lbs.
L = Length in feet.

EXAMPLE II.—If a body offers a constant resistance to motion in *any direction* of **P** lbs., and if it be forced along a distance of **L** ft., then the work done is **P** × **L** ft.-lbs.

Or,	Work done	=	Force	×	Distance
i.e.	**Foot-pounds**	=	**P lbs.**	×	**L feet.**

Suppose a cart with its load weighs **W** lbs. and offers a constant resistance of **P** lbs. to traction along a road, and that it is pulled through a distance of **L** feet; then,

$$\text{The work done} = P \times L \text{ (ft.-lbs.)}$$

EXAMPLE III.—In drawing a loaded cart along a level road, a horse has to exert a constant pull of 100 lbs.; how much work will be done in 10 minutes supposing the horse to walk at the rate of 6000 yards an hour?

$$\left.\begin{array}{l}\text{Distance in feet through}\\ \text{which the resistance of}\\ \text{100 lbs. is overcome in 10}\\ \text{minutes.}\end{array}\right\} = \frac{6000 \text{ (yds.)} \times 10 \text{ (m.)} \times 3 \text{ (ft.)}}{60 \text{ (m.)}}$$

$$\text{,, ,,} = 3000 \text{ ft.}$$

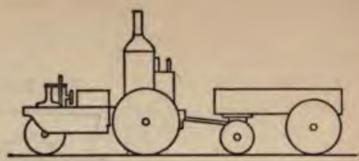
$$\text{Work done in 10 minutes} = P \times L.$$

$$\text{,, ,,} = 100 \times 3000.$$

$$\text{,, ,,} = \mathbf{300{,}000 \text{ ft.-lbs.}}$$

EXAMPLE IV.—A traction engine is employed to draw a loaded waggon along a level road where the resistance to be overcome is

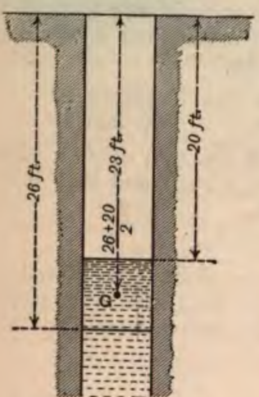
100 lbs. per ton. How many foot-pounds of work are expended in drawing 10 tons over 100 yards?



TRACTION ENGINE AND LOAD.

1. Tractive force = 100 lbs. per ton.
2. Total pull, P, = 100 (lbs.) × 10 (tons) × 2240 (ft.)
3. Distance, L, = 100 (yds.) × 3 (ft.)
4. Work done = P × L.
   „ „ = 100 × 10 × 2240 × 100 × 3.
   „ „ = **672,000,000 ft.-lbs.**

Work Done against a Variable Resistance.—If the resistance varies whilst the force overcoming it acts through a known distance, then the work done will be measured by the product of the average resistance and the distance. If the resistance varies uniformly, its average can be found by adding its values at the commencement and end of the motion, and dividing by two.



WORK VARYING UNIFORMLY.

EXAMPLE V.—Explain the method of estimating the work done by a force, and define the unit of work. The surface of the water in a well is at a depth of 20 feet, and when 500 gallons have been pumped out, the surface is lowered to 26 feet. Find the number of units of work done in the operation, the weight of a gallon of water being 10 lbs. (S. and A. Exam. 1887.)

For an answer to the first part of this question refer to the previous part of this lecture.

1. Weight of water raised = weight of 500 gallons.
   „ „ „ = 500 × 10 lbs.

   Or, . . . P, = 5000 lbs.

2. Mean height water is lifted = { Distance through which the centre of gravity, **G** (of raised water), has been elevated.

" " " $= \frac{20 + 26}{2}$ ft.

Or, . . . L, = 23 ft.

3. Work done . . = P × L.

" " . . = 5000 × 23.

" " . . = **115,000 ft.-lbs.**

Diagrams of Work.—(1) *Against a Uniform Resistance.*—If the resistance overcome is uniform, then the work done may be graphically represented by the area of a rectangle.

To find the work done in overcoming a uniform resistance of 5 lbs. through a distance of 10 ft.: Plot down a vertical line to any convenient scale to represent P (or 5 lbs.) and a horizontal line to the same scale to represent L (or 10 ft.) Then complete the *rectangle.*

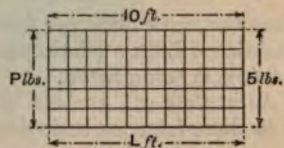


DIAGRAM OF UNIFORM WORK.

The area P × L or 5 × 10 = 50 ft.-lbs. of work.

In the accompanying figure a scale of $\frac{1}{10}$th inch has been used to represent both pounds and feet, consequently each of the small squares represents to scale one foot-pound of work.

(2) *With a Gradually Increasing Resistance.*—If the resistance gradually increases — for example, in the raising of a length of rope or chain vertically by one end from the ground, then the work done may be graphically represented by the area of a right-angled triangle, where P represents the total weight of chain in lbs., and L its total length in feet.

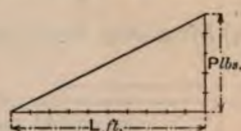


DIAGRAM OF WORK FOR AN INCREASING RESISTANCE.

$$\therefore \text{ The Total Work done} = \frac{P \times L}{2} \text{ ft.-lbs.}$$

Here the work done per foot of length of chain lifted, gradually increases from a minimum to a maximum, until the whole rope or chain is off the ground. When any known length, l, has been lifted, then the area enclosed by the triangle whose horizontal side is l, and vertical side p represents the work done.

(3) *With a Gradually Decreasing Resistance.*—If the resistance gradually decreases, as in the case of winding a rope or chain upon the barrel of a winch or crane, then the work done will also be represented graphically by the area of a right-angled triangle, where P represents the total weight of rope or chain in pounds being lifted at the start, and L its length in feet.

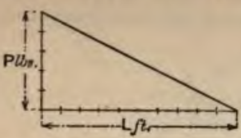


DIAGRAM OF WORK FOR A DECREASING RESISTANCE.

$$\therefore \text{The Total Work done} = \frac{P \times L}{2} \text{ ft.-lbs.}$$

Here the work done per foot of length of chain lifted, gradually diminishes from a maximum at the start to a minimum, when the last foot is being lifted.

As in Case (2), you can at any time know the work done or still to be done from the scale diagram, if you know the length of chain lifted or to be lifted.

For example, if l feet have still to come on to the barrel, then the vertical ordinate p on the scale diagram will represent the pull being exerted at the time, and consequently $\frac{p \times l}{2}$ represents the work still to be done.

Or, generally, with any gradually increasing or decreasing resistance the work done is equal to the mean of the average resistance in lbs. × the distance through which it acts in feet.

(4) *Diagram of Work for Example V.*—In the case of Example V. the distance through which each pound of water is lifted gradually increases, not from zero, but from a depth of 20 feet at the commencement, to a final depth of 26 feet. Hence, if P represent the resistance due to the weight of water lifted, and $L_1 = 20$ ft. and $L_2 = 26$ ft. drawn to scale in accordance with the foregoing directions, the area of the accompanying trapezoid represents to scale the work done.*

DIAGRAM OF WORK FOR EXAMPLE V.

; of the Author's "Elementary Manual on Steam and the Steam
r how to find the Area of a Trapezoid.

$$\text{Or: The work done} = P \times \frac{L_1 + L_2}{2}$$
$$= 5000 \times \frac{20 + 26}{2}$$
$$= 5000 \times 23 = \mathbf{115{,}000\ ft.\text{-}lbs.}$$

(5) *With a Combination of Uniform and Variable Loads.*—When one part of a load is fixed and another part variable, as in the case of lifting a fixed weight with a chain, by winding the chain on the barrel of a winch or crane, the diagram of work for the fixed load is naturally a rectangle, and for the chain a triangle if the chain is completely wound on to the barrel, or a trapezoid if there is still some portion of it to be lifted.*

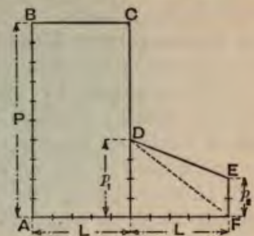


DIAGRAM OF WORK FOR UNIFORM AND VARIABLE LOADS.

Let P . = the uniform pull required to lift the load or overcome the uniform resistance.
L . = the distance the weight is lifted.
p_1, p_2 = the weights of chain hanging at the commencement and at the finish of the lift.

$$\text{Work done in lifting the fixed weight} = P \times L$$

$$\text{Work done in lifting the variable weight} = \frac{p_1 + p_2}{2} \times L$$

$$\therefore \text{Whole work} = P \times L + \frac{p_1 + p_2}{2} \times L = \left(P + \frac{p_1 + p_2}{2}\right)L$$

" " = Area of rectangle and trapezoid.
" " = Area of the figure, ABCDEF.

* Instead of splitting up the figure for the diagram of work into two distinct parts, viz., into a rectangle and a trapezoid, the trapezoid might have been placed on the top of the rectangle with the line representing p_1 as an extension of AB or P. Then the diagram would not only point out the work done when the load was lifted any known distance, but also how the resistance varied during the operation. At first the resistance $= P + p_1$, at the finish, it is $= P + p_2$.

$$\therefore \text{Total work} = \frac{(P+p_1)+(P+p_2)}{2} \times L = \left(P + \frac{p_1+p_2}{2}\right)L$$

The resistances when the load has been lifted $\frac{1}{4}$, or $\frac{1}{2}$, or $\frac{3}{4}$ of the full distance, will be represented by vertical lines drawn from the horizontal base line at A to the inclined line DE (when the trapezoid is placed on the top

Power or Activity *is the rate of doing work.**—In estimating or testing the power of any agent the time in which the work is done must be noted and taken into account. Consequently, we speak of the activity or power of a man, of a horse, or of an engine, as capable of doing so many foot-pounds of work per minute.

Units of Power.†—The unit of power adopted in this country is called the *horse-power*. *It is the rate of doing work at* 33,000 *foot-pounds per minute.*

The Horse-power Unit was introduced by James Watt, the great improver of the steam engine, for the purpose of reckoning the power developed by his engines. He had ascertained by experiment that an average cart-horse could develop 22,000 foot-pounds of work per minute, and being anxious to give good value to the purchasers of his engines, he added 50 per cent. to this amount, thus obtaining (22,000 + 11,000) the 33,000 foot-pounds per minute unit, by which the power of steam and other engines has ever since been estimated.

To find the Horse-power of any Working Agent.—*Divide the number of foot-pounds of work which it does in one minute by* 33,000.

Let P = Pull exerted or resistance overcome in pounds.
L = Length or distance through which P acts.
M = Minutes the agent is at work.
H.P. = Horse-power.

Then,

$$\text{H.P.} = \frac{P \times L}{33000 \times M}\,;\ P = \frac{\text{H.P.} \times 33000 \times M}{L}\,;\ L = \frac{\text{H.P.} \times 33000 \times M}{P}$$

EXAMPLE VI.—In what way is the rate of doing work measured in horse-power?

If 40 cubic feet of water be raised per minute through 330 feet, what horse-power of engine will be required, supposing that there is no loss of friction or other resistances? *Note.*—1 cubic foot of water weighs $62\frac{1}{2}$ lbs. (S. and A. Exam. 1892).

of BC) at points $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the length L from A. We purposely divided the diagram into two parts with the view of helping the student to see how the two portions of the work could be treated separately, but he should now draw the diagram for himself in the way just indicated.

* The word *power* is very frequently misapplied by writers and students, for they often call the mere pull, pressure, or force exercised on or by an agent the power. Students should strenuously avoid this misuse of the word power, and never employ it in any other sense than as expressing *a rate of doing work, or activity.*

† I[illegible] trical Engineering the Unit of Power is called the *Watt*, and it eq[illegible] s per second, or 746 Watts = 1 horse-power.

ANSWER.—The rate of doing work, as measured in horse-power, is equivalent to 33,000 foot-pounds of work done per minute.

1st. 1 cubic foot of water weighs $62\frac{1}{2}$ lbs.

$\therefore$ 40 cubic feet of water weigh $40 \times 62\frac{1}{2} = 2500$ lbs.

2nd. Work done per minute $= \dfrac{P \times L}{M} = \dfrac{2500\text{ (lbs.)} \times 330\text{ ft.}}{1}$

$$\text{3rd. } \therefore \text{ H.P.} = \frac{P \times L}{33000 \times M} = \frac{2500 \times 330}{33000 \times 1} = \frac{825000}{33000} = \mathbf{25}$$

Note.—Students will find it a great advantage, as well as a saving of time not to multiply figures together until the last stage of the answer has been reached, and then to cancel all common factors in numerator and denominator. For example, in the answer to the above question we might proceed thus—

1st. 40 cubic feet of water $= 40 \times 62\frac{1}{2}$ lbs.

2nd. Work done per minute $= 40 \times 62\frac{1}{2} \times 330$ ft.-lbs.

$$\text{3rd. } \therefore \text{ Horse-power of engine} = \frac{\overset{4}{\cancel{40}} \times 62\frac{1}{2} \times \overset{1}{\cancel{330}}}{\underset{\underset{10}{\cancel{100}}}{\cancel{33000}}}$$

$$\therefore \text{ H.P.} = \frac{4 \times 62{\cdot}5}{10} = \frac{250}{10} = \mathbf{25}$$

The process consists in this—the factor, 330, can be cancelled from numerator and denominator, leaving 100 as the denominator. The factor, 10, can then be divided out of 40 in the numerator and from the 100 in the denominator, thus leaving $4 \times 62\frac{1}{2}$ as the numerator and 10 as the denominator. The remainder of the work is evident.

LECTURE II.—QUESTIONS.

1. Define the unit of work. What name is given to this unit? In drawing a load a horse exerts a constant pull of 120 lbs.; how much work will be done in 15 minutes, supposing the horse to walk at the rate of 3 miles an hour? (S. and A. Exam. 1891.) *Ans.* 475,200 ft.-lbs.

2. How is the work done by a force measured? The resistance to traction on a level road is 150 lbs. per ton of weight moved; how many foot-pounds of work are expended in drawing 6 tons through a distance of 150 yards? *Ans.* 405,000 ft.-lbs.

3. Distinguish between *force* and *work done* by a force. How is each respectively measured? A traction engine draws a load of 20 tons along a level road, the tractive force on the load being 150 lbs. per ton. Find the work done upon the load in drawing it through a distance of 500 yards. (S. and A. Exam. 1888.) *Ans.* 4,500,000 ft.-lbs.

4. Find the number of units of mechanical work expended in raising 136 cubic feet of water to a height of 20 yards. The weight of a cubic foot of water is 62½ lbs. *Ans.* 510,000 ft.-lbs.

5. A weight of 4 tons is raised from a depth of 222 yards in a period of 45 seconds; calculate the amount of work done. *Ans.* 5,967,360 ft.-lbs.

6. A hole is punched through a plate of wrought-iron one-half inch in thickness, and the pressure actuating the punch is estimated at 36 tons. Assuming that the resistance to the punch is uniform, find the number of foot-pounds of work done. *Ans.* 3360 ft.-lbs.

7. How is work done by a force measured? Give some examples. Set out a diagram of the work done in drawing a body weighing 10 lbs. up a smooth incline 4 feet high, marking dimensions. (S. and A. Exam. 1889.)

8. A train of 12 coal waggons weighing 133 tons is lifted by hydraulic power (two waggons being raised at a time) through 20 feet in 12 minutes. Estimate the work done in foot-tons. Taking the average of work done, how many foot-pounds are done per minute? (S. and A. Exam. 1890.) *Ans.* 2660 ft.-tons; 496,533·3 ft.-lbs per minute.

9. The plunger of a force-pump is 8¾ inches in diameter, the length of the stroke is 2 feet 6 inches, and the pressure of the water is 50 lbs. per square inch; find the number of units of work done in one stroke, and plot out a diagram of work to scale. *Ans.* 7516·5 ft.-lbs.

10. A chain 30 feet long, and weighing 100 pounds per yard, lies coiled on the ground. Find by calculation and by a scale diagram of work how many units of work would be expended in just raising it by the top end from the ground. *Ans.* 15,000 ft.-lbs.

11. A chain, weighing 30 lbs. to the fathom, is employed to lift 1 ton to a height of 30 ft. by winding the chain on a barrel. Find by calculation and by a scale diagram of work, how many units of work will be expended —(*a*) when the outer end of the chain is brought home to the barrel; (*b*) when 18 feet of it are still hanging free with the weight at the end of it.

12. Define the following mechanical terms:—Force, work, unit of work, power, activity, and horse-power. A horse drawing a cart at the rate of 2 miles per hour exerts a traction of 156 lbs.; find the number of units of work done in one minute. *Ans.* 27,456 ft.-lbs.

13. In what way is the rate of doing work measured in horse-power? If 100 cubic feet of water be raised per minute through 330 feet, what horse-power of engine will be required, supposing that there is no loss by friction or other resistances? *Ans.* 62·5 h.p.

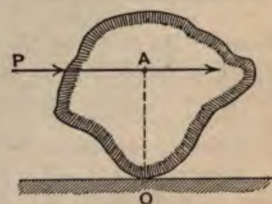
... *se, walking* at the rate of 2½ miles per hour, draws 104 lbs.
... *means* of a cord going over a wheel, how many units of
... *perform* in one minute? *Ans.* 22,880 ft.-lbs.

LECTURE III.

CONTENTS.—The Moment of a Force—Principle of Moments applied to the Lever—Experiments I. II. III.—Pressure on and Reaction from the Fulcrum—Equilibrant and Resultant of two Parallel Forces—Couples—Centre of Parallel Forces or Position of Equilibrant and Resultant—Centre of Gravity—Examples of Centre of Gravity—The Lever when its weight is taken into Account—Examples I. II.—Position of the Fulcrum—Example III.—Questions.

The Moment of a Force *is equal to the force multiplied by the perpendicular distance from a point on its line of action.*

For example, suppose a body to be resting on the point O, and a force, P, to be applied to the body in the direction PA. Then, if the perpendicular distance from O on the line of action of the force be OA, the *moment of the force* P, tending to turn the body about the point O, is P × AO. If the force be reckoned in pounds, and the perpendicular distance in feet, the product will be in pounds-feet. The student must therefore avoid confusing the answer with ft.-lbs. of work.



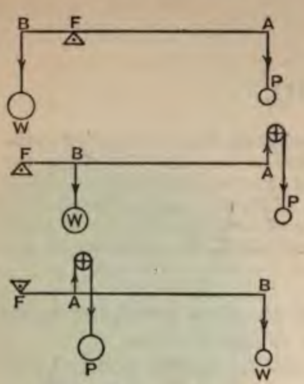
MOMENT OF A FORCE.

Principle of Moments.—If any number of forces act in one plane on a rigid body, and if these forces are in equilibrium; then the *principle of moments* asserts *that the sum of the moments of those forces which tend to turn the body in one direction about a point, is equal to the sum of the moments of the forces which tend to turn the body in the opposite direction about the same point.*

Or, to state the principle more concisely, *the opposing moments about the point are equal.*

If the moments of those forces which tend to turn the body to the right hand (*i.e.*, in the direction of the motion of the hands of a clock) be called *positive* (+), and the moments of the remaining set of forces which tend to turn the body to the left hand (*i.e.*, in the opposite direction to the movement of the hands of a clock) be called *negative* (−), *then the algebraical sum of the moments of the forces which act in one plane, and which are in equilibrium about a point, is zero.*

Principle of Moments applied to the Lever.—A lever is simply a rigid rod, bar, or beam, capable of turning about a fixed point called the fulcrum (F). Acting on the lever in one direction is a force or set of forces which we shall term the pull or pressure (P), and in the other direction there is the resistance or set of resistances to be overcome, which we shall term the weight (W). The pressure, P, and the weight, W, produce a reaction at the fulcrum, which is called the equilibrant (E).



Levers in Equilibrium.

The parts of the lever between the fulcrum and the pressure and between the fulcrum and the weight are called the arms of the lever.

The accompanying three figures show three ways in which F, P and W may be arranged with a straight lever.* *In each case, the opposing moments about the fulcrum are equal, when the lever is in equilibrium.*

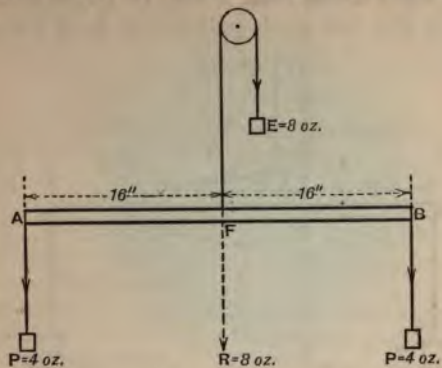
Or, . . $P \times AF = W \times BF$

satisfies the conditions for equilibrium in the case of a lever.

Experiment I.—To prove the foregoing statements, take a rigid homogeneous bar, AB, of uniform section. Let the bar be of yellow pine, 1 inch deep, $\frac{1}{2}$ inch broad, and 32 inches long. Attach to the ends, A and B, light flexible cords with small hooks at their lower ends, and attach to the middle of the bar at F another light flexible cord, and pass this cord over a pulley having a minimum of friction at its bearings. Fix such a weight to the free end of this middle cord as will just counterpoise the bar and cords. Test the accuracy of this preliminary adjustment by

* The levers represented by the above three figures are assumed to be without weight. A force, P, acts through a perfectly flexible, weightless cord at A, and another force, W, acts also through an exactly similar cord at B, with the fulcrum at F in each case. In the second and third case the cord attached at A passes over a frictionless pulley in order to give the necessary direction to the force P. These three relative positions of P, W and F used to be termed the first, second and third order of levers; but there is no necessity for any such distinction, since all the student has to remember is this, that when equilibrium exists the opposing moments about the fulcrum are equal, *i.e.*, ($P \times AF = W \times BF$), or, $\frac{P}{W} = \frac{BF}{AF}$, or $\frac{W}{P} = \frac{AF}{BF}$ The ratio W to P is termed the *theoretical advantage* of the lever.

observing whether the bar hangs horizontal, and, if pulled down or up a little, whether the weight balances the bar and cords. Now affix equal weights, P, of, say, 4 oz., to the cords hanging



EXPERIMENT I. ON PARALLEL FORCES.

from the ends A and B, and add an equilibrating weight, E, of 8 oz. to the end of the central cord. You will find that the bar will come to rest in a horizontal position, thus proving that—

$$P \times AF = P \times BF$$

i.e., $$4\text{ (oz.)} \times 16'' = 4\text{ (oz.)} \times 16''$$

Or, $$P : P :: BF : AF$$

i.e., $$\frac{P}{P} = \frac{BF}{AF} = \frac{16}{16} = \frac{4}{4} = \frac{1}{1}$$

Also, that the equilibrant,

$$E = P + P$$
$$8\text{ oz.} = 4\text{ oz.} + 4\text{ oz.}$$

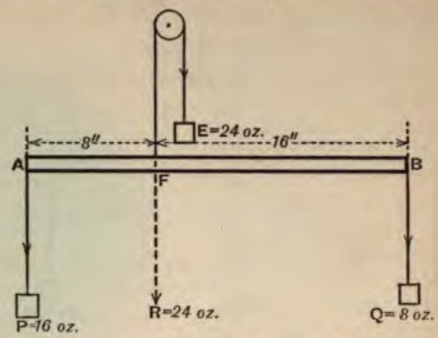
If P and P are now removed from the ends A and B, and a single weight, R, of 8 oz. be hung from F (as represented by the vertical dotted line and arrow), the result as far as the balancing of the system is concerned will be unaffected.

Consequently, $$R = E = P + P$$

i.e., $$8\text{ oz.} = 8\text{ oz.} = 4\text{ oz.} + 4\text{ oz.}$$

Or, the resultant of two equal parallel forces acting in the same direction is equal to the sum of the two forces, and acts midway between them and parallel to them—*i.e.*, at the same point as the equilibrant, and in the same line therewith, but in the opposite direction.

Experiment II.—Take another rigid homogeneous bar, AB, of the same uniform section as the previous bar, but let its length be 24 inches. Attach cords with hooks to the ends A and B, and to a point F, say 8 inches from A and 16 inches from B. Pass this latter cord over the guide-pulley, and fix it there until you



Experiment II. on Parallel Forces.

have just added sufficient weight to the end A to balance the longer end BF; then unfix the end of the middle cord, and attach such a weight to it as will counterpoise the whole system. Now attach to the cord at A a weight P = 16 oz.; to the other end, B, a weight Q; and a weight at E, so as to again balance the whole system. It will be found that Q equals 8 oz. and E equals 24 oz., thus proving that—

$$\begin{array}{ccccccc} P & \times & AF & = & Q & \times & BF \\ 16\ (\text{oz.}) & \times & 8'' & = & 8\ (\text{oz.}) & \times & 16'' \end{array}$$

Or,
$$P : Q :: BF : AF$$

i.e.,
$$\frac{P}{Q} = \frac{BF}{AF} = \frac{16}{8} = \frac{2}{1}$$

Or, the point F is twice the distance from the end B that it is from the end A, and P has twice the value of Q.

Also, that the equilibrant,

$$E = P + Q$$

For,
$$24\ \text{oz.} = 16\ \text{oz.} + 8\ \text{oz.}$$

If P and Q be now removed from the cords at A and B, and a single weight, R, of 24 oz., be hung from F (as represented by the vertical dotted line and arrow), the result, as far as the balanc- *of the system* is concerned, will be unaffected.

Consequently, $R = E = P + Q$
For, $24 \text{ oz.} = 24 \text{ oz.} = 16 \text{ oz.} + 8 \text{ oz.}$

Or, *the resultant of any two parallel forces acting in the same direction is equal to the sum of the two forces, and acts parallel to them and at a point between them, so that the ratio of the forces is inversely proportional to their distances from the point;* or so that—

$$P : Q :: BF : AF$$

Pressure on, and Reaction from, the Fulcrum.—You may also conclude from these two experiments, if the lever had been balanced on a knife-edge or journals, that the *pressure* on the fulcrum *due to the forces* P and Q would have been equal to and act in the direction of the *resultant* R, and that the *reaction* from the fulcrum would have been equal to and act in the direction of the *equilibrant* E.

EXPERIMENT III.—Supposing that in the last experiment, after adjusting the lever by placing a counterpoise weight at A, in order to bring the beam to a horizontal position, and after balancing the weight of the beam and cords by an equivalent weight at position E, you added a weight Q, of 8 oz., to the cord at B, and a weight E, of 24 oz., to the cord attached at F, the beam would turn, and would only be brought to a horizontal position by attaching a weight at A of 16 oz. Hence you observe that P acts at A as the equilibrant both in direction and magnitude to the two unequal parallel *non-concurrent* forces Q and E. Consequently a force equal and opposite in direction to P would be the resultant of the two forces Q and E; and it would replace their combined effect on the balanced beam.

Further, $P = E - Q;$
for, $16 \text{ oz.} = 24 \text{ oz.} - 8 \text{ oz.}$

And the moments about the position where the equilibrant acts are equal,
for $Q \times BA = E \times FA$
i.e., $8 \text{ (oz.)} \times 24'' = 24 \text{ (oz.)} \times 8''$

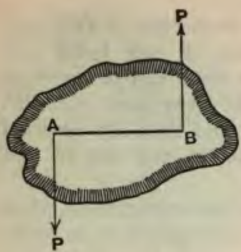
Equilibrant and Resultant of Two Parallel Forces.—From the above experiments you conclude that the equilibrant and the resultant of *any* two *concurrent* parallel forces are equal to their sum, and any two *unequal non-concurrent* parallel forces are equal to their difference.

Couples.—When the two parallel forces are equal and act in opposite directions upon a body, they are termed a *couple*. The perpendicular distance between the two forces is termed "*the arm of the couple*," and the "*moment of the couple*" is the product of

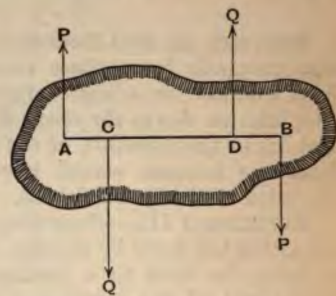
one of the forces and the arm. *A couple simply tends to cause rotation of the body upon which it acts, for it has no resultant, since*

$$R = P - P = 0.$$

One couple can, however, be equilibrated or balanced by another couple of an equal moment, acting in the same plane, and tending to



A COUPLE.



TWO BALANCING COUPLES.

turn the body in the opposite direction. In the accompanying figure the couple P,AP,P will be balanced by the couple Q,CD,Q if their moments are equal; *i.e.*, if

$$P \times AB = Q \times CD.*$$

We shall frequently have to refer to practical examples of couples, such as in a ship's capstan, the screw press used in copying manuscript, pressing bales of goods, and the fly press for punching holes in thin plates, or for stamping or embossing metals, &c.

Centre of Parallel Forces, or Position of Equilibrant and Resultant.—From Experiment III. and the accompanying figure to Experiment II., you conclude that the position where the equilibrant and resultant act is such, with respect to the positions where the forces act, that the moments of the forces *about that position* are equal and opposite in effect upon the lever.

$$\text{For, } Q \times BA = E \text{ (or R)} \times FA\text{; or, } \frac{Q}{E} = \frac{FA}{BA}$$

* *Let* $P = 8$ lbs., and $Q = 10$ lbs.; $AB = 10$ ft. and $CD = 8$ ft.
Then, $P \times AB = Q \times CD$
Or, $8 \times 10 = 10 \times 8$
$80 = 80$

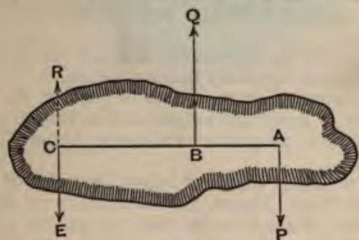
$$i.e., \ 8 \text{ (oz.)} \times 24'' = 24 \text{ (oz.)} \times 8''; \text{ or, } \frac{8}{24} = \frac{8}{24} = \frac{1}{3}$$

$$\text{And } P \times AB = E \text{ (or R)} \times FB; \text{ or, } \frac{P}{E} = \frac{FB}{AB}$$

$$i.e., \ 16 \text{ oz.} \times 24'' = 24 \text{ (oz.)} \times 16''; \text{ or, } \frac{16}{24} = \frac{16}{24} = \frac{2}{3}$$

The fulcrum F, where the equilibrant and resultant act, is termed the *centre* of the two parallel forces, and it is $\frac{1}{3}$ of the length of the lever from one end, and $\frac{2}{3}$ from the other end.

Reasoning generally from this particular case, if you have any two unequal *non-concurrent* parallel forces, P and Q, acting on a body in the directions AP and BQ respectively, and of which Q



CENTRE OF PARALLEL NON-CONCURRENT FORCES.

is the greater force, then if the line AB be drawn perpendicular to the directions of these forces, and prolonged, a single force E, parallel to P and Q and equal to Q – P, will balance these two forces at a point C, so *that the moments about C are equal and opposite; or,*

$$P \times AC = Q \times BC$$

Further, a force R, equal and opposite to E, acting at C, will represent the resultant of P and Q. *This point, C, is termed the centre of the parallel forces.*

The position of the point C, which is determined by the above equation, is not affected by the directions of the forces so long as they act at the same points A and B, and have the same magnitudes.

You may imagine any number of parallel forces acting in one plane being replaced by a single force. For in the above case you have formed a resultant, R, for the two forces P and Q; *consequently you could find* a resultant, R_1, for R and any other *parallel force—say* S; and so on for any number.

The final resultant of the whole of the forces would act at a point which would be the centre of the system of the whole of the parallel forces acting on the body.

Centre of Gravity.—Since gravity attracts towards the earth each particle of matter of which a body is composed, the *weight* of a body may be considered as the sum of a system of parallel forces. *The centre of these parallel forces is called the centre of gravity of the body, and is the point where the resultant of the weights of all the particles composing the body acts.*

The following statements in small type, which are generally proved as propositions and corollaries in books on Elementary Theoretical Mechanics, should be remembered by the student:—

1. If a body is symmetrical, the centre of gravity (or *c.g.* of the body) coincides with the centre of the mass.

2. If a body be uniform, the *c.g.* coincides with the centre of volume.

3. In a plate of uniform thickness and density the *c.g.* coincides with the centre of surface.

4. If the *c.g.* of a body be determined for any one position of the body, the same point is the *c.g.* for every other position.

5. If a body be supported on its centre of gravity, the body will balance in any position. Or, a body will balance about its *c.g.* in all positions.

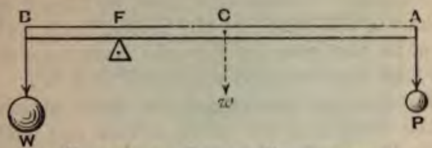
6. If a body balance in all positions about a straight line through it, the *c.g.* lies in that line.

7. If the *c.g.* be vertically above or below the point of support, the body will rest in that position. Hence, if you balance or support a body from three different points, the *c.g.* lies in the intersection of the three vertical lines from the three points respectively. Or, if you balance a body on an edge, the *c.g.* is in the vertical plane passing through that edge. Balance it again on a different edge, thus finding another plane which passes through the *c.g.* Then the *c.g.* lies in the *straight line* constituting the intersection of the two planes. Balance the body for a third time in another position, then the *point* where this third vertical plane intersects with the *straight line* will be the *c.g.* of the body.

8. The *c.g.* of regular geometrical bodies may easily be found by mere inspection when they are of uniform density.

For Example.—The *c.g.* of a *line* is at the middle of the line; of a *circle* at its centre; of a *sphere* at its centre; of the *surface* of a *uniform cylinder* and of a *solid cylinder* at the centre of the axis; of a *parallelogram* at the intersection of its diagonals; of a *triangle* at the intersection of straight lines drawn from two of the angles to the middle points of the opposite sides—*i.e.*, at a distance from one of the angles along one of these lines equal to $\frac{2}{3}$ of the line; of the *perimeter of a triangle* (*i.e.*, of three uniform rods forming a triangle) at the intersection of the two straight lines which bisect two of the angles of the triangle formed by joining the centres of the three uniform rods; of a *polygon* at the point of application of the resultant of the parallel forces represented by the areas of the respective triangles into which the polygon may be formed, and where each of these forces is considered to act at the *c.g.* of its own triangle; of a *pyramid* at $\frac{3}{4}$ of the line from the vertex to the *c.g.* of the base; of a *cone* at $\frac{3}{4}$ of the axis from the vertex; of the *curved surface of a cone* at $\frac{2}{3}$ of the axis from the vertex; of a *prism* at the middle of the line connecting the *c.g.*'s of its

The Lever when its Weight is taken into Account.—In this case we have to add the moment due to the weight of the lever, to the moment of P or of W according as it acts along with the one force or with the other; *i.e.*, according as the *c.g.* of the lever is on the same side of the fulcrum as P or W. When the lever is of uniform section and density throughout, then the *c.g.* of the lever is at its middle point, and consequently the whole weight of the lever may be considered as concentrated and acting at that point.



WEIGHT OF LEVER CONSIDERED.

Let AB be a uniform lever, of weight w, acting at its *c.g.* or middle point C, let a weight, W, be attached to the end B, then the force P, which will have to be applied to the other end A, in order to balance the whole about the fulcrum F, will be found by taking moments about F.

Thus, $$P \times AF + w \times CF = W \times BF$$

Or,* . . . . $$P = \frac{W \times BF - w \times CF}{AF}$$

EXAMPLE I.—A uniform lever, 5 ft. long, of 30 lbs. weight, is placed on a fulcrum 10 in. from one end, and has a weight of 100 lbs. attached to the short end. What force must be applied, and in what direction, in order to produce equilibrium? Also, what is the pressure on the fulcrum, and in what direction does the reaction from the fulcrum act?

1. Referring to the above figure, we find from the question that

$$AB = 5 \text{ ft.} = 60 \text{ in.}; \; BF = 10 \text{ in.} \; \therefore \; AF = 50 \text{ in. and } CF = 20 \text{ in.}$$
$$W = 100 \text{ lbs. and } w = 30 \text{ lbs.}$$

2. By the principle of moments—

The Opposing Moments about the Fulcrum are equal.

Consequently, $$P \times AF + w \times CF = W \times BF$$

$$\therefore P = \frac{W \times BF - w \times CF}{AF}$$

Substituting the numerical values—

$$P = \frac{100 \times 10 - 30 \times 20}{50} = 8 \text{ lbs.}$$

* If the *c.g.* of the lever was on the opposite side of the fulcrum—on the *side of W, then* $P \times A = W \times BF + w \times CF$.

3. P acts vertically *downwards*, since the moment due to the weight of the lever is not sufficient to equalise the moment due to the weight W about the point F.

4. *The pressure on the fulcrum* is evidently equal to the sum of all the forces, since all the forces act in one direction, or vertically downwards. It is therefore equal to

$$W + w + P = 100 + 30 + 8 = \mathbf{138\ lbs.}$$

5. *The reaction from the fulcrum* is equal and opposite in direction to this resultant. It therefore acts vertically upwards, and is the equilibrant of the whole of the forces, for a vertical force of 138 lbs. applied to the lever at F would counterpoise or just lift the whole bar with the attached weights P and W.

EXAMPLE II.—Suppose everything the same as in the previous example but the weight of the lever, which you may consider as now equal to 60 lbs.; what force P would be required, and in what direction would it have to act, in order to produce equilibrium? Also, what would be the resultant or downward pressure at F.

1. You observe at once that the moment of the weight of the lever is greater than the moment of W about the fulcrum.

For, $w \times CF > W \times BF$

Since, $60 \times 20 > 100 \times 10$

Consequently by the principle of moments P must act against w, or vertically upwards, so as to assist W, in order that *the opposing moments about the fulcrum may be equal.*

2. The formula therefore becomes

$$w \times CF - P \times AF = W \times BF$$

Or, $$w \times CF = W \times BF + P \times AF$$

$$\therefore \frac{w \times CF - W \times BF}{AF} = P$$

Substituting the numerical values, we have

$$\frac{60 \times 20 - 100 \times 10}{50} = P = \mathbf{4\ lbs.}$$

3. The *resultant pressure* at F is equal to the algebraical sum of the forces, or

$$W + w - P = 100 + 60 - 4 = \mathbf{156\ lbs.}$$

And acts vertically *downwards*. The *equilibrant* would therefore be 156 lbs. acting on the lever at F and vertically upwards.

Position of the Fulcrum.—In answering questions which give the magnitude of the forces with which they act, and require only answer for the position of the fulcrum, the student has

simply to employ the general formula for the principle of moments, and then to substitute the known numerical values in order to get the unknown. Or, he may reason out the formula into the following shape, and then interpolate the numerical values. Referring to the last figure, suppose that the distance AF is required :

Then, *neglecting the weight of the lever*, we have by the principle of moments—

$$P \times AF = W \times BF = W\ (BA - AF) = W \times BA - W \times AF.$$

Or, $$P \times AF + W \times AF = AF\ (P + W) \ = W \times BA$$

$$\therefore\ AF = \frac{W \times BA}{P + W}$$

Now, *taking the weight of the lever into account*, we have by the principle of moments :

$$P \times AF + w \times CF = W \times BF.$$

Or,

$$P \times AF + w\ (AF - AC) = W (BA - AF) = W \times BA - W \times AF.$$

Or,

$$P \times AF + w \times AF + W \times AF = W \times BA + w \times \frac{BA}{2} = BA\ (W + \tfrac{1}{2}w)$$

$$\therefore\ AF = \frac{BA\ (W + \frac{1}{2}w)}{P + w + W}$$

Example III.—Where should the *fulcrum* be placed under a uniform lever in order to produce equilibrium, if the lever is 5 ft. long, weighs 30 lbs., and has weights of 100 and 8 lbs. respectively hung at its ends.

From the above general equation for equilibrum—viz. :

$$P \times AF + w \times CF = W \times BF$$

We get $$AF = \frac{BA\ (W + \frac{1}{2}w)}{P + w + W}$$

$$AF = \frac{60\ (100 + 15)}{8 + 30 + 100} = 50 \text{ inches.}$$

Which proves the data given in Example I. to be correct.

Lecture III.—Questions.

1. Define what is meant by "the moment of a force," and give an example with a sketch.

2. State "the principle of moments," and apply it to the case of a simple straight lever.

3. A weight of 10 lbs. on the end of a lever 100 inches from the centre of motion is found to balance a weight of 100 lbs. at a distance of 10 inches. Explain the natural law which governs matter and motion, upon which the above mechanical fact depends. (*Answer this by giving the definition of the principle of moments.*)

4. Describe an experiment to prove the equality of the moments when the pull is between the weight and the fulcrum and acts in the opposite direction to the weight.

5. In the case of a straight lever, how would you ascertain the pressure on and the reaction from the fulcrum?

6. Three forces, of 12, 10 and 2 lbs., act along parallel lines on a rigid body; show by a sketch how they may be adjusted so as to be in equilibrium? *Ans.* The force of 12 lbs. must act as the equilibrant to the forces 2 and 10 lbs.—*i.e.*, in a line with their resultant, but in the opposite direction.

7. Two parallel forces of 10 and 12 lbs. act in opposite directions on a rigid body, and at 2 feet apart. Where is the centre of the two forces, and what is their resultant? *Ans.* 10 feet from the force of 12 lbs., 2 lbs.

8. Define the "centre of gravity" of a body, and show how you would find it experimentally in the case of any irregular body. Give an example.

9. State the rule which applies when two unequal forces balance on opposite sides of the fulcrum of a straight lever, the weight of the lever being neglected. A uniform straight lever, 4 feet long, weighs 10 lbs., the fulcrum is at one end; find what upward force acting at the other end will keep the lever horizontal when a weight of 10 lbs. is hung at a distance of 1 foot from the fulcrum. Find also the pressure on the fulcrum and the direction in which it acts. (S. and A. Exam. 1891.) *Ans.* 7·5 lbs.; 12·5 lbs. downwards.

10. A uniform bar, 4 feet long and weighing 4 lbs., can turn about a fulcrum at one end, and a weight of 10 lbs. is hung upon the bar at a distance of 1 foot from the fulcrum. Find the upward force at the free end which will keep the bar horizontal. (S. and A. Exam. 1887.) *Ans.* 4·5 lbs.

11. A uniform bar of metal 10 inches long weighs 4 lbs., and a weight of 6 lbs. is hung from one end. Find the fulcrum or point upon which the bar will balance. *Ans.* 2 inches from 6 lbs.

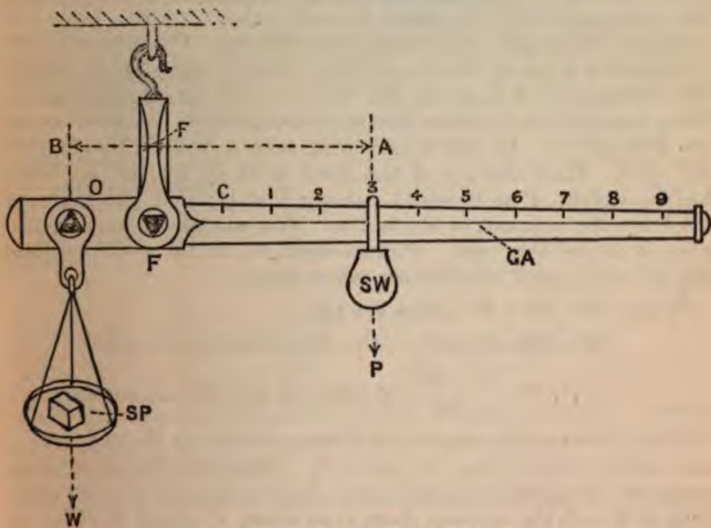
12. Two parallel forces whose magnitudes are 8 and 12 lbs. respectively, act in the same direction on a rigid body at points 10 inches apart. Find the magnitude and line of action of the resultant of the two forces. *Ans.* 20 lbs. at a point 6 inches from the force of 8 lbs.

13. A uniform lever is 5 feet long, and weighs 10 lbs., the fulcrum being at one end. A weight of 30 lbs. is hung at a distance of 4 feet from the fulcrum; what upward force acting at the middle point of the lever will keep it in a horizontal position? *Ans.* 58 lbs.

LECTURE IV.

CONTENTS.—Practical Applications of the Lever—The Steelyard, or Roman Balance—Graduation of the Steelyard—The Lever Safety Valve—Example I.—Lever Machine for Testing Tensile Strength of Materials —Straight Levers acted on by Inclined Forces—Bent Levers—The Bell Crank Lever—Bent Lever Balance—Duplex Bent Lever, or Lumberer's Tongs—Turkus, or Pincers—Examples II. and III.—Questions.

IN this Lecture we shall give a number of examples of the application of the lever.



STEELYARD, OR ROMAN BALANCE.

INDEX TO PARTS.

F	represents	Fulcrum.
GA	"	Graduated arm.
SW	"	Sliding weight.
P	"	Pull due to SW
SP	"	Scale pan.
W	represents	Weight in SP.
AF	"	Distance of P from F.
BF	"	Distance of W from F.

The Steelyard, or Roman Balance, is a straight lever with unequal arms, having a movable or sliding weight on the longer arm. It is very much used by butchers for weighing the carcasses of cattle and sheep, and in such cases it generally has two fulcra and two scales of division corresponding to them, the one set being, say, for hundredweights and the other for pounds.

Graduation of the Steelyard.—The practical method of graduating the steelyard is to put unit weight (say 1 lb.) into the scale pan, SP (or attach it to the hook on the shorter arm if there should be no such pan), and mark the position where the sliding weight, SW, has to be placed in order to cause equilibrum. Mark this position 1 on the scale. Then put in two units (say 2 lbs.) into SP, and adjust SW as before, marking its new position as 2 on the scale; and so on until SW is at the end of the longer arm.

In this form of steelyard, if the differences of the weights W, corresponding to successive distances, 1 to 2, to 3, &c., be the same, the graduations will be equal to each other. If the lever was unloaded, the longer arm would preponderate. Consequently, let O be such a point on the shorter arm that, if the sliding weight SW be suspended from it, the beam would become horizontal. Then a pull, P at O, causes the whole beam to be balanced about the fulcrum F. In the longer arm take a point C, such that $FC = FO$. Then the *c.g.* of the lever is at C, for, if the beam had no weight, P at C would balance P at O; and the moment $P \times CF =$ the moment of the beam. Now remove P from O, and place it on the long arm. Put a weight, W, in the scale pan, and slip SP along until equilibrium takes place.

$$\text{Then, } W \times BF = P \times AF + P \times CF$$

$$W \times BF = P\,(AF + CF) = P\,(AF + FO) = P \times AO$$

$$\therefore\ W = \frac{P \times AO}{BF}\text{, of which P and BF are constants.}$$

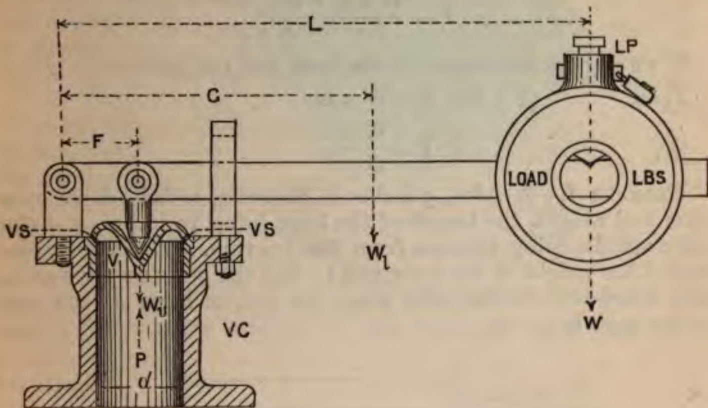
Hence the weights attached at B vary directly as the distances of the sliding weight from the point O. Therefore the graduations commence virtually at O, and not at F, or O is the zero of the scale. For, if $W = P$, the distance from O to where P would have to be placed to balance it would be equal to AC. If $W = 2P$, the distance from O to where P would have to be placed to balance it would equal twice AC; and so on.

The Lever Safety Valve.*—The lever safety valve is a simple

* For a more detailed description of safety valves and their action, refer to Lecture XXVII. of the author's Elementary Manual on "Steam and the Steam Engine."

contrivance fixed on the top of a boiler for the purpose of automatically preventing the steam exceeding an agreed-upon working pressure.

Referring to the above figure, VC is a cast-iron valve chest, containing a tightly-fitted gun-metal valve seat, VS, on which rests a steam-tight gun-metal valve, V. On the centre of the upper side of this valve rests a conical steel pin attached to a straight lever by an eye and bolt. One end of this lever is free to turn on a fulcrum fixed to the upper flange of the valve chest, and a lock-fast cast-iron weight is placed near the other end, so



LOCKFAST LEVER SAFETY VALVE.

INDEX TO PARTS.

VC represents Valve chest.	V represents Valve.
VS " Valve seat.	LP " Locking pin.

that the downward moment of the weight about the fulcrum balances the upward moment of the steam pressure on the valve about the same fulcrum.

Let L = length of lever in inches from fulcrum to the *c.g.* of the weight, W.
F = Distance in inches from fulcrum to centre line of valve, V.
G = " " " to *c.g.* of the lever.
W = Weight in lbs. of the cast-iron counterpoise block.
W_l = " " lever.
W_v = " " valve.
P = Pressure of steam in lbs. per square inch.

d = Diameter of valve in inches.

A = Area of valve in square inches $= \frac{\pi}{4}d^2$.

$P \times A$ = Total pressure in lbs. on the valve.

Then, by taking moments about the fulcrum, we find the pressure of steam per square inch which will balance the several forces.

For the upward moment = the sum of the downward moments.

$$(P \times A - W_v)F = (W \times L) + (W_l \times G).$$

Or, $$(P \times A) \times F = (W \times L) + (W_l \times G) + (W_v \times F)$$

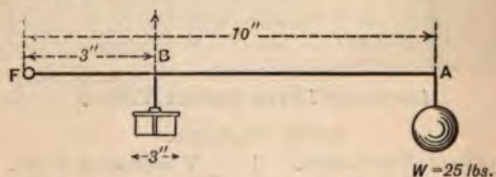
$$\therefore P = \frac{W \times L + W_l \times G + W_v \times F}{A \times F}$$

If we neglect the weight of the lever and the valve—

Then, $$(P \times A) \times F = W \times L$$

Or, $$P = \frac{W \times L}{A \times F}$$

EXAMPLE I.—A valve, 3 inches in diameter, is held down by a lever and weight, the length of the lever being 10 inches, and the valve spindle being 3 inches from the fulcrum. You are to disregard the weight of the lever and to find the pressure per square inch which will lift the valve when the weight hung at the end of the lever is 25 lbs.



Referring to the previous figure as well as to the accompanying one, we see from the question that

$$d = 3'' \therefore A = \frac{\pi}{4}d^2 = \cdot 7854 \times 3 \times 3 = 7{\cdot}07 \text{ sq. ins.};$$

$$BF = 3'', AF = 10'' \text{ and } W = 25 \text{ lbs.}$$

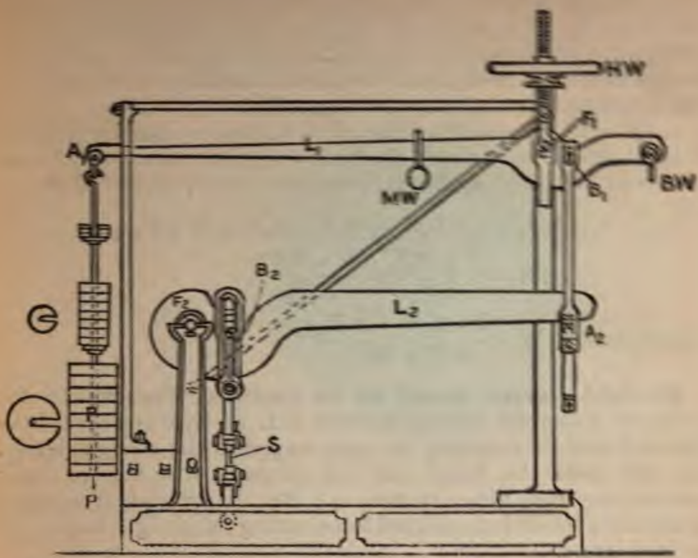
Taking moments about F, we get—

$$(P \times A) \times BF = W \times AF$$

$$P \times 7{\cdot}07 \times 3 = 25 \times 10$$

$$P = \mathbf{11{\cdot}8} \text{ lbs. per square inch.}$$

Testing Machine.—The following figures illustrate a machine which is used for testing the tensile strength of iron, steel and like materials. It consists of a combination of levers. After



LEVER MACHINE FOR TESTING TENSILE STRENGTH OF MATERIALS.

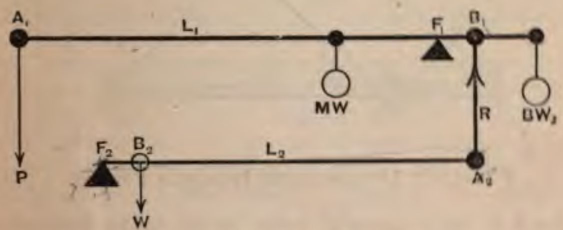


DIAGRAM OF THE LEVERS.

INDEX TO PARTS.

L_1L_2	represent	Levers.
F_1F_2	"	Fulcra.
P	"	Pull, or dead weights.
A_1	"	Where P acts on L_1.
S	"	Specimen under test.
B_1,A_2	represents	Where R acts on L_1,L_2.
B_2	"	Where W acts on R.
MW	"	Movable weight.
BW	"	Balance weight.
HW	"	Hand-wheel and screw for elevating F_1, &c.

mastering the general arrangement of the machine by comparing the index to parts with the side elevation, the student should refer to the accompanying skeleton diagram (where the same index letters have been used), from which he will readily understand how the stresses are transmitted and magnified.

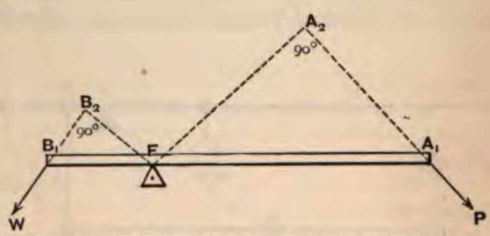
Looking at the second of the above figures, or skeleton diagram of the levers, it will be seen that when equilibrium exists between the stress W on the specimen S, and the pull P, applied at A_1,

$$P \times A_1F_1 = R \times B_1F_1, \text{ and } R \times A_2F_2 = W \times B_2F_2$$

$$\therefore R = \frac{P \times A_1F_1}{B_1F_1} = \frac{W \times B_2F_2}{A_2F_2}$$

Consequently, $W = \dfrac{P \times A_1F_1 \times A_2F_2}{B_1F_1 \times B_2F_2}$

Straight Levers Acted on by Inclined Forces.—In the previous Examples and in Lecture III. we have considered the forces P and W as acting at right angles to the straight levers. In such cases the forces had the greatest advantage, or their turning moments were a maximum. But the *principle of moments* is equally applicable to inclined forces acting on straight levers and to bent levers.



STRAIGHT LEVERS WITH INCLINED FORCES.

For, let A_1B_1 be a straight lever acted on by inclined forces, P and W. Draw from the fulcrum, F, lines at right angles to the produced directions of the forces as shown by the dotted lines in the above figure.

Then, the effective arms for the forces P and W are respectively A_2F and B_2F; and equilibrium takes when their moments about F are equal;

i.e., when $\quad P \times A_2F = W \times B_2F$

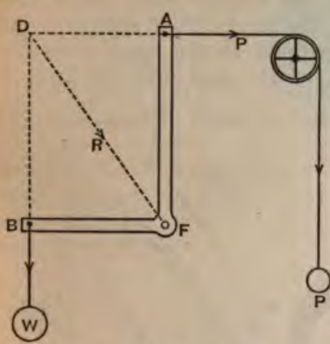
Or, $\quad P : W :: B_2F$, or $B_1F : A_2F$.

Bent Levers.*—The Bell Crank Lever.*—The same principle *nd action hold good* in the case of bent levers. Take an ordi-

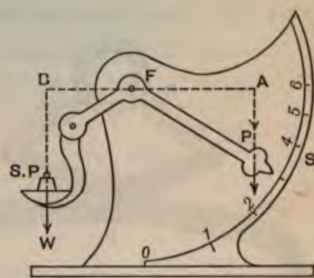
nary right-angle bell crank lever, as shown by the accompanying figure. Here the effective arms are equal to the actual arms of the lever, because the forces have been shown as acting at right angles to their respective arms, or with maximum turning moments.

Therefore, $P \times AF = W \times BF.$

But, if the lever be turned round through any angle by, say, an extra pull at P, then, in order to ascertain the virtual moments we should have to draw lines at right angles from F on the directions of P and W in order to calculate their effective arms.



BELL CRANK LEVER



BENT LEVER BALANCE.

Bent Lever Balance.—Examine an ordinary bent lever balance, such as is frequently used for weighing letters and light parcels, where the force P is a constant quantity, and the variable forces W is represented by the article to be weighed. As shown by the accompanying figure, the effective arms change with each weight to be ascertained, and consequently the scale S of this balance has to be graduated by trial, or by introducing standard pounds, such as SP, or other units, and marking the values on the scale opposite the position where the end of the pointer on P comes to rest. Or, the graduation might be done by plotting the various positions of the arms and values of the forces to scale. In the illustration we have evidently got equilibrium when

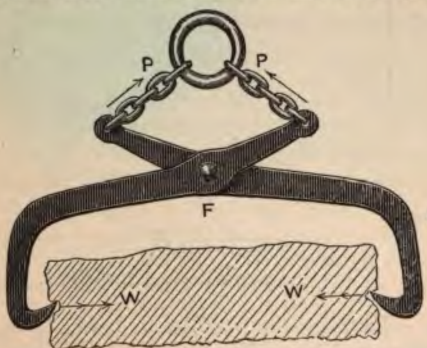
$$P \times AF = W \times BF.$$

Duplex Bent Lever, or Lumberer's Tongs.—The accompanying illustration shows a very useful and simple application of the bent lever, which is used at the end of a winch or crane chain,

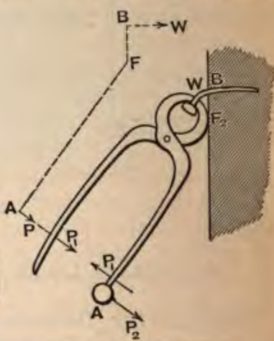
for affixing to and holding fast stones, logs of wood, blocks of ice, or other heavy articles when they have to be lifted.

P, P indicate the directions of the pulling forces on the short chains between the ends of the shorter arms and the common link which is attached to the crane chain. F is the common fulcrum, and W, W show the directions of the forces with which the article is gripped. The student will be able to draw a diagram of the forces and calculate their effective moments for himself for any particular case.

The Turkus, or Pincers.—The ordinary carpenter's turkus, or pincers, which is frequently used for extracting nails from wood, is another familiar illustration of the duplex bent lever. As shown by the accompanying figure, the forces P, P represent the forces with which the pincers is gripped by the hand after the jaws have been closed on the neck of the nail, and the force P



TONGS OR DUPLEX BENT LEVER.



TURKUS, OR PINCERS.

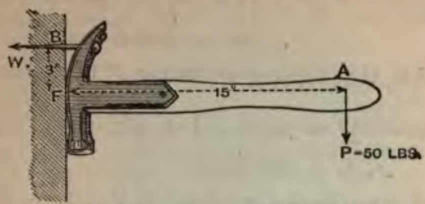
the pressure which has to be exerted by the arm and body in order to extract the nail from the wood—*i.e.*, to overcome the frictional resistance, W, between the wood and the nail. As shown by the separate diagram of forces in dotted lines, straight lines have been drawn, not from the joint of the pincers, but from a position representing the fulcrum F (or point where the nose of the pincers rests on the wood), perpendicular to the directions of the forces P and W, in order to obtain the lengths AF and BF of the effective arms of the bent lever.

$$\text{Here again, } P \times AF = W \times BF.$$

EXAMPLE II.—The handle of a claw-hammer is 15 inches long, and the claw is 3 inches long. What resistance of a nail would be overcome by the application of a pressure of 50 lbs. at the end of the handle?

You are required to show, by a diagram, the manner in which you arrive at your result. (S. and A. Exam. 1892.)

ANSWER.—Here we have a simple case of a bent lever, with fulcrum at F, and effective arms, AF, BF, 15 and 3 inches long



EXAMPLE OF A BENT LEVER.

respectively. Let W represent the resistance in lbs. offered by the nail at B. Then, by taking moments about F, we get

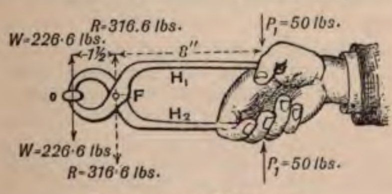
$$W \times BF = P \times AF$$

$$\text{Or, } W \times 3 = 50 \times 15$$

$$\therefore W = \frac{50 \times 15}{3} = \textbf{250 lbs.}$$

EXAMPLE III.—State the mechanical law known as the *Principle of the Lever*. In a pair of pincers the jaws meet at $1\frac{1}{2}$ inches from the pin forming the joint. The handles are grasped with a force of 50 lbs. on each handle at a distance of 8 inches from the pin. Find the compressive force on an object held between the jaws, and also the pressure upon the pin. (S. and A. Exam. 1888.)

Let P denote the force of 50 lbs. with which the handles are grasped at a distance of 8 inches from F, the pin. Let W denote



PINCERS OR NIPPERS.

the compressive force on the object O, and R the resultant reaction or pressure on the pin or fulcrum F. Although there are two levers here, each having a common fulcrum, F, it is best to con-

Note.—It is a mistake to speak of the "Principle of the Lever"; what is evidently meant is the Principle of Moments as applied to the lever.

sider the action of one lever only. Suppose the lower handle, H_2, to be fixed, and consider the action of the upper handle, H_1. It then becomes a simple question on the lever.

(1) To find W, take moments round F, then

$$W \times 1\tfrac{1}{2}'' = 50 \times 8''$$
$$\therefore W = \mathbf{266{\cdot}\dot{6}\ lbs.}$$

(2) To find R, the pressure on the pin F, take moments round O then

$$R \times 1\tfrac{1}{2}'' = 50 \times (1\tfrac{1}{2} + 8'') = 50 \times 9\tfrac{1}{2}''$$
$$\therefore R = \mathbf{316{\cdot}\dot{6}\ lbs.}$$

Or, since R must be the resultant of P and W, we get

$$R = P + W = 50 + 266{\cdot}\dot{6} = \mathbf{316{\cdot}\dot{6}\ lbs.}$$

LECTURE IV.—QUESTIONS.

1. Sketch and describe the steelyard, or Roman balance, and explain fully how the graduations on the scale are equal for equal differences in the weights applied to the shorter arm.

2. Sketch and describe a lockfast lever safety valve. A valve, 3 inches in diameter, is held down by a lever and weight, the length of the lever being 30 inches, and the valve spindle being 4 inches from the fulcrum. You are to disregard the weight of the lever and to find the pressure per square inch which will lift the valve when the weight hung at the end of the lever is 56 lbs. *Ans.* 52·5 lbs.

3. The diameter of a safety valve is 3″, its weight $3\frac{1}{2}$ lbs.; length of lever is 30″, and its weight 16 lbs.; the distance from fulcrum to centre of valve is 3″, and to *c.g.* of lever 12″. Find where a weight of 50 lbs. must be placed on the lever in order that steam may just blow off at 70 lbs. per square inch by gauge. *Ans.* 25.65 inches from the fulcrum.

4. The safety valve of a boiler is required to blow off steam at 100 lbs. per square inch by gauge. The dead weight is 100 lbs., weight of lever 10 lbs., and of valve 5 lbs.; diameter of valve $3\frac{1}{2}$″, distance from centre of valve to fulcrum 4″, from *c.g.* of lever to fulcrum 15″. Where should you place the weight on the lever? *Ans.* 36·8 ins. from fulcrum.

5. Sketch and describe a lever machine for testing the tensile strength of materials. If the advantage, or ratio of pull P to resistance R in the first lever, is 56 to 1, and of the second lever 40 to 1, what stress will be produced on the test specimen when $P = 100$ lbs.? *Ans.* 100 tons.

6. A force of 100 lbs. acts at one end of a straight lever, but at an angle of 60° to it. What force acting at the other end of the lever, at an angle of 45° to it, will keep the lever in equilibrium if the fulcrum be placed half the distance from the first force that it is from the second? Draw a diagram of the forces and their effective arms. *Ans.* 61.25 lbs.

7. Sketch a bell crank lever, to convey a small movement from one line to another, cutting each other at 60°; the spaces moved through to being as 1 to 2.

8. The handle of a claw-hammer is 12 inches long, and the claw is 2 inches long. What resistance of a nail would be overcome by the application of a pressure of 40 lbs. at the end of the handle? Show, by a diagram, the manner in which you arrive at your result. *Ans.* 240 lbs.

9. In a pair of pincers the jaws meet at $1\frac{1}{4}$ inches from the pin forming the joint. The handles are grasped with a force of 30 lbs. on each handle at a distance of $7\frac{1}{2}$ inches from the pin. Find the compressive force on an object held between the jaws, and also the pressure upon the pin. Sketch the apparatus and show the direction and values of all the forces. *Ans.* 180 lbs; 210 lbs.

LECTURE V.

CONTENTS.—The Principle of Work—Work put in, Work lost, Useful Work—Efficiency of a Machine—Principle of Work applied to the Lever—Experiments I. II.—Wheel and Axle—The Principle of Moments applied to the Wheel and Axle—The Principle of Work applied to the Wheel and Axle—Experiment III.—The Winch Barrel—Example I.—Ship's Capstan—The Fusee—Questions.

The Principle of Work.*—The principle of work is applicable to all machines, and may be stated as follows:—

The work put into a machine is equal to the work absorbed by the machine plus the work given out by the machine.

Or, WORK PUT IN = LOST WORK + USEFUL WORK.

This is an *axiom*. But, nevertheless, many deluded would-be inventors have spent much time and money in devising "*perpetual motion*" appliances, or machines which should turn out as much work as, or even more than, was put into them!

1. When a machine is employed to perform mechanical work, a certain force must be applied to one part of it in order to move the machine and to perform work at another part.

The product of this applied force and the distance through which it acts constitute *the whole work put into the machine.*

2. Some of this work *must* be expended in merely keeping the different parts in motion, against natural resistances due to friction at the fulcra or journals, and friction between moving parts and the air or water in the case of an hydraulic apparatus. The work so absorbed is termed *lost work.*

The mean value of the frictional resistances, multiplied by the mean distance through which they are overcome, constitute *the work lost in the mechanism.* One great object to be kept in view, in designing most machines, is to minimise this *lost work* by minimising the internal resistances to motion in the machine

* The *Principle of Work* is usually stated as follows in books on Mechanics, but I find that engineering students *much* prefer the *above* definition. "*If a system of bodies be at rest under the action of any forces, and be moved a very little,* no work will be done." "Conversely: If no work is [illegible] *during this small* movement, the forces are in equilibrium."—Prof. [illegible]'s "*Manual of Mechanics,*" p. 73.

itself; but you must remember that these can *never be entirely disposed of*, as has only too often been conjectured by "perpetual motion" faddists.

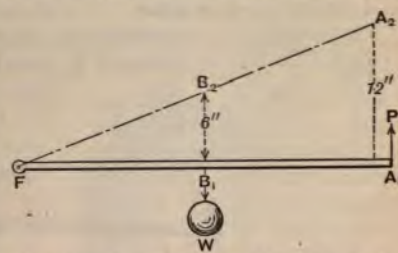
3. The remainder goes to do the *useful work* for which the machine was designed, and therefore—

4. *The efficiency of a machine* $= \dfrac{\text{the work got out.}}{\text{the work put in.}}$

To impress these facts on the mind of the student we present them in the following condensed form :—

1. *Work put in = force applied × the distance it acts.*
2. *Work lost = force absorbed in overcoming internal resistances × the distance it acts.*
3. *Useful work = force given out × the distance it acts.*
4. Efficiency = *ratio of work got out to work put in.*
5. Work put in = *lost work + useful work.*

Principle of Work applied to the Lever.—In applying the above "principle of work" to the lever, we will take the liberty of neglecting the lost work. We shall therefore assume that the friction at the fulcrum is so small that it may be neglected for the purpose we have in view.



PRINCIPLE OF WORK APPLIED TO A LEVER.

EXPERIMENT I. — Let A_1F be a straight lever without weight, having its fulcrum at F, a force, W, acting vertically downwards from the point B_1, and a force, P, acting vertically upwards at the end A_1, keeping W in equilibrium. Now imagine the lever elevated to the position A_2F.

The work put in at $A_1 = P \times$ the vertical distance from A_1 to A_2.
The work got out at $B_1 = W \times$ the vertical distance from B_1 to B_2.
Therefore, since we neglect all frictional resistances—

$$\textit{The work put in} = \textit{the work got out}$$

Or, $$P \times A_1A_2 = W \times B_1B_2$$

i.e., $$\frac{P}{W} = \frac{B_1B_2}{A_1A_2}$$

But by Euclid the triangles A_1FA_2 and B_1FB_2 are similar in every respect.

Therefore, $$\frac{B_1B_2}{A_1A_2} = \frac{B_1F}{A_1F}$$

Hence, $$\frac{P}{W} = \frac{B_1F}{A_1F}$$

Or, $$P \times A_1F = W \times B_1F$$

But this is the equation we proved in Lecture III. with respect to the lever as complying with the "principle of moments." Hence the "*principle of work*" and the "*principle of moments*" are in agreement.

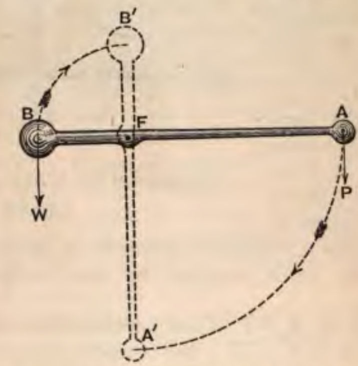
In the accompanying figure the force P has been shown as elevated through 12″, and the force W as elevated through 6″.

Therefore, $$P \times 12'' = W \times 6''$$

Or, $$\frac{P}{W} = \frac{6}{12} = \frac{1}{2}$$

P being half the magnitude of W, it has to be elevated through double the distance in order that the same amount of work may be done in the same time.

Experiment II.—Consider the case of a simple lever, where a weight, W, at B is balanced by another weight, P, at A, around



Principle of work applied to a Lever.

a fulcrum at F, without friction. Let the lever be turned through 90°, or a quarter of a revolution—*i.e.*, from a *horizontal* position, AB, to a *vertical* position, A′B′.

Then by the definition of work—

The work put in at $A = P \times A'F$, and
The work got out at $B = W \times B'F$.

It does not matter in the slightest degree how circuitous the paths P and W take in passing from their original to their new positions in this case, since all we require to know is the vertical distances through which P is depressed and W elevated.

Consequently, by the "*Principle of Work*,"

$$P \times A'F = W \times B'F$$

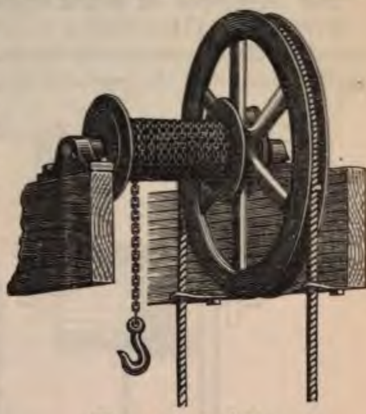
But, $A'F = AF$, and $B'F = BF$,

$\therefore$ Substituting AF for A′F, and BF for B′F,

We get, $P \times AF = W \times BF$

But this is the equation for the "*principle of moments*," which we have again deduced from the "*principle of work*" by another and simpler form of reasoning. We find that this latter method appeals more directly to the minds of young engineering students than the proofs usually found in books on Mechanics.

The Wheel and Axle.—The wheel and axle has been used for centuries for drawing water by a bucket from a well. It is used by the navvy for lifting the material which he excavates from the earth, by the mason for raising stones, bricks and mortar, and by many other tradesmen for a variety of purposes; as well as by the quartermaster as a steering-gear, and the able seaman as a capstan. The accompanying illustration shows the form it takes when used for elevating goods in a store or mill.* It is simply a practical arrangement for continuing the action of the lever as long as required. So long as a sufficient pull is applied to the rope, which fits into the grooved wheel, to overcome the resistance of the load attached to the chain hook, the weight will be raised. The wheel and axle is therefore a form of lever by which a weight may be raised through any desired height.

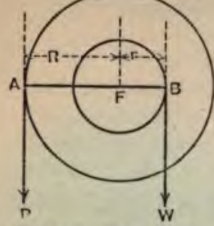


WHEEL AND AXLE.

The Principle of Moments applied to the Wheel and Axle.—In the diagram let the larger circle represent the circumference of a wheel of radius, R, to the periphery of which a force,

* *The above figure represents* a wheel and axle as supplied by Messrs. *P. & W. MacLellan, of Glasgow.*

P, is applied. Let the smaller circle represent the circumference of the axle or barrel of radius, r, to the periphery of which is applied a resistance W. Let the forces P and W act in the same direction and vertically downwards. Join the points where the lines of action of the forces are tangents to the wheel and axle by a straight line, AB. Then, AB passes through the common centre of the circles—*i.e.*, through their common centre of motion or fulcrum F, and AF is the effective arm for the force P, whilst BF is the effective arm for the force W. In fact, AFB is a straight lever in equilibrium, with the fulcrum at F.



WHEEL AND AXLE.

Therefore, taking moments about F, we have—

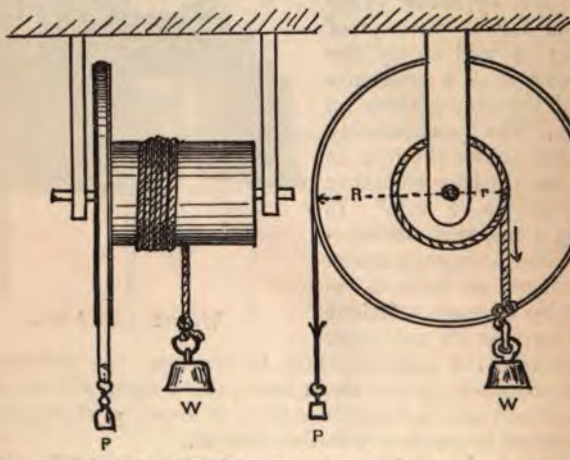
$$P \times AF = W \times BF$$

Or,

$$P \times R = W \times r$$

The Principle of Work applied to the Wheel and Axle.

EXPERIMENT III.—Take a model of the wheel and axle as illustrated by the accompanying figure. Let forces, P and W, act in equilibrium, as in the previous case, at radii R and r respectively.



MODEL TO TEST THE PRINCIPLE OF WORK APPLIED TO THE WHEEL AND AXLE.

Now mark carefully with a piece of coloured chalk or ink the exact *itions where the tape* supporting P is a tangent to the wheel,

and where the cord supporting W is a tangent to the barrel. Pull P until the wheel and barrel have just made one complete revolution. Then, neglecting any force required to overcome friction at the bearings of the spindle—

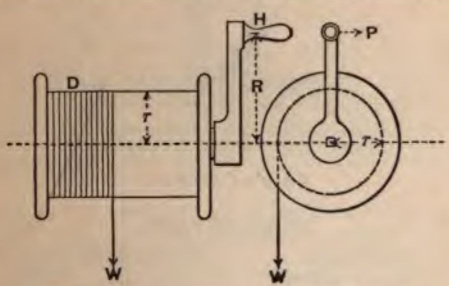
$$\begin{aligned} \text{The work } \textit{put in} \text{ by P} &= \text{P} \times 2\pi\text{R} \\ \text{The work } \textit{got out} \text{ in raising W} &= \text{W} \times 2\pi r \\ \textit{But the work put in} &= \textit{the work got out} \\ \therefore \text{P} \times 2\pi\text{R} &= \text{W} \times 2\pi r \end{aligned}$$

Cancelling 2π from each side of the equation—

We have $$\text{P} \times \text{R} = \text{W} \times r.$$

But this is the same equation as we obtained above by applying the "*principle of moments.*" Therefore, we see that the "*principle of moments*" and the "*principle of work*" harmonise.

The Winch Barrel.—The wheel may be replaced by a handle H, and the mere axle by a barrel or drum D, of any desired size.



SIDE VIEW. END VIEW.

WINCH BARREL AND HANDLE.

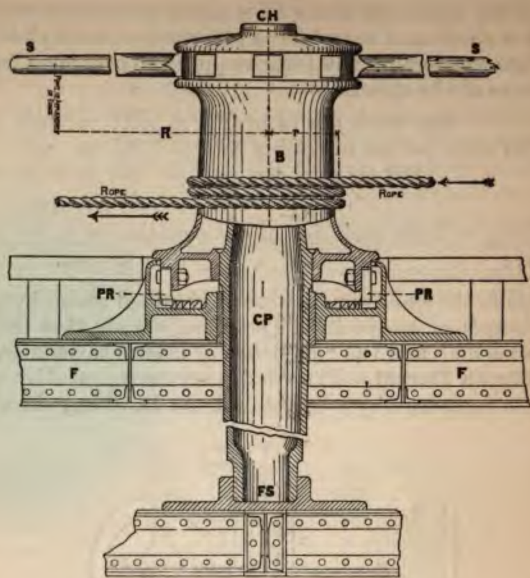
EXAMPLE I.—A man exerts a constant force of 30 lbs. on a winch handle of 15″ radius; what weight will he be able to lift attached to a rope hanging from a barrel of 5″ radius?

By the principles of moments and of work; and interpolating the numerical values—

$$\text{P} \times \text{R} = \text{W} \times r$$
$$30 \times 15 = \text{W} \times 5$$
$$\therefore \text{W} = \frac{30 \times 15}{5} = \mathbf{90\ lbs.}$$

Ship's Capstan.—A partly sectional partly outside view of this useful machine is illustrated by the following figure:

A capstan is generally fixed upon the forecastle of ships, or near to the side of a quay or dock, for the purposes of warping and



SHIP'S CAPSTAN.

INDEX TO PARTS.

CH	represents	Capstan head.	PR	represents	Pall and Ratchet.
SS	„	Spokes or arms.	F	„	Frame.
R	„	Radius of S.	CP	„	Capstan pillar.
B	„	Barrel.	FS	„	Footstep of CP.
r	„	Radius of B.			

berthing the vessels. The above illustration shows a capstan as built into a forecastle, where the round turned footstep, FS, of the vertical cast-iron capstan pillar, CP, bears in a cast-iron or cast-steel shoe fitted upon the steel or wrought beams of the main deck. The frame F, which supports the casing for the pall and ratchet gear, may be the beams of the upper or forecastle deck. A strong rope made fast on shore is passed several times round the capstan barrel B, and the slack end of the rope is coiled on deck. The addition of the rope to the barrel increases the effective arm or radius r, at which the resistance of the ship acts by half the diameter of the rope. Eight or any desired less number of *wooden spokes*, S, S, having their inner ends squared and tapered, *are fixed into* hollow square holes in the cast-iron capstan *head CH*. Then, just as many sailors as may be required to

overcome the resistance of the ship apply themselves to the outer rounded ends of the spokes, and push away as hard as they can.

It will be observed that, calling, p, the force applied by each sailor at radius R; then, when we have two sailors acting on diametrically opposite spokes p, $2R$, p forms a couple tending to cause rotation of the capstan in one direction. Consequently, from the property of couples (as we showed in Lecture III.) this couple can only be balanced by another couple acting in the opposite direction and having an equal moment. Such another couple exists, when the resistance of the ship, W, acting with an arm, r (equal to the distance from centre of capstan to centre of rope), balances the corresponding reaction at the centre of the capstan barrel. Hence, when the force applied by the *two* sailors is balanced by the resistance to motion of the ship, we have the one couple just balancing the other one.

Or Couple p, $2R$, p balancing couple W, r, W

i.e.,
$$p \times 2R = W \times r$$

In the same way, with two, three, or four pairs of sailors, each pair being supposed to act on diametrically opposite spokes, we have two, three, or four couples acting in one direction, balanced by one couple, viz., the resistance of the ship into the distance from the centre of the capstan barrel and the reaction from that centre.*

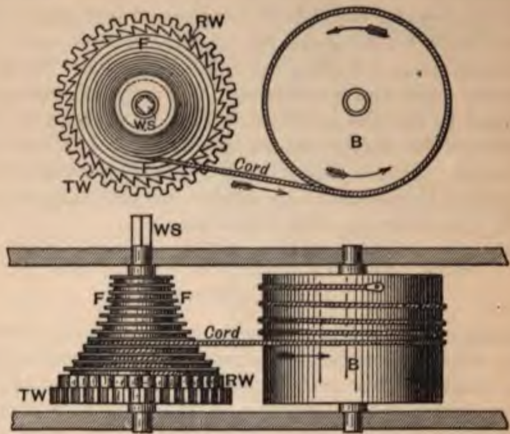
In the case of four sailors just being able to move the ship, two couples, p, $2R$, $p+p$, $2R$, p, balance one couple, W, r, W;

i.e.,
$$p \times 2R + p \times 2R = W \times r$$

The Fusee.—As an illustration of the lever action and of work put into and got out of a machine, we cannot do better than finish this lecture by a description of the construction and action of the simple yet most ingenious contrivance termed the fusee. In good watches and clocks, where the elastic force of a coiled spring is used to drive the works, the fusee is used for the purpose of compensating the gradually diminishing pull of the uncoiling spring. The driving of the works at a constant rate is the object for which a watch or clock is designed. This naturally entails a *constant* resistance to be overcome, but since one of the most compact and convenient forms of mechanism into

* The student should draw a plan of the capstan barrel, and show radial lines to indicate one, two, or more pairs of diametrically opposite spokes, with forces, p, acting at their ends, all tending to turn the barrel in one direction. He will then see that a couple formed by resistance to the stress on the rope, and an equal reaction from the centre of motion, will be required to act in the other direction in order that equilibrium may take place.

which mechanical force can be stored is that of a coiled spring, and since the very nature of the spring is such that its force decreases as it uncoils, we must employ some compensating device between this variable driving force and the constant resistance. The fusee does this in a most accurate and complete



THE FUSEE FOR A CLOCK OR WATCH.

INDEX TO PARTS.

F	represents Fusee.	TW	represents Toothed wheel.
B	" Barrel.	WS	" Winding square.
RW	" Ratchet wheel.		

manner. Looking at the accompanying figures and index to parts, we see that the barrel B, which contains the watch or clock spring, is of uniform diameter, and that between the outside of this barrel and the fusee, or spirally grooved cone, there passes a cord or chain. When the winding key is applied to the winding square WS, and turned in the proper direction, a tension is applied to the cord, and it is wound upon the spiral cone, thus coiling up the spring inside the barrel B; for the outer end of this spring is fixed to the periphery of the barrel, and the inner end to its spindle or axle, which is in direct gear with the works of the watch or clock. When the spring is fully wound up it has the greatest force, but it acts with the least advantage, since then the cord is on the smallest groove of the cone pulley. When the spring is almost uncoiled it acts with the greatest advantage, for then the cord is on the largest groove of the cone. Conse-

quently the radii of the grooves of this cone are made to increase in proportion as the force applied to the cord decreases, so that $(P \times R)$ shall be a constant moment applied to overcome the constant moment $(W \times r)$ or the resistance of the works into the radius of the barrel B.

The work put in when winding up the coiled spring, is given up by it in overcoming the frictional resistances of the different parts of the mechanism.

Or the work put in = lost work, for the whole of the work put in, is devoted to simply keeping the parts of the machine in motion, leaving nothing for other work, unless the clock is used to strike a bell or do some other kind of work.

Lecture V.—Questions.

1. State the "Principle of Work," and explain the manner in which it is applied in determining the relation of a P to W in the lever. A lever, centred at one end, is 15 feet long, and a weight of W lbs. hangs from the opposite end. The weight W is supported by an upward pressure of 28,270 lbs. at 13 feet from the fulcrum. Find W. *Ans.* 24,500·6 lbs.

2. Define work put in, lost in, and got out of a machine, and prove that the work put in = lost work plus the useful work. How is the "advantage" and the efficiency of a machine reckoned?

3. Sketch and describe the wheel and axle. Apply both the "principle of moments" and the "principle of work" to find the relation between the force applied and the weight raised by aid of this machine. A wheel and axle is required so that the force applied at the circumference of the wheel in moving through a distance of 10 feet shall raise a weight of 4 cwts. through a height of 2 feet. If the diameter of the axle is 10 inches, find the force applied in lbs., and the radius of the wheel in feet. *Ans.* 89·6 lbs.; 2 feet 1 inch.

4. The crank or handle which turns a windlass is 14 inches in length; what must be the diameter of the axle when a man exerting a force of 60 lbs. upon the handle raises a tub of coals weighing 2 cwt.? *Ans.* $7\frac{1}{2}$ inches.

5. In a windlass the barrel is 8 inches diameter, the rope is $1\frac{1}{4}$ inches diameter, and the crank handle $15\frac{1}{2}$ inches long. What force must be applied at the handle to raise 2 cwt.? Also, what weight would be raised by a constant force of 30 lbs. applied at the handle? *Ans.* 66·8 lbs.; 100·5 lbs.

6. A capstan is worked by four men; each man exerts a constant force of 30 lbs. at a distance of 4 feet from the axis. A rope of $\frac{3}{4}$-inch diameter is wound round the drum, of $5\frac{5}{8}$ inches radius. Find the pull on the rope which balances the pressure on handles. Make a diagram showing the action of the forces, and find the pressure on the central shaft of the capstan. *Ans.* 921·6 lbs.; 921·6 lbs.

7. Describe, with a sketch, the spring-barrel and fusee of a clock or of a watch. Explain its action by reference to the principle of moments.

LECTURE VI.

CONTENTS.—Pulleys—Snatch Block—Block and Tackle—Theoretical Advantage—Velocity Ratio—The Principle of Work applied to the Block and Tackle—Actual or Working Advantage—Work put in—Work got out—Efficiency—Percentage Efficiency—Example I.—Questions.

Pulleys.—Suppose you had to elevate a sack of flour from the ground to an upper storey of a mill or store, you might place it upon your back and carry it up the stairs. In doing so, you would expend so many foot-pounds of work. Let the sack of flour be 100 lbs., your own weight 150 lbs., and the height to which it is raised be 30 feet. Then the

Work done in elevating the flour	$= 100$ lbs. $\times 30' = 3000$	ft.-lbs.
,, ,, yourself	$= 150$,, $\times 30' = 4500$	,,
Total work done	$= 250$,, $\times 30' = 7500$	,,

And your efficiency as a machine would be found thus—

$$\text{Mechanical efficiency} = \frac{\text{useful work}}{\text{total work}}\text{; or, } \frac{\text{work got out}}{\text{work put in}} = \frac{3000 \text{ ft.-lbs.}}{7500 \text{ ft.-lbs.}} = \cdot 4$$

Or, your percentage efficiency would be found from the proportion—

$$7500 : 3000 :: 100 : x$$

$$x = \frac{3000 \times 100}{7500} = 40\ \%$$

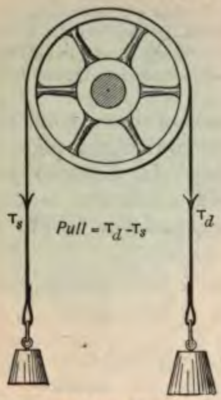
In other words, 60 per cent. of the total work done is *lost* work, and only 40 per cent. is *useful* work.

If instead of carrying the sack upstairs, you found ready to hand a long rope (with its two ends close to the ground) that had been passed over a smooth iron hook fixed to the outside wall above an outside landing for the particular storey of the building, and, if you attached one end of this rope to the sack and found that by pulling with all your strength (or say with a force of 150 lbs., *i.e.*, equal to your weight) on the other end, you could just lift the sack. *Then,* if by this means you elevated the sack *to the landing, you would* have expended less work than by the *former method; for,*

$$
\begin{array}{lll}
\text{Work done in elevating flour} & = 100\text{ lbs.} \times 30' = 3000 & \text{ft.-lbs.} \\
\quad ,, \quad \text{against friction, \&c.} & = 50 \;,, \times 30' = \underline{1500} & ,, \\
\quad \text{Total work done} & = 150 \;,, \times 30' = \underline{\underline{4500}} & ,,
\end{array}
$$

$$\therefore \text{Mechanical efficiency} = \frac{\text{useful work}}{\text{total work}}; \text{ or, } \frac{\text{work got out}}{\text{work put in}} = \frac{3000}{4500} = \cdot\dot{6}$$

And the percentage efficiency is therefore 66·6.



PULLEY AND WEIGHTS.

$$\text{For, } 4500 \;:\; 3000 \;::\; 100 \;:\; x$$

$$x = \frac{3000 \times 100}{4500} = \mathbf{66 \cdot \dot{6}} \;\%$$

Hence $33\cdot\dot{3}$ per cent., or $\frac{1}{3}$ of the total work put in by you in pulling at one side of the rope, is spent in overcoming the friction between the rope and the hook and bending the rope over the hook, whilst only $66\cdot\dot{6}$ per cent., or $\frac{2}{3}$, remain for elevating the sack of flour.

If, instead of the iron hook you had found a double-flanged deep **V**-grooved pulley with a rope over it, as in the accompanying illustration, and that this pulley revolved so easily on its bearings that you had only to pull with a constant force of 110 lbs. in order to lift the sack of flour from the ground up to the 30-feet level, then—

$$
\begin{array}{lll}
\text{Work done in elevating flour} & = 100\text{ lbs.} \times 30' = 3000 & \text{ft.-lbs.} \\
\quad ,, \quad \text{against friction, \&c.} & = 10 \;,, \times 30' = \underline{300} & ,, \\
\quad \text{Total work done} & = 110 \;,, \times 30' = \underline{\underline{3300}} & ,,
\end{array}
$$

$$\therefore \text{Mechanical efficiency} = \frac{\text{useful work}}{\text{total work}}; \text{ or, } \frac{\text{work got out}}{\text{work put in}} = \frac{3000}{3300} = \cdot\dot{9}0\dot{9}$$

And the percentage efficiency is 90·9

$$\text{For} \quad 3300 \;:\; 3000 \;::\; 100 \;:\; x$$

$$x = \frac{3000 \times 100}{3300} = \mathbf{90 \cdot 9} \;\%$$

Hence only 9·1 per cent. of the total work put in is *lost* work in overcoming friction at the pulley bearing and in bending the rope over the pulley.

You see, therefore, what a useful machine a pulley is, not only *for enabling* you to change the direction of a force, but also for *e saving of* labour.

A pulley is simply a wheel and axle wherein their radii are one and the same, or a lever with equal arms. Hence the principles of moments and of work may be applied to it in the same way as we applied them to the lever and to the wheel and axle.

Snatch Block.—If you should require to put the bend of a rope on a pulley, and at the same time prevent the possibility of the rope coming out of the groove, without having to reeve the end of the rope between its cheeks, you would use what is called a snatch block. One form of snatch block is illustrated by the accompanying figure, where on the side of one cheek there is a sneck or snatch, which is turned to one side, to enable the bend of the rope to be placed around the U groove of the pulley. The snatch then falls down and closes upon the central pin. Another form has a hinged snatch which can be lifted up at right angles to the face of the cheek, and after the rope has been put on the pulley the snatch is closed down and locked by a pin attached to a short chain fixed to the side of the cheek, just like an ordinary front hinge for closing a chest. The single movable pulley, which is used for supporting the load to be lifted by a Chinese windlass or by a jib crane, is sometimes called a snatch block (see the illustration of the wheel and compound axle in this Lecture, and of jib cranes in Lectures VIII. and XIII.). In the latter case the chain passes from the barrel of the crane over the pulley at the point of the jib, then vertically down, underneath the snatch-block pulley, and vertically upwards to a point on the under side of the jib where it is fixed by an eye-shackle with a bolt and nut. If the load, including the weight of the snatch-block, be W, then, neglecting friction, the pull P on the chain will be $\frac{W}{2}$; for W is supported by two vertical or parallel parts of the chain, each part carrying half the load, or $W = 2P$. If the load be elevated any distance L, then the chain will have to be pulled in on the barrel a distance of 2L, for by the principle of work

SNATCH BLOCK.

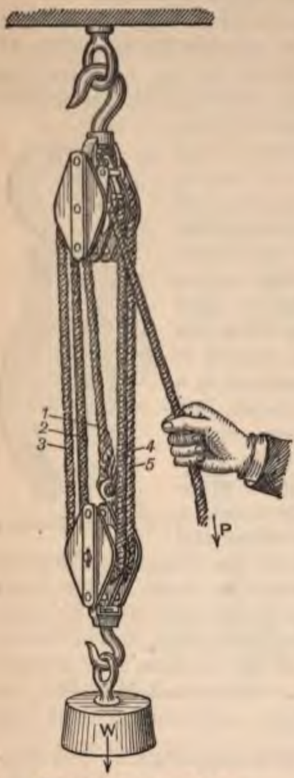
The pull × *its distance* = *the load* × *its distance.*

Or, $$P \times 2L = W \times L.$$

The theoretical advantage is therefore 2 to 1, or a certain force would lift *double the weight*, neglecting friction.

Block and Tackle.—Passing over the various arrangements *of pulleys for lifting* weights which are treated of in theoretical

mechanics, we come to this well-known and useful contrivance. As will be seen from the accompanying sketch, it consists of a number of pulleys (or sheaves as they are technically termed) free to run round on a turned central iron or steel spindle, and inserted in a block, having their iron divisions between each pulley, and strong iron cheeks fixed to a swivel joint terminating in an iron hook hung from an eye bolt. Three sheaves are shown in this block, but the number may range from one upwards, according to the size and work to be done. There is a similarly constructed block with two sheaves, from which the weight to be raised, or the body to be pulled, is attached, and this is called the movable block, whereas the upper or home one is termed the fixed block. Around the pulleys of both blocks there is reeved a rope with the inner end made fast to an eye on the movable block, whilst the free end hangs from one of the outside sheaves; but this arrangement is frequently reversed, for the inner end of the rope may be attached to an eye on the fixed block, and the free end may spring from the other one (see the figure in connection with Example I. of this Lecture). The free end of the rope is then ready to be pulled by the hands or by aid of a winch.



BLOCK AND TACKLE.

Now, neglecting friction, and supposing the rope to be perfectly flexible, a force, P, applied to the free end of the rope would be transmitted throughout it to the other end at the movable block. Hence the effect of this force in overcoming a resistance, W, *is multiplied by the number, n, of parts of the rope which spring from the movable block.*

Or,
$$W = nP$$

And (1) *The theoretical advantage* $= \dfrac{W}{P} = \dfrac{n}{1}$

(2) *The velocity ratio*, or *ratio of the distance through which* P acts, to that through which W *is overcome in the same time.*

Or, $$\text{Velocity ratio} = \frac{\text{P's distance}}{\text{W's distance}} = \frac{n}{1}$$

In the figure there are shown three pulleys in the upper block and two in the lower, with five parts of rope springing from the latter; therefore in this case $n = 5$.

Here $$W = nP = 5P; \text{ or, } P = \frac{W}{n} = \frac{W}{5}$$

since P must pass through five times the distance that W does in the same time.

$$\text{The velocity ratio} = \frac{\text{P's distance}}{\text{W's distance}} = \frac{n}{1} = \frac{\mathbf{5}}{\mathbf{1}}$$

So that the theoretical advantage and the velocity ratio have the same numerical value.

The Principle of Work applied to the Block and Tackle.—Using the very kind of block and tackle represented by the previous figure, attach a light Salter's spring balance by its hook to the rope where the hand is shown. Fix such a weight to the lower block that the weight of rope between the blocks, the movable block, and the load are 60 lbs. Call this W. Now pull the ring of the spring balance until the load rises slowly and uniformly, and note the reading on the balance; let it be 18 lbs., and let the weight of this balance and the hanging free end of the rope, which is assisting the arm, be 2 lbs. Call this total pull of 20 lbs. P; then:

(3) *The actual or working advantage* $= \frac{\text{weight raised}}{\text{pull applied}} = \frac{W}{P} = \frac{60 \text{ lbs.}}{20 \text{ lbs.}} = \frac{\mathbf{3}}{\mathbf{1}}$.

Lift W up through one foot exactly, and measure the length of rope which you have pulled out from the upper block, and you will find that it is five feet; hence,*

(4) *The work put in* $= P \times n = 20 \text{ lbs.} \times 5 \text{ ft.} = \mathbf{100}$ ft.-lbs.

(5) *The work got out* $= W \times 1 = 60 \text{ lbs.} \times 1 \text{ ft.} = \mathbf{60}$ ft.-lbs.

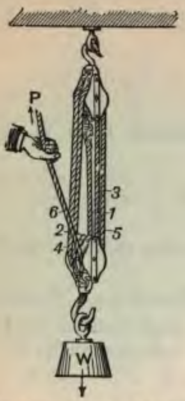
(6) *The efficiency* $= \frac{\text{Work got out}}{\text{Work put in}} = \frac{60 \text{ lbs.}}{100 \text{ lbs.}} = \mathbf{\cdot 6}$

(7) *The percentage efficiency* $= \cdot 6 \times 100 = \mathbf{60}$ %

In the same way the efficiency of any other block and tackle may be found, and the student should carry out a series of

* The above results were obtained by the Author from a block and tackle of the same kind as that shown by the previous figure, at a demonstration in his Junior Applied Mechanics class.

experiments in a laboratory or workshop so as to impress the various measurements and the results on his memory. He will find that if the efficiency is over 50 per cent. a comparatively small load will run down and overhaul the free end of the rope, unless it has some restraining force applied to it, or be fixed to some rigid body. It is for this reason that sailors, who work very much with ordinary block and tackle, always "belay" the free end of the rope when they have adjusted their sails or have heaved up a body to the required height.



BLOCK AND TACKLE.
2ND CASE, EXAMPLE I.

EXAMPLE I.—A tackle, consisting of an ordinary double and treble block, is employed for lifting a weight of 600 lbs. attached to the double block. What force is required, neglecting friction? If the tackle is reversed, so that the weight is attached to the treble block, the free end of the rope being pulled upwards, what force would now be required to lift the weight? (S. and A. Exam. 1892.)

ANSWER.—*First Case.*—By an inspection of the previous figure in this Lecture, it will be apparent that the weight W is supported by *five* parts of the rope, or $n = 5$.

$$\therefore P_1 = \frac{W}{n} = \frac{W}{5} = \frac{600}{5} = 120 \text{ lbs.}$$

Second Case.—Here the system is inverted, so that the block with the three pulleys is lowermost, as shown by the accompanying figure. In this case it is evident that there are *six* parts of the rope supporting W, or $n = 6$.

$$\therefore P_2 = \frac{W}{n} = \frac{W}{6} = \frac{600}{6} = 100 \text{ lbs.}$$

LECTURE VI.—QUESTIONS.

1. Suppose that your weight is 10 stone 10 lbs., and that you lift a weight of $\frac{1}{2}$ cwt. on your shoulder, and walk upstairs with it to a height of 20 ft.; what work have you expended, and what will be your efficiency as a machine? *Ans.* 4120 ft.-lbs.; 27 per cent.

2. Suppose that you had a rope passed round a beam of wood, and that you attached $\frac{1}{2}$ cwt. to one end and pulled with a force of 84 lbs. on the other end and then elevated it 10 ft.: (*a*) what work have you put in? (*b*) what is the percentage efficiency of the arrangement? (*c*) what is the percentage of lost work? *Ans.* (*a*) 840 ft.-lbs.; (*b*) $66 \cdot \dot{6}$; $33 \cdot \dot{3}$.

3. Suppose that a weight of $\frac{1}{2}$ cwt. is attached to one end of a rope passed round a pulley, and that you lift it 10 ft. by pulling on the other end of the rope with a force of 70 lbs.: what percentage of the work done is *lost* in overcoming the friction at the pulley? *Ans.* 20 per cent.

4. What will be the difference, and why, in the tension on the chain of a crane when a *snatch-block* is used, and when the weight is lifted directly? Sketch a snatch-block, and describe its construction and action.

5. In a rope and pulley lifting block with three sheaves in the upper block, and two sheaves in the lower block, find the theoretical advantage gained. Give the reason for your answer, and sketch the arrangement, showing where the rope is to be attached. *Ans.* W : P :: 5 : 1.

6. Sketch an arrangement of 5 equal pulley sheaves for lifting a weight of 1 ton. What force is exerted on the rope in your arrangement? Explain the mode of arriving at this numerical result by the principle of work. (S. and A. Exam. 1891.) *Ans.* With 3 pulleys in upper block and 2 in lower block, P = 448 lbs.

7. A tackle is formed of two blocks, each weighing 15 lbs., the lower one being a single movable pulley, and the upper or fixed block having two sheaves; the parts of the cord are vertical, and the standing end is fixed to the movable block; what pull on the cord will support 200 lbs. hung from the movable block, and what will then be the pressure on the point of support of the upper block? Give a sketch. *Ans.* 71·6 lbs.; 301·6 lbs.

8. A weight of 400 lbs. is being raised by a pair of pulley blocks, each having two sheaves. The standing part of the rope is fixed to the upper block, and the parts of the rope, whose weight may be disregarded, are considered to be vertical. Each block weighs 10 lbs.; what is the pressure at the point from which the upper block hangs? *Ans.* 522·5.

9. A tackle, consisting of an ordinary double and treble block, is employed for lifting a weight of 1000 lbs. attached to the double block. What force is required, neglecting friction? If the tackle is reversed, so that the weight is attached to the treble block, the free end of the rope being pulled upwards, what force would now be required to lift the weight? Sketch the two arrangements. *Ans.* 200 lbs.; 166·6 lbs.

10. Apply the "principle of work" to find the relation between the force applied and the weight raised by an ordinary set of block and tackle. State what is meant by the following terms:—(1) velocity-ratio; (2) theoretical mechanical advantage; (3) actual or working advantage; (4) work put in; (5) work got out; (6) efficiency of an apparatus or machine; (7) percentage efficiency.

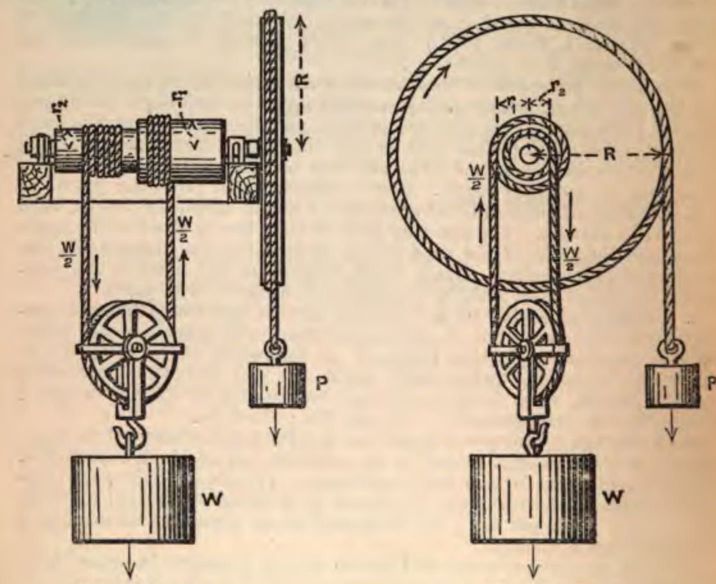
11. With an ordinary block and tackle having 3 pulleys in upper block and 2 in lower block—*i.e.*, 5 ropes attached to lower block—it is found that a pull of 50 lbs. *is required to* raise a weight of 165 lbs. Find—(1) Theo*retical advantage and velocity* ratio = 5 : 1; (2) Actual advantage = 3·3 : 1; *(3) Efficiency of apparatus* = ·66; (4) Percentage efficiency of apparatus = 66.

LECTURE VII.

CONTENTS.—The Wheel and Compound Axle, or Chinese Windlass—The Principle of Moments applied to the Wheel and Compound Axle—The Principle of Work applied to the Wheel and Compound Axle—Examples I. II.—Weston's Differential Pulley Block—The Principle of Work applied to Weston's Differential Pulley Block—Experiment I.—Cause of the Load not overhauling the Chain—Questions.

The Wheel and Compound Axle, or Chinese Windlass.—This ingenious contrivance was first devised by the Chinese for the purpose of lifting weights. The theoretical mechanical advantage is very great, but it possesses the disadvantage of requiring a long length of rope to lift the weight a small height.

Its construction and action will be easily understood from the accompanying side and end views, which are taken from a model



SIDE VIEW.

END VIEW (Without End Bearing).

THE WHEEL AND COMPOUND AXLE.

made in the author's engineering workshop for the purpose of demonstrating its action and efficiency to his students.

The Principle of Moments applied to the Wheel and Compound Axle.—Taking moments about the axle, we have, when there is equilibrium between P and W,

$$P \times R + \frac{W}{2} \times r_2 = \frac{W}{2} \times r_1$$

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore P = \frac{W}{2}\frac{(r_1 - r_2)}{R}$$

The Principle of Work applied to the Wheel and Compound Axle.—Neglecting friction, and supposing the rope to be perfectly flexible, cause the wheel to make one complete revolution in the direction shown by the arrow near its circumference on the end view.

Then, by the principle of work,

The work put in = the work got out.

Or, P × its distance = W × its distance ;*

i.e., P × circumference of wheel = W × $\frac{1}{2}$ of the difference of the circumferences of the larger and smaller axles.*

Or, $$P \times 2\pi R = W \times \tfrac{1}{2}(2\pi r_1 - 2\pi r_2)$$

(Dividing both sides of the equation by 2π)—

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore P = \frac{W}{2}\frac{(r_1 - r_2)}{R}$$

Which is the same result as the one above; consequently the principle of moments and the principle of work agree.

EXAMPLE I.—In a compound wheel and axle, where the weight hangs on a single movable pulley, the diameters of the two portions of the axle are 3 and 2 inches respectively, and the lever handle which rotates the axle is 12 inches in length. If a force

* If $\frac{W}{2}$ is raised the circumference of the larger circle on one side, then $\frac{W}{2}$ is lowered at the same time on the other side, the circumference *of the smaller axle; consequently* W will be elevated a distance equal to *half the difference of the* circumferences of two axles, or $= \frac{1}{2}(2\pi r_1 - 2\pi r_2)$.

of 10 lbs. be applied to the end of the lever handle, what weight can be raised?

ANSWER.—Here P = 10 lbs.; R = 12″; $r_1 = 1.5''$ and $r_2 = 1''$.

By the principles of moments and work—

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$10 \times 12 = W \times \tfrac{1}{2}(1.5 - 1) = W \times \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}W$$

$$\therefore W = 10 \times 12 \times 4 = \mathbf{480\ lbs.}$$

EXAMPLE II.—In a compound wheel and axle, let the diameter of the large axle be 6 inches, and that of the smaller axle 4 inches, and the length of the handle 20 inches; find the ratio of the velocity of the handle to that of the weight raised.

ANSWER.—Here R = 20″; $r_1 = 3''$; $r_2 = 2''$.

By the principle of moments and work—

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore \frac{P}{W} = \frac{\frac{1}{2}(r_1 - r_2)}{R}$$

$$\frac{P}{W} = \frac{\frac{1}{2}(3-2)}{20} = \frac{1}{40}$$

But by the principle of work—

P × its distance = W × its distance

1 × P's distance = 40 × W's distance

∴ The velocity ratio,

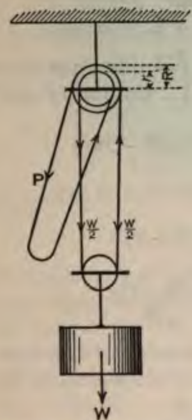
Or,
$$\frac{\text{P's distance}}{\text{W's distance}} = \frac{\mathbf{40}}{\mathbf{1}}$$

Weston's Differential Pulley Block.—This practical application of the Chinese windlass is simply a compound axle without the wheel. Or, where $R = r_1$.

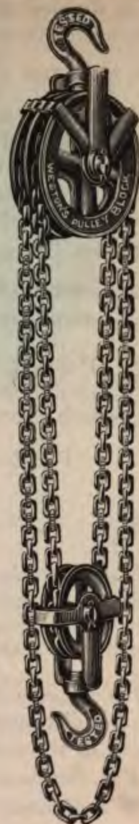
Hence,
$$P \times R = \frac{W}{2}(R - r)$$

where R is the radius of the larger axle or pulley, and r the radius of the smaller one. After describing Weston's differential pulley block, we will deduce this formula from the "principle of work" by the same kind of reasoning as we adopted in the case of the wheel and compound axle. We leave the student, however, *to apply the* "principle of moments," whereby he should get the *same results.*

As will be gathered from an inspection of the accompanying outside view and the small diagram showing the directions of the forces and their arms, it will be seen that the apparatus consists of three parts—(1) an upper block; (2) an endless chain; (3) a movable lower block or snatch-block. The upper block has a hook with swivel joint, from which the iron frame is suspended. In the centre of this frame is a turned steel axle on which rotates a couple of pulleys cast in *one piece*, and therefore rigidly connected together. The one pulley is slightly larger than the other, and both pulleys have **V**-grooved peripheries with side ridges or teeth cast on the inner sides of the grooves, so as to fit the pitch of the links of the chain, which passes over them and thereby prevent it slipping over the surface of the pulleys. The lower or movable pulley is simply an ordinary smooth **V**-grooved pulley with swivel and hook like that already described under the heading "Snatch Block." The endless chain is an ordinary open-linked chain of uniform pitch and size of link. It passes from the position where the hand or pull, P, is applied, over the larger pulley of the upper block, underneath the lower pulley, over the smaller of the upper block pulleys, and back to the starting-point. (See also the small figure.) When a pull, P, is applied at this part *of the chain* (*if there were no friction*), it would be transmitted with *undiminished value* throughout its whole length where the tension



SKELETON FIGURE OF WESTON'S DIFFERENTIAL PULLEY BLOCK.



WESTON'S DIFFERENTIAL PULLEY BLOCK. (BY HOLT & WILLETT.)

can act; but, as we shall see afterwards, a large proportion of this force is absorbed in overcoming friction. The stress due to the load W is divided equally between the two vertical parts of the chain connected to the lower block, and if W is moved through any distance, the stress $\frac{W}{2}$ must act through double that distance.

The Principle of Work applied to Weston's Differential Pulley Block and Tackle.—Theoretically (*i.e.*, leaving friction out of account, the weight of the hanging part of the chain and the weight of the lower block), we have by the *principle of work*, in one revolution of the upper pulleys—

$$P \times \text{its distance} = W \times \text{its distance}.$$

$$P \times \left\{\begin{matrix}\text{circumference of} \\ \text{the larger pulley}\end{matrix}\right\} = \frac{W}{2}\left\{\begin{matrix}\times \text{ difference of the circumferences of} \\ \text{the larger and smaller pulleys.}\end{matrix}\right\}$$

$$P \times 2\pi R \quad = \frac{W}{2}(2\pi R - 2\pi r)$$

(Dividing each side of the equation by 2π)

$$P \times R \quad = \frac{W}{2}(R - r)$$

$$\therefore P \quad = \frac{W(R - r)}{2R}.$$

(1) *The Theoretical Mechanical Advantage* or ratio of W to P is found directly from the above equation by simple transposition.

$$\therefore \frac{W}{P} = \frac{2R}{R - r}$$

(2) *The Velocity Ratio* (or ratio of the distance passed through by P to the distance passed through by W in the same time) is also found in the same way.

$$\therefore \frac{\text{P's distance}}{\text{W's distance}} = \frac{2\pi R}{\frac{1}{2}(2\pi R - 2\pi r)} = \frac{2R^*}{R - r}$$

Or, the velocity ratio has the same numerical value as the theoretical advantage.

EXPERIMENT I.—With a Weston's differential pulley block, having in the upper block one pulley with an effective radius of 4″ (*i.e.*, from the centre of the pulley to the centre of the chain which passes round it), and a smaller pulley with an effective radius of $3\frac{1}{2}$″, you can just lift a total load of 100 lbs. (including the dead weight, the lower block, and the hanging parts of the chain) by a pull of 20 lbs. on the chain.

* Dividing numerator and denominator by π does not alter the fraction.

In this case the *theoretical advantage* and the velocity ratio are each equal to—

$$\frac{2R}{R-r}=\frac{2\times 4''}{4''-3\cdot5''}=\frac{8}{\cdot5}=\frac{\mathbf{16}}{\mathbf{1}}$$

Or, the pull on the forward side of the chain must act through 16 ft. for every foot the load is raised.

(3) *The Actual or Working Advantage* of the machine is, however, only as—

$$\frac{W}{P}=\frac{100\text{ lbs.}}{20\text{ lbs.}}=\frac{\mathbf{5}}{\mathbf{1}}$$

(4) *The Work put in* in lifting W 1 ft. is

$$P\times 16=20\text{ lbs.}\times 16'=\mathbf{320\ ft.\text{-}lbs.}$$

(5) *The Work got out* is $=W\times 1=100\text{ lbs.}\times 1'=\mathbf{100\ ft.\text{-}lbs.}$

(6) *The Efficiency is* $=\dfrac{\text{Work got out}}{\text{Work put in}}=\dfrac{100\text{ ft. lbs.}}{320\text{ ft. lbs.}}=\cdot3125.$

(7) *The Percentage Efficiency* is

$$=\cdot3125\times 100=31\cdot25\ \%.$$

This is a very low efficiency for a machine, but it accounts for one of the useful properties of the Weston's differential pulley block—viz., that you can lift a weight by it, then let go your hold of the chain, and the weight will remain hanging in the exact position you left it, without overhauling the chain in the slightest degree. It is therefore an extremely useful appliance in engineering workshops where, for example, a slide valve and its valve casing port face have to be scraped so as to fit each other. After rubbing the valve on the port face, you can lift the valve by aid of a Weston's block, and leave it hanging, without any fear of its overhauling the chain which supports it, until you have scraped off the high or hard parts from the port face, when you can lower it for another rub. Or, in the case of having to adjust the centres of a heavy job to be turned in a lathe, you can lift the job from the lathe by a Weston's block, and leave it hanging quite free at the most convenient height to be acted upon, until you are ready to lower it again into position. Of course, with such apparatus, although the theoretical advantage is great, the actual or working advantage is small; yet this property of not overhauling is of such importance that appliances possessing it are constantly being used in every engineering workshop.

Cause of the Load not Overhauling the Chain.—In the *first place, the chain* cannot slip round the pulleys of the upper *block, because the links* of the chain fit into the notches or

LECTURE VIII.*

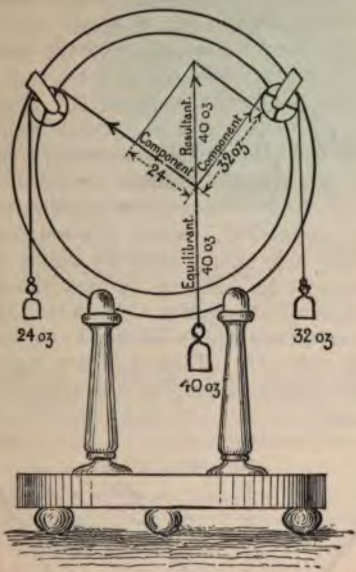
CONTENTS.—Graphic Demonstration of Three Forces in Equilibrium—Parallelogram of Forces—Triangle of Forces—Three Equal Forces in Equilibrium—Two Forces acting at Right Angles—Resolution of a force into Two Components at Right Angles—Resultant of Two Forces acting at any Angle on a Point—Resultant of any number of Forces acting at a Point—Example I.—Stresses in Jib Cranes—Examples II. III.—Stresses on a Simple Roof—Example IV.—Questions.

IN Lecture I. we explained and illustrated how a force may be represented by a straight line both in direction and magnitude, and we defined the terms components, equilibrant, resultant, resolution, and composition of forces. We will now discuss briefly the case of three forces in equilibrium when acting towards or from a point, as well as the parallelogram and the triangle of forces with examples, before taking up the inclined plane and friction.

Graphic Demonstration of Three Forces in Equilibrium. —EXPERIMENT I.—Take a black board which (for convenience of handling and demonstration before a class) may be of the form shown by the accompanying figure. Select two movable clamps, each fitted with a small **V**-grooved pulley about 2 inches in diameter, with a minimum of friction at their bearings, and fix them to the outside of the board as indicated. Pass a very fine flexible cord over the pulleys, and attach to the ends of this cord **S** hooks. Hang from these hooks weights of say 24 oz. and 32 oz., and from the cord (anywhere between the pulleys) another cord with an **S** hook and a weight of, say, 40 oz. After a few up-and-down oscillations these three weights will come to rest in the definite position shown by the figure, and if you disturb them from this position they will invariably return to it again. Consequently, you conclude that the three forces acting from their common point of attachment are in equilibrium, and that the force 40 oz. is the *equilibrant* of the two forces 24 oz. and 32 oz.

* This Lecture may require two meetings of a class when the students have had no previous training in Theoretical Mechanics. In any case, it will be well to spend at least one with a revisal hour before the written examination, which should now take place upon the work gone over since the beginning of the session, prior to the Christmas holidays.

With a piece of finely pointed white chalk, draw lines (from the point where the three forces act) on the black board parallel to the cords, and plot off from this point to any convenient scale (say by aid of a two-foot rule) distances along them to represent their respective magnitudes. Extend from the same point in an upward vertical direction another line, and mark it off to represent 40 oz. *This line* evidently corresponds, in point of application, direction, and magnitude, to the *resultant* of the *components* (24 oz. and 32 oz.), for it is equal and opposite in direction to their *equili-*



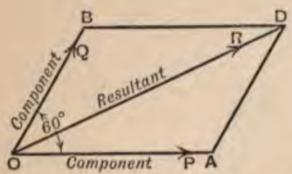
GRAPHIC REPRESENTATION OF FORCES IN EQUILIBRIUM.

brant. From the extremity of this resultant draw lines joining the outer ends of the components (24 oz. and 32 oz.), and you have a parallelogram whose adjacent sides from the point of application represent, both in direction and magnitude, the component forces, and whose diagonal represents, also both in direction and magnitude, their resultant.

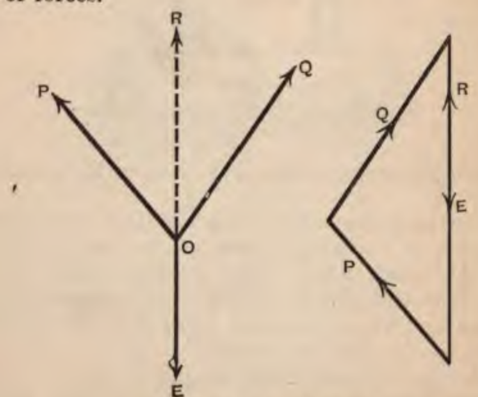
If any other pair of convenient weights be selected and applied in the same way, you can find an equilibrant and resultant for them. From these experiments you conclude that a general principle, termed the "parallelogram of forces," is true without having recourse to any special mathematical reasoning.

Parallelogram of Forces.—*If two forces, acting simultaneously towards or from a point, be represented in direction and magnitude by the adjacent sides of a parallelogram, then the resultant of these forces will be represented in direction and magnitude by the diagonal of the parallelogram which passes through their point of intersection.*

For example, let any two forces, P and Q, act from the point O at any convenient angle, say 60°, then, if OA and OB be plotted to scale to represent these forces in direction and magnitude, the diagonal OD of the parallelogram OADB will represent in direction and to the same scale their resultant R. But the resultant R is equal and opposite in direction to a force E, which would exactly balance the effect of P and Q, or to a force represented in *direction* and in *magnitude* by the line DO. Further, since the side AD is equal and parallel to the side OB, it may be taken to represent Q in direction and magnitude. Hence we have the three sides of the triangle OAD taken in the order OA, AD, DO, representing in direction and magnitude three forces, P, Q, E, in equilibrium, acting from the point O. Hence we have a general proposition termed the "triangle of forces," or a deduction from the "parallelogram of forces."



PARALLELOGRAM OF FORCES.



TRIANGLE OF FORCES.

Triangle of Forces.—*If three forces acting towards or from a point are in equilibrium, and a triangle be drawn with its sides*

respectively parallel to those forces taken in due order, then the forces will be represented to scale by the sides of the triangle.

CONVERSELY:—*If three forces acting towards or from a point are represented in direction and to scale by the sides of a triangle taken in due order, these three forces are in equilibrium.*

For example, let the three forces P, Q and E act from the point O, and be in equilibrium. Draw a triangle with its sides, P, Q, E, respectively parallel to these forces; then the sides of this triangle, taken in that order, represent to the same scale these forces. Or, if the triangle, whose sides are respectively P, Q and E, represent in direction and to scale the three forces P, Q and E, as they act from a point O, these forces are in equilibrium. We have shown by a dotted line the resultant R, and its direction as opposed to E, by the same side of the triangle.

It is quite evident that if the forces P, Q and E acted *towards* the point O, instead of from it, the triangle P, Q, E would still represent these forces in magnitude, but the direction of all the arrows would have to be pointed the opposite way.

SPECIAL CASES.—**Three Equal Forces in Equilibrium.**—It can easily be proved by the apparatus used for Experiment I., or by construction, that if you have three equal forces in equilibrium they must act at 120° from each other, and that the triangle representing their directions and magnitudes will be an equilateral triangle, or a triangle whose angles are each equal to 60°.

Two Forces acting at Right Angles.—In this case it can be proved by the same apparatus, or by Euclid, Book I. Prop. 47, that any two forces P and Q, acting at right angles to each other, have a resultant R, or are balanced by a third force E, of such magnitude that—

$$E^2 = R^2 = P^2 + Q^2$$

Consequently, if you have any two forces in the proportion of 3 to 4 acting at right angles to each other, their resultant must have a value of 5.

For,
$$R^2 = P^2 + Q^2$$
$$R^2 = 3^2 + 4^2 = 25$$
$$\therefore R = \sqrt{25} = 5$$

Or,
$$\frac{R}{P} = \frac{5}{3}, \text{ and } \frac{R}{Q} = \frac{5}{4}$$

Conversely, if any two forces in the proportion of 3 to 4 units are balanced by a third force proportionally of 5 units, the forces 3 and 4 must be acting at right angles to each other.

Also,
$$P^2 = R^2 - Q^2 \qquad \text{And} \quad Q^2 = R^2 - P^2$$
$$\therefore P = \sqrt{R^2 - Q^2} \qquad \therefore Q = \sqrt{R^2 - P^2}$$

Or,
$$3 = \sqrt{25 - 16} \qquad \text{Or,} \quad 4 = \sqrt{25 - 9}$$
$$3 = \sqrt{9} \qquad 4 = \sqrt{16}$$

Resolution of a Force into Two Components at Right Angles to each other.*—Let R be the force to be resolved, P and Q the components, and let R make an angle, θ, with the force Q.

Then . . . . $R \cdot \text{Cos}\ \theta = Q$; for $\text{Cos}\ \theta = \frac{Q}{R}$

And . . . . $R \cdot \text{Sin}\ \theta = P$; for $\text{Sin}\ \theta = \frac{P}{R}$

Also . . . . $\frac{R \cdot \text{Sin}\ \theta}{R \cdot \text{Cos}\ \theta} = \frac{P}{Q} = \text{Tan}\ \theta$

Resultant of Two Forces acting at any Angle on a Point.—The proof of this general case must be left to the Author's Advanced Treatise on Applied Mechanics, but the formula may be given here, viz.:

$$R^2 = P^2 + Q^2 + 2P \times Q\ \text{Cos}\ \alpha$$

where P and Q are any two forces, R their resultant, and α the angle between the directions of the forces P and Q.

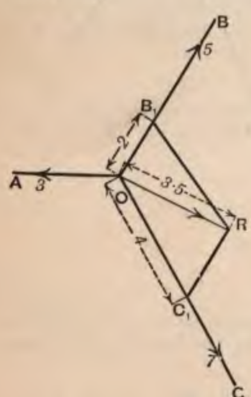
If P = Q, then—

$$R^2 = P^2 + P^2 + 2P^2\ \text{Cos}\ \alpha = 2P^2 + 2P^2\ \text{Cos}\ \alpha$$

$$R^2 = 2P^2\ (1 + \text{Cos}\ \alpha) \qquad = 4P^2\ \text{Cos}^2 \frac{\alpha}{2}$$

$$\therefore R = 2P\ \text{Cos}\ \frac{\alpha}{2} \qquad (\text{Since Cos}\ \alpha = 2\ \text{Cos}^2 \frac{\alpha}{2} - 1)$$

Resultant of any Number of Forces Acting at a Point.—Let P_1, P_2, P_3, &c., be any number of forces acting at a point; then, by the parallelogram of forces find a resultant, R_1, for P_1 and P_2; and a resultant, R_2, for R_1 and P_3; and so on. The last resultant will be the resultant of all the forces.

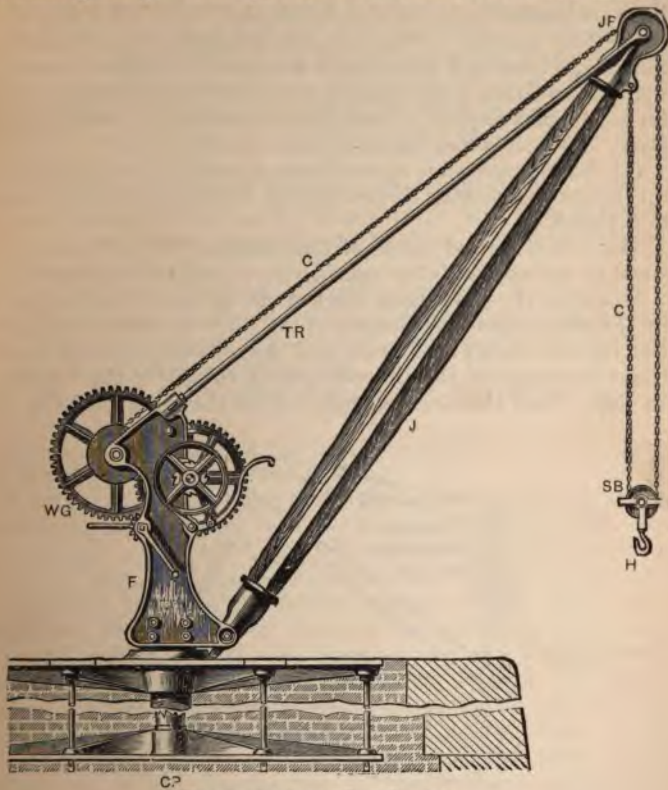


Example I.—Forces 3, 5 and 7 units act from a central point O at equal angles. Find the resultant.

ANSWER.—Let OA, OB and OC represent the forces in direction and magnitude. Then you can follow out the above rule and find a resultant for, say, 3 and 5—call this R_1; and finally find a resultant for R_1 and 7. But it is obvious that you may subtract 3 units from each of them without affecting the result, since the forces are acting at equal angles from each other. This will destroy one of them, and leave OB_1 to represent 2 units, and OC_1 to represent 4 units. Then, by the parallelogram of forces you find the resultant $R = 3{\cdot}5$ units.

* The reverse of this may be applied to the composition of two or more forces acting at a point in one plane, but we will leave the demonstration of such problems, as well as that of the polygon of forces, to our Advanced Book on Applied Mechanics.

Stresses in Jib Cranes.—As a practical example of the application of the "triangle of forces," take the case of an ordinary hand-worked jib crane. The load is suspended from the hook H of the snatch-block SB; or, in the case of a crane for lifting light loads quickly, to a simple hook with a swivel attached directly



HAND-WORKED JIB CRANE.

INDEX TO PARTS.

CP	represents	Central post.	J	represents	Jib.
F	,,	Framing.	JP	,,	Jib pulley.
WG	,,	Wheel gear.	SB	,,	Snatch-block.
C	,,	Chain.	H	,,	Hook.
TR	,,	Tie-rods.			

to the end shackle of the chain C, as it comes down from the jib pulley JP, instead of the chain passing round a snatch-block pulley, and up to an eyebolt near the point of the jib.

(1) The load produces a tension on the chain C.

(2) A thrust along the jib J, from the jib pulley to the eye-bolt connecting the shoe of the jib to the bottom of the framing F.

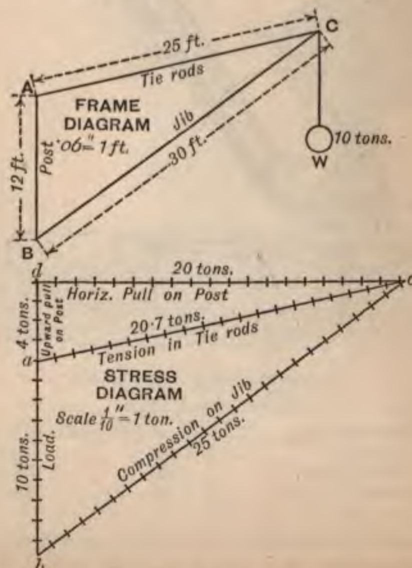
(3) A tension in the tie-rods from the top of the framing to their connection with the top of the jib.

(4) This tension on the tie-rods produces a horizontal pull, tending to bend and break the crane-post CP, where it leaves the upper foundation plate-bearing and joins the framing. Cranes for heavy lifts require a back balance weight to counteract this force. (See the third figure in Lecture XIII.)

(5) It also causes an upward stress, tending to unship or lift up the crane-post from its bearings in the upper and lower foundation plates.

The directions and values of these stresses will be better understood by the student after considering a particular example.

EXAMPLE II.—In a hand-worked jib crane of the form shown by the above figure, the length of the jib is 30 feet, the lengths of the tie-rods are 25 feet each, and the vertical distance between the attachments of the tie-rods and of the jib to the framing, is 12 feet. Find the stresses produced on these parts of the crane



FRAME AND STRESS DIAGRAM FOR A JIB CRANE.

by a load of 10 tons hung from the hook, neglecting all other stresses produced by the weight of the several parts of the crane.

ANSWER.—*First*, draw a "frame diagram," or figure to scale, representing the directions and the lengths of centre lines of the various parts under stress. As shown by the frame diagram of the accompanying upper figure, AB represents the 12 feet vertical distance between the foot of the jib and inner ends of the tie-rods marked post, BC represents the 30 feet *jib*, AC the 25 feet *tie-rods*, and CW the 10-ton *load*—all to the same scale.

Now, it is evident from an inspection of this figure that the load W causes—

(1) A vertical downward tension on the chain from C to W.

(2) A thrust or compression along the jib from C to B, which produces an equal and opposite reaction from the framing at B along the jib from B to C.

(3) A tension on the tie-rods from A to C.

(4) This tension on the tie-rods may be resolved into a horizontal pull or stress from A towards the direction of W, tending to bend or break the post about B.

(5) Also, a vertical upward stress or pull in the post from B towards A.

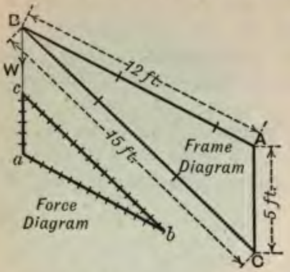
The student should mark the directions of these various stresses by arrow-heads on his frame diagrams.

Second, draw a "stress diagram," viz., *ab*, vertical and to a convenient scale, to represent the downward force of the 10-ton load on the point C; *bc*, parallel to the *reaction* along the jib from B to C; and *ca*, parallel to the tension in the tie-rods from C to A.

Then, by "the triangle of forces," since we have three forces acting from the point C (viz., load, reaction along jib, and tension in tie-rods) in equilibrium, and since we have drawn a triangle with its sides respectively parallel to these forces taken in due order; the forces will be represented to one scale by the sides of this triangle. Consequently, *ab* represents the load to scale; *bc* the reaction along the jib; and *ca* the tension in the tie-rods. Now, this tension on the tie-rods may be resolved into vertical and horizontal components by the method already described in this Lecture; therefore, a vertical line, *ad*, represents the vertical component or upward pull on the *post*, and *dc* the horizontal pull on the same, both in direction and to the same scale as *ab* represents the load. By applying the scale to which *ab* has been drawn to represent 10 tons (viz., $\frac{1}{10}''$ to 1 ton), *bc* shows 25 tons; *ca*, 20·7 tons (*which would be* 10·35 tons on each tie-rod *if they were parallel to each other*, but more if inclined from the jib-head *to the outside of the* framing); *cd*, 20 tons; and *ad*, 4 tons.

EXAMPLE III.—In a common crane the jib is 15 feet long, and the tie-rod 12 feet. The tie-rod is attached to the crane post at a point 5 feet above the foot of the jib. If a weight of 6 tons be hung from the point of the jib, find the tension in the tie-rod and the thrust in the jib.

ANSWER.—*First* draw the frame diagram as explained in Example II., marking the lengths of the parts by dotted lines and arrow-heads. (The student in his diagrams should also mark the directions of the stresses.)



TENSION IN THE TIE-ROD AND THRUST IN THE JIB OF A CRANE.

Second, on the line of action of the weight W draw *ca* to scale to represent the direction and magnitude of the weight, 6 tons. Then draw *cb* parallel to the jib, and *ab* parallel to the tie-rod. The triangle, *cab*, represents by its sides to one scale the magnitudes of the forces —viz., 14.4 tons tension in the tie-rods and 18 tons thrust or reaction in the jib.*

Stresses on a Simple Roof.—EXAMPLE IV.—The weight of and on each principal of a simple triangular roof is 1 ton. Find the stresses on the points of support and in the several members of the principal.

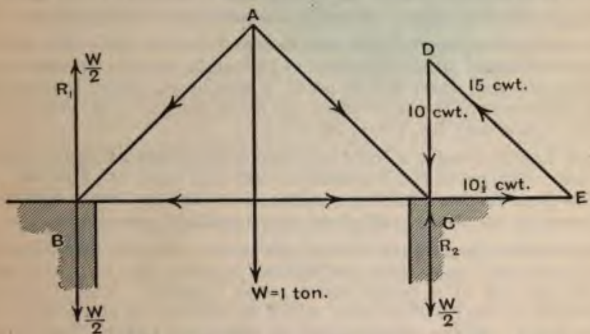
ANSWER.—*First*, draw a frame diagram of the principal, where AB and AC represent the direction and length of the rafters, and BC represents the tie-beam.

Then, the whole weight may be supposed to act in a vertically downward direction, AW, from the junction of the rafters through the middle of the tie-beam. This weight naturally produces a pressure at A and at B of $\frac{W}{2}$. It also produces at these points reactions R_1 and R_2, each equal to $\frac{W}{2}$, since the whole is symmetrically balanced about the central vertical line AW. Further, there is a stress of compression along the rafters in the directions AB and AC, and consequently an equal and opposite reaction

* We have purposely used the letters ABC and *abc* differently placed from the previous figures in Example II., and have drawn the stress diagram in a different position, in order to teach the student that he must not depend upon his memory with regard to letters, but upon a clear understanding of the "triangle of forces." Students should draw their frame and stress diagrams to as large a scale as their exercise books will admit.

along those members from B to A, and from C to A. Also, there is an equal tension on the tie-beam from its centre towards B and towards C.

Second, draw the stress diagram for the three forces that are in equilibrium at the bearing C (viz., the vertical downward pressure $\frac{W}{2}$, the horizontal tension along the tie-beam and the reaction along the rafter from C to A) by plotting DC as a vertical line to scale to represent $\frac{W}{2}$, or 10 cwt., and drawing DE parallel to AC, and producing the tension on the tie-beam until it meets this line DE. Therefore, the other sides of the triangle DCE represent in



FRAME AND STRESS DIAGRAMS OF A SIMPLE ROOF.

direction and to the same scale as DC represents 10 cwt.; the tension on the tie-beam by CE, equal to $10\frac{1}{2}$ cwt.; and the reaction along the rafter by ED, equal to 15 cwt.

In a precisely similar manner the stresses at the bearing B may be found by the "triangle of forces."

We will leave the more complicated questions in graphic statics to our book on the Advanced Stage of Applied Mechanics, since we believe the elementary or first year's student will find that what has been included in this Lecture is sufficient to enable him to understand what will be brought before him in the future Lectures of this book, as well as prepare him for answering the various problems which are likely to be asked of him.

Lecture VIII.—Questions.

1. State the principle of the parallelogram of forces, and explain how you would prove the truth thereof by experiment. A vertical force of 50 lbs. is balanced by two forces of 30 lbs. and 40 lbs. Find their directions and the angle between them.

2. Represent the point of application, the direction and the magnitude (to a scale of $\frac{1}{10}$ inch to a pound) of the following forces:—10 lbs. acting northwards, 15 lbs. acting eastwards, 20 lbs. acting southwards, and 25 lbs. acting westwards, all from one point. Find their resultant, and its direction. *Ans.* 14·11 lbs. acting south-west.

3. State the principle of the triangle of forces. Three forces, P, Q and R, act from or towards a point, and are in equilibrium. Show graphically how you would represent their magnitude and direction by the three sides of a triangle taken in order. Explain the converse of this question.

4. Two ends of a piece of cord are fastened to two nails in a wall 8 ft. apart in a horizontal line. The cord is 10 feet in length, and has a knot 4 ft. from one end, from which point a weight of 25 lbs. is suspended. Find by construction the stresses on the nails, and indicate their direction by arrows. *Ans.* 22·5 lbs.; 17·5 lbs.

5. Show how to resolve a force into two components at right angles to each other. A force of 100 lbs. acts at (1st) 30°, (2nd) 45°, (3rd) 60°, (4th) 75° to the horizontal. Find by construction the vertical and horizontal components for each case, and prove your results by calculation.

6. Sketch an ordinary hand-worked jib-crane. Explain its action by an index to parts, and show how the various stresses due to a load on the chain act, by aid of a frame and a stress diagram. Nine tons is hung from the end of the jib of a crane, which is inclined to the horizontal at an angle of 60°. If the compression on the jib is 16 tons, find by frame and stress diagrams the tension on the tie-rod. *Ans.* 9·4 tons.

7. In a crane, show the method of estimating the tension of the tie-rod and thrust on the jib when a given weight is hung from the end of the jib. If the load = 6 tons, and the tension of the tie-rod (which makes an angle of 60° with the vertical) = 18 tons, find by a diagram drawn to scale the thrust on the jib. (S. and A. Exam. 1890.) *Ans.* 21·6 tons.

8. In a common crane the jib is 30 ft. long and the tie-rod 24 ft. The tie-rod is attached to the crane-post at a point 10 ft. above the foot of the jib. If a weight of 10 tons be hung from the point of the jib, find by constructing a frame and a stress diagram—(*a*) the tension on the tie-rod; (*b*) the thrust on the jib; (*c*) the horizontal pull on the post; (*d*) the upward pull on the same. *Ans.* (*a*) 24 tons; (*b*) 30 tons; (*c*) 21·2; (*d*) 11·2 tons.

LECTURE IX.

CONTENTS.—Inclined Planes—The Inclined Plane without Friction—When the Force acts Parallel to the Plane—Example I.—When the Force acts Parallel to the Base—Example II.—When the Force acts at any Angle to the Inclined Plane—Example III.—The Principle of Work applied to the Inclined Plane—Example IV.—Questions.

Inclined Planes.—An inclined plane is a plane surface inclined to the horizontal, whereby a certain force may be used to raise a greater weight to a desired height than could be done by applying it directly to elevate the weight vertically. Inclined planes are also used for easing down weights with less retarding force than would be necessary to lower them vertically. In another form, called the wedge, inclined planes are employed for splitting bodies, or different parts of the same body, asunder, as in the case of the steel wedge used by the woodman to split up logs for firewood and other purposes. Wedges are also used for forcing bodies together, and for fixing them tightly in a desired position ; or for elevating them through a small distance, as in the case of the levelling of the heavy cast-iron sole-plate of an engine. And further, as we shall have occasion to prove, the well-known screw, in whatever form it may be applied, is simply an inclined plane of a particular shape.

The Inclined Plane without Friction.—In the first place, we will consider the inclined plane with a body placed thereon and kept in position by a force applied to the body, *when all friction between the plane and the body is supposed to be absent or negligible* —*i.e.*, both the plane and the body are assumed to be perfectly smooth. There are three cases of this statical problem.

(1) When the force supporting the body acts parallel to the inclined plane.

(2) When the force acts parallel to the base of the plane.

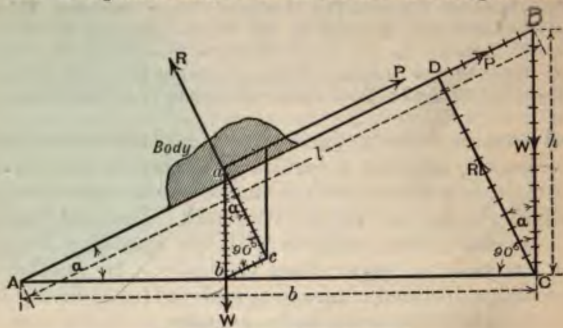
(3) When the force acts at any angle to the plane.

Case I.—*Let the force P act parallel to the plane*, and let the accompanying figure represent a vertical section through the plane and the *c.g. of the body*. Let a be the c.g. of the body ; W its weight, *acting vertically downwards* along the line aW ; P the necessary *pull (to keep the* body in position) applied along the line aP,

parallel to the plane AB; and R the reaction from the plane (due to the weight of the body resting thereon), acting along the line aR, at right angles to the plane.

Also, let the length of the plane AB be indicated by l; its height, BC, by h; its base, AC, by b; and the angle of the plane to the horizontal by a.

Now, by the "triangle of forces," since we have three forces, W, P and R, acting at a, the *c.g.* of the body, and since these forces are in equilibrium, if we construct a triangle whose sides



INCLINED PLANE, CASE 1.
WHEN **P** ACTS PARALLEL TO PLANE **AB.**

are parallel to these forces, they will represent them in direction and in magnitude.

Therefore, plot off along the line aW a distance ab, to represent the weight of the body W, to any convenient scale. From, b, draw a line bc parallel to P, and from, a, extend the direction of R to, c, by the line ac.

Then, $$\mathrm{W} : \mathrm{P} : \mathrm{R} :: ab : bc : ca$$

But by Euclid the triangle abc is similar to the triangle ABC.

$$\therefore \quad ab : bc : ca :: \mathrm{AB} : \mathrm{BC} : \mathrm{CA}$$

And, $$\mathrm{AB} : \mathrm{BC} : \mathrm{CA} :: l : h : b$$

Consequently, $$\mathrm{W} : \mathrm{P} : \mathrm{R} :: l : h : b$$

Or, $$\frac{\mathrm{P}}{\mathrm{W}} = \frac{h}{l}; \qquad \frac{\mathrm{R}}{\mathrm{W}} = \frac{b}{l}; \text{ and } \frac{\mathrm{P}}{\mathrm{R}} = \frac{h}{b}.$$

Or, $$\frac{\mathrm{P}}{\mathrm{W}} = \mathrm{Sin}\, a; \; \frac{\mathrm{R}}{\mathrm{W}} = \mathrm{Cos}\, a; \text{ and } \frac{\mathrm{P}}{\mathrm{R}} = \mathrm{Tan}\, a$$

Precisely the same results will be arrived at if (as shown by the *right-hand* side of the figure) we considered the vertical side BC

of the triangle ABC as representing W, and then have drawn a line from C on the direction of AB, parallel to R. It will form a useful exercise for the student if in every case he will plot down both methods, and mark along the *sides* of the triangle of forces the respective *forces* which they respectively represent.

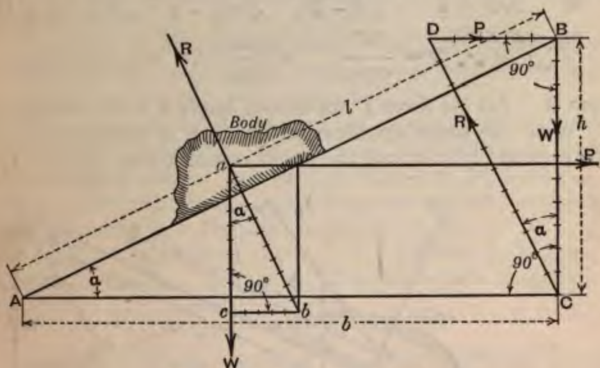
EXAMPLE I.—A weight of 100 lbs. is supported on a smooth inclined plane by a force P, acting parallel to the plane. If the incline be 1 in 10, find P, and give the reasoning by which you establish the result.

ANSWER.—Draw a figure exactly the same as that accompanying Case 1, and mark W = 100 lbs., l = 10, and h = 1. Then by the "triangle of forces":

$$\frac{P}{W} = \frac{BC}{AB} = \frac{h}{l} = \frac{1}{10}$$

$$P = W\frac{1}{10} = \frac{100}{10} = 10 \text{ lbs.}$$

Case 2.—*Let the force P act parallel to the base*, with the same



INCLINED PLANE, CASE 2.
WHEN P ACTS PARALLEL TO BASE AC.

signification for each of the forces and parts of the inclined plane, and the same assumptions. Then plot off ac, along aW, to represent W; draw cb parallel to P, and extend the direction of R backwards along ab,

Then $\quad W : P : R :: ac : cb : ba$

But by Euclid the triangle acb is similar to the triangle ACB.

$$\therefore\ ac : cb : ba :: AC : CB : BA$$

And, $\quad AC : CB : BA :: b : h : l$

Consequently, $W : P : R :: b : h : l$

Or, $\frac{P}{W} = \frac{h}{b}$; $\frac{R}{W} = \frac{l}{b}$; and $\frac{P}{R} = \frac{h}{l}$

Or, $\frac{P}{W} = \text{Tan } \alpha$ $\frac{R}{W}$; $= \text{Cosec } \alpha$; and $\frac{P}{R} = \text{Sec } \alpha$

Precisely the same results will be arrived at if (as shown by the right-hand side of the figure) we considered the vertical side BC of the triangle ABC as representing W, and then draw a line from C parallel to R, and a line BD, parallel to P, to meet the line CD.

Example II.—A force of 100 lbs. is supported on a smooth inclined plane by a force P acting parallel to the base. If the incline be 1 in 10, find P.

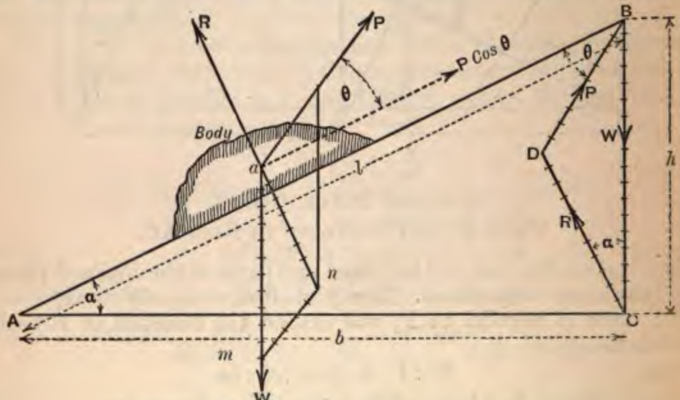
Answer.—Draw a figure exactly the same as that accompanying Case **2**, and mark W = 100 lbs., $l = 10$, and $h = 1$.

Then, by the "triangle of forces":

$$\frac{P}{W} = \frac{CB}{AC} = \frac{h}{b} = \frac{1}{\sqrt{l^2 - h^2}} = \frac{1}{\sqrt{100 - 1}} = \frac{1}{\sqrt{99}} = \frac{1}{9 \cdot 95}$$

$$\therefore P = \frac{W}{9 \cdot 95} = \frac{100}{9 \cdot 95} = 10 \cdot 05 \text{ lbs.}$$

Case 3.—*Let the force P act at any angle θ to the inclined plane* AB. With the same signification for each of the forces and parts of the inclined plane, and the same assumptions, plot off from a,



Inclined Plane, Case 3.
When P acts at any Angle to Plane AB.

along the line aW, a distance am, to any convenient scale to represent the weight of the body W. From this point, m, draw a line mn parallel to P, and extend the direction of R backwards to meet this line. This small triangle, amn, will be a "triangle of forces," for W, P and R, which are in equilibrium about the *c.g.* of the body at a.

But in this case the student will probably realise the proof of the problem more easily if he considers BC as representing to scale the weight W, and then draws CD parallel to R, and DB parallel to P,

When $$W : R : P :: BC : CD : DB,$$

or the triangle BCD is the "triangle of forces," representing the forces W, R and P in direction and magnitude by the sides BC, CD and DB respectively.

Or, $$\frac{P}{W} = \frac{DB}{BC}; \ \frac{R}{W} = \frac{CD}{BC}; \text{ and } \frac{P}{R} = \frac{DB}{CD}$$

If we resolve the force P (which acts at the angle θ to the inclined plane) parallel to the plane, then we can treat the components of P *exactly* in the same way as we did the simple force P in Case 1.

If we resolve P into the direction of R, then this component acts with R, and is evidently balanced by the resolved part of W in the same direction—*i.e.*, along the line, an.

EXAMPLE III.—A weight of 100 lbs. is supported on a smooth inclined plane by a force P, acting at 60° in an upward direction from the inclined plane. If the incline be 1 in 10, find P.

ANSWER.—Draw a figure exactly the same as that accompanying Case 3, and mark $W = 100$ lbs., $l = 10$, $h = 1$, and $\theta = 60°$.

Then by the "triangle of forces," BC represents W, and DB represents P to scale. Measuring their respective lengths we get

$$P = W\frac{DB}{BC} = 100\frac{20}{100} = \mathbf{20\ lbs.}$$

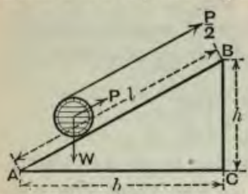
Principle of Work applied to the Inclined Plane.—Referring to the figure for Case 1, let the body, whilst under the action of the three forces W, P and R, be moved the whole length of the incline. Therefore P acts from A to B, and at the same time W acts through a vertical height CB. Consequently, *neglecting friction* as before, we have by the "principle of work"—

$$\text{The work put in} = \text{The work got out}$$
$$P \times \text{its distance} = W \times \text{its distance}$$
$$P \times AB = W \times CB$$
$$P \times l = W \times h$$
$$\therefore \ \frac{P}{W} = \frac{h}{l}; \text{ and } P = W\frac{h}{l}.$$

But this is precisely the same result as we got by applying the principle of the "triangle of forces." Hence, the "principle of work" agrees with the "triangle of forces" in respect to the inclined plane.

Cases **2** and **3** may be treated by the student in exactly the same way, and the correct results will be the same as those found by the "triangle of forces."

Example IV.—An inclined plane is used for withdrawing barrels from a cellar by securing two ropes to the top of the incline at B, then passing them down the incline, half round the barrel, and up to the horizontal platform at the top of the incline, where two men pull on the ropes in a direction parallel to the plane. If the weight, W, of the barrel is 200 lbs., the length, l, of the incline 20 ft., and the height 10 ft., find, by the principle of work, the least force which must be exerted by the two men, and the work expended, *neglecting friction*, in drawing the barrel from the cellar.



Raising Barrels by Return Rope and Inclined Plane.

Let the accompanying figure represent a vertical cross section through the middle of the barrel and the inclined plane. Then a statical force, P, applied at the *c.g.* of the barrel, would just balance its weight, W, and the reaction from the plane (not shown).

By the principle of work, *neglecting friction*—

The work put in = The work got out.

$$P \times \text{its distance} = W \times \text{its distance}.$$

$$P \times l = W \times h.$$

$$P = W\,\frac{h}{l} = 200\frac{10'}{20'} = 100 \text{ lbs.}$$

But by passing the rope round the barrel, as explained in the question, this force P is halved on the ropes (see Lecture V. on the pulley and snatch-block). Therefore the least force which the two men must exert in order *just* to move the barrel will be—

$$\frac{P}{2} = \frac{100}{2} = \mathbf{50\ lbs.}$$

But this force acts through a distance $2\,l = 40'$ ft.; therefore the work expended will be—

$$\frac{P}{2} \times 2\,l = 50 \text{ lbs.} \times 40' = \mathbf{2000} \text{ ft.-lbs.}$$

Or, work got out $= W \times h = 200 \text{ lbs.} \times 10' = \mathbf{2000}$ ft.-lbs.

In this question we have a combination of the pulley and the inclined plane. The inner ends of the two ropes being fixed at the top of the inclined plane, the force with which the men act on the free ends is communicated throughout the ropes, so that the stress in the ropes on each side of the barrel balances the force P, that would be required to move the barrel up the incline if applied at its centre of gravity.

Or, the *theoretical advantage* due to the pulley part of the system is, . . . $\dfrac{P}{\frac{P}{2}}=\dfrac{2P}{P}=\dfrac{2}{1}$

Then for the inclined plane part we have by the "triangle of forces," or by the "principle of work," a *theoretical advantage* of—

$$\frac{W}{P}=\frac{l}{h}=\frac{20}{10}=\frac{2}{1}$$

Therefore, the *total* theoretical advantage is the *product* of the two separate advantages, viz.—

$$\frac{2P}{P}\times\frac{W}{P}=\frac{2}{1}\times\frac{2}{1}=\frac{4}{1}$$

Consequently, a force of 1 lb. applied at the free end of the rope would balance a weight of 4 lbs. on the incline. Or, as in the question, and, *neglecting friction*, a barrel weighing 200 lbs. requires a pull of 50 lbs. to move it up the inclined plane.

We have simply split up the total advantage in this way to show the student that the combined advantages of the several parts of a compound machine must equal the advantage of the whole. We might have said at once, as we have done before in other cases—

$$\textit{The Theoretical Advantage}=\frac{W}{P}=\frac{2000}{50}=\frac{4}{1}$$

NOTE.—I have this day (Sept. 9, 1892) witnessed the interesting operation of lowering four very large 35-ton steam boilers of the marine type, down an incline of about 100 feet in length by the method described in the foregoing question. *One man*, by aid of an ordinary block and tackle, supplied the requisite restraining force on the free end of the rope.

LECTURE IX.—QUESTIONS.

1. Prove by the triangle of forces (drawn to scale) the relation between the weight W of a body resting on a smooth inclined plane, the reaction, R, from the plane, and the force, P, necessary to just balance the weight—(1) when the force, P, acts parallel to the plane ; (2) when it acts parallel to the base ; (3) when it acts at an angle, θ, to the plane.

2. A ball, weighing 100 lbs., rests on an inclined plane, being held in position by a string which is fastened to a bracket so as to be parallel to the plane. The height of the plane being $\frac{1}{3}$ of the length, find the tension of the string and the pressure perpendicular to the plane. Establish your results by reasoning on known principles, such as the principle of work or that of the parallelogram of forces. *Ans.* P=33·3 lbs., and R=93·3 lbs.

3. Prove the relation between W, P and R, acting on a body resting on a smooth inclined plane by the "*principle of work*" for cases **1**, **2** and **3** in this Lecture. An incline is 1 ft. in vertical height for 15 in length. A weight of 100 lbs. rests on the plane and is held up by friction ; make a diagram for estimating the pressure on the plane, and find its amount. *Ans.* 99·7 lbs.

4. Friction being neglected, find the force, acting parallel to the plane, which will support 1 ton on an incline of 1 ft. vertical and 10 ft. along the incline. Prove the formula which you employ. If the incline were 1 ft. vertical and 280 ft. along the incline, find the force in pounds which would support 1 ton. (S. and A. Exam. 1890.) *Ans.* 224 lbs., and 8 lbs.

5. A smooth incline plane has a vertical side of 1 ft., and a length of 10 ft. ; what work is done in pulling 10 lbs. up 8 ft. of the incline ? *Ans.* 8 ft.-lbs.

6. When a body is raised through a given height, how is the work done estimated ? A body weighing 8 cwt. is drawn along 100 ft. up an incline, which rises 2 ft. in height for every 5 ft. along the incline ; the resistance of friction being neglected, find the work done. *Ans.* 35,840 ft.-lbs.

7. A smooth incline is 8 ft. long, and the total vertical rise from the bottom to the top thereof is 2 ft. What amount of work is performed in drawing a weight of 100 lbs. up 4 ft. of the incline, and what is the least force which will do this work ? *Ans.* 100 ft.-lbs. ; 25 lbs.

8. Friction is neglected, and it is found that a force acting horizontally will move 10 lbs. up 5 ft. of an incline rising 1 in 4. Find the work done, and find also the force parallel to the plane which will just support the weight of 10 lbs. *Ans.* 12·5 ft.-lbs. ; 2 lbs.

9. A car laden with 20 passengers is drawn up an incline, one end of which is 160 ft. above the other ; the car, when empty, weighs 3 tons, and the average weight of each passenger is 140 lbs. Find the number of ft.-lbs. of work done in ascending the incline, neglecting friction. *Ans.* 1,164,800 ft.-lbs., or 520 ft.-tons.

10. It will be observed that draymen sometimes lower heavy casks into cellars by means of an inclined plane and a rope. One end of the rope is secured to the upper end of the inclined plane, and is then passed under and over the cask, the men holding back by means of the loose end. Now, supposing the incline to be at an angle of 45 degrees, explain the mechanical principles that are here applied, and find the advantage. *Ans.* $2\sqrt{2} : 1$.

11. A barrel weighing 5 cwt. is lowered into a cellar down a smooth slide inclined at an angle of 45 degrees with the vertical. It is lowered by means of two ropes passing under the barrel, one end of each rope being fixed, while two men pay out the other ends of the ropes. What pull in lbs. must each man exert in order that the barrel may be supported at any point ? (S. and A. Exam. 1889.) *Ans.* 99 lbs. nearly.

LECTURE X.

CONTENTS.—Friction—Heat is Developed when Force overcomes Friction—Laws of Friction—Apparatus for Demonstrating First and Second Laws of Friction—Experiment I.—Example I.—Angle of Repose or Angle of Friction—Experiment II.—Diagram of Angles of Repose—Limiting Angle of Resistance—Experiment III.—Apparatus for Demonstration of the Third Law of Friction—Experiment IV.—Lubrication—Anti-Friction Wheels—Ball Bearings—Work done on Inclines, including Friction—Example II.—Questions.

Friction.—Whenever a body is forced to slide over another body, an opposing resistance is at once experienced. This natural resistance is termed *friction*.* The true cause of friction is the roughness of the surfaces in contact. The smoother the sliding surfaces are made the less will be the friction. Friction cannot, however, be entirely eliminated by any known means, for even the most microscopical protuberances on the smoothest of surfaces seem to fit into corresponding hollows on other equally smooth places, so that some force is required to make the one body slide over the other.

Friction has its advantages as well as its disadvantages. For example, if it were not for friction we could not walk. Let anyone put on a pair of hard, very smooth-soled boots, and try to walk over a sheet of very smooth ice. He will at once experience the difficulty of doing so with any degree of speed and comfort. Just imagine the boots to be as hard and smooth as the ice, and where would the experimenter be landed? On the other hand, if he put on a rough pair of tacketed boots and walked along a rough road he would soon become tired.

It is the duty of the engineer to reduce friction to a minimum in the case of the bearings of engines, shafting, and machines generally, in order that a minimum of work may be expended in working them. He has, however, also to devise means of producing a maximum of friction in the case of certain pulleys, grips, clutches, brakes, and such like appliances, where motion has to be transmitted by aid of friction, or bodies in motion (such as a

* French writers call friction a *passive resistance*, because it is only apparent when one body tends to move or pass over another.

moving train) have to be brought to rest quickly when nearing a station.

Heat is Developed when Force overcomes Friction.—Whenever a body is set in motion by a force, part (or in certain cases it may be the whole) of the mechanical force is expended in overcoming frictional resistance. This *lost work* is directly transformed into *heat* in the act of overcoming the frictional resistance through a distance. *For example*:—A person slips down a vertical rope by holding it between his hands and his legs. The force of gravity impels him downwards, overcoming the frictional resistance between his hands and limbs and the rope, with the consequence that they become severely heated, especially if he happens to slip down quickly. A boy takes a run, and then slides along a level piece of ice. The foot-pounds of work stored up in him just before he begins to slide are expended partly in overcoming the frictional resistance between the soles of his boots and the ice, and partly in the frictional resistance between his clothes and the air. As a consequence, he will find that by the time he gets to the end of the slide his soles are considerably warmed. If the ice were perfectly level, infinitely long, and if there were absolutely no friction between it and his boots, and if there were no frictional resistance between him and the air, then he would slide on *for ever!* If we could diminish the frictional resistance between the skin of a ship and the water, and between the exposed parts of the ship and the air, *to nothing*, then all that would be required to transport her across the Atlantic would be a strong force applied at the start until she attained the desired speed, when she would proceed forward, and arrive at her destination with undiminished velocity! In reality, however, we find it necessary to employ steam engines of 10,000 horse-power continuously in order to propel an Atlantic "greyhound" of 5000 tons at twenty knots an hour in the calmest of weather. About one-half of this power is absorbed in overcoming the frictional resistance of the ship through the water and air, and the other half in the frictional and other losses due to the working of the propelling machinery. Examples of the conversion of mechanical work into heat are so familiar to you all, being in fact brought prominently before your notice every day of your existence, that we need not further enlarge upon this question except to remind the student of Dr. Joule's discovery of the rate of exchange between heat and work. He found by experiment that if work is transformed into heat, every 772 ft.-lbs. of work will produce 1 heat unit, or that *quantity of heat* which would raise 1 lb. of water 1° Fahr.*

* *For further* examples and an explanation of Dr. Joule's experiments *see the Author's* Treatise on Steam and the Steam Engine.

Laws of Friction.—There are three primary laws of friction :

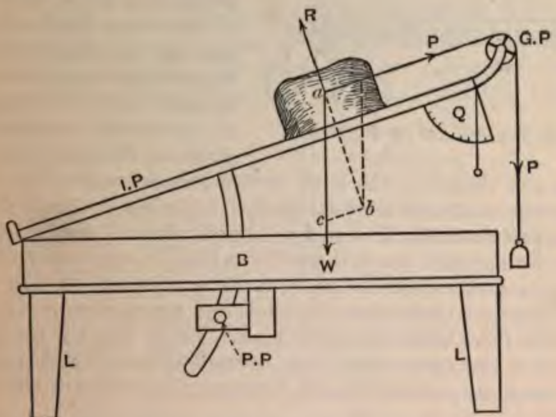
1st Law. *Friction is directly proportional to the pressure between two surfaces, if they remain in the same condition.*

2nd Law. *Friction is independent of the areas in contact.*

3rd Law. *Friction is independent of velocity.*

Almost every law has, however, exceptions, and these are most certainly only true within narrow limits. For considerable variations from the first law take place when the pressure on the surfaces becomes so great that the irregularities on the one body bight into the other, or when a lubricant is squeezed so thin that it cannot be evenly maintained between the surfaces. The second law does not hold good when the areas in contact are greatly altered, and the third law is not reliable at high speeds. In fact, to obtain even a fair knowledge of the variations of friction with alterations of pressure, area, velocity, and different materials, requires a careful study of a vast number of experiments.

Apparatus for Demonstrating First and Second Laws of Friction.—Nevertheless, it will be both interesting and instructive to students to have these three laws demonstrated by the following simple apparatus :—



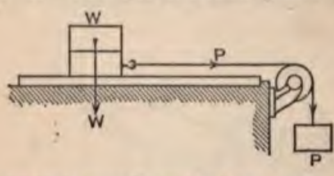
APPARATUS FOR DEMONSTRATING THE FIRST AND SECOND LAWS OF FRICTION.

INDEX TO PARTS.

IP represents *Inclined plane.*	B represents Box for planes.
GP „ *Guide pulley.*	PP „ Pinching pin.
Q „ *Quadrant.*	LL „ Legs.

The inclined plane IP is fitted with a joint at its left-hand end, and after slackening the pinching pin PP, it may be raised to any desired angle or fixed in a level position by tightening the pin. The desired position is found by reading off the angle opposite the plumb-ball line on the graduated degree scale of the quadrant Q. In the box B may be kept planes of glass, brass, iron, steel, &c., as well as the different kinds of wood to be experimented upon. These planes are fixed on IP, in a central position, by means of a catch, and the bodies to be laid upon them should be fitted with a small hook opposite their *c.g.*, to which a fine flexible silk cord can be attached and passed over the guide-pulley GP, which should turn very freely on its bearings.* The pull P is best effected by attaching to this cord a small tin pail into which shot may be dropped one by one until the body moves freely on the plane. The pail and shot may then be unhooked and weighed in a balance.

Demonstrations of the First Law of Friction.—EXPERIMENT I.—Fix the inclined plane IP in a truly *horizontal* position. Take from the box B, say, a long strip of planed yellow pine and a small block of the same kind of wood, and let its weight be W. Adjust the strip along the middle line of IP by means of the sneck or catch, and place the block therein. Attach the silk thread to the hook on the forward side of the block, and pass the same over the practically frictionless pulley. Hang a little tin pail from the free end of the silk thread, and drop small shot one by one into the pail until the block moves freely over the yellow pine strip when aided by a little tapping on the table. Unhook the pail containing the shot, and weigh it as carefully as you weighed the block of yellow pine. Let it equal P units.



PROOF OF FIRST LAW OF FRICTION.

Then P is the force which just overcomes the *directly opposing passive resistance*, called friction, between the surface of the yellow

* The guide-pulley bracket should be fitted with a stiff joint and with a telescope arm, so that the pulley may be raised or lowered in order to bring the direction of the pull P on the cord parallel to the plane, or parallel to its base, or adjusted to any desired angle with respect to the plane, in order to demonstrate Cases 1, 2 and 3 of the inclined plane in Lecture IX. By having, say, a $\frac{1}{2}''$ slot along the middle of the plane, and by lowering the pulley, Case 2, wherein the pull on the body is parallel to the base, may be readily demonstrated; and by pulling out the telescope arm of the bracket, and turning up the bracket, Case 3, wherein the pull *makes an angle, θ,* with the plane, may be verified.

pine block and strip; and the ratio $\frac{P}{W}$ is termed the *co-efficient of friction.* Now put another block of weight W on the top of the one just tested, so as to double the pressure on the sliding surface, and put in shot until the block moves when aided by a little vibration, so as to overcome the greater resistance to starting the body in motion than to keep it moving.* You find on weighing the pail and shot that it is now 2P. Consequently the co-efficient of friction has not altered, for $\frac{2P}{2W}$ is the same fraction as $\frac{P}{W}$.

EXAMPLE I.—Suppose you take a very small block of wood (say $\frac{1''}{10}$ thick, 2″ long and 1″ broad; in fact, so light that its weight is negligible), and place a 1-lb. weight on the top of it; you will find that 5·75 oz. are required to cause motion of this piece of wood over the surface of the yellow pine strip. You therefore conclude that the *co-efficient of friction* is

$$\frac{P}{W} = \frac{5{\cdot}75 \text{ oz.}}{16 \text{ oz.}} = {\cdot}353, \text{ or friction} = {\cdot}353 \text{ W}.$$

Now, place a 2-lb. weight on the upper piece of wood, and you find that it requires more shot in the pail to move it. Weigh the pail and the shot again just after you have obtained free movement of the one body over the other, and you will find that it amounts to 11·5 oz.

Consequently, $$\frac{W}{P} = \frac{11{\cdot}5}{32} = {\cdot}353 \text{ as before.}$$

If, however, you put a 10-lb. weight on the upper piece of wood, you will obtain a different result, thus proving the first law and the variation therefrom; because in this latter case the pressure is so great, compared with the first and second experiments, that the grains of the upper piece of wood enter those of the lower, and bring into play another condition of affairs—viz., the gripping action of the one set of grains on the other set. If you had taken a large plank of yellow pine, weighing, say, 100 lbs., and had placed it on another similar plank, the co-efficient of friction would have a certain value. If you had even put a 100-lb. weight on the upper plank, the co-efficient of friction might not have varied perceptibly. But if you placed a weight of 1000 lbs.

* Statical friction, or the friction of repose, is that resistance which opposes the *commencing* of the motion. If a body be allowed to rest on *another for some time,* it requires more force to move it than if it had only *been stationary for a few seconds.*

on the upper plank, the co-efficient of friction would be considerably altered. Hence you observe that this first law only holds good between narrow limits.*

Angle of Repose, or Angle of Friction.—EXPERIMENT II.—Another way of proving the first law of friction is to disconnect the silk thread and the shot-pail from the upper body, and tilt up the inclined plane to such an angle, a, with the horizontal that (with the aid of a little tapping) the weighted block of yellow pine just slides slowly down the incline. Here we have simply the force of gravity acting on the body and overcoming friction. At the moment the body just begins to slide we have the weight, W, of the body acting vertically downwards, R the reaction from the plane at right angles to the surface, and F, the passive resistance of friction, acting parallel to the plane in the direction of aP in the first figure in this Lecture. Now, these three forces act from the *c.g.* of the body, and they are in equilibrium. R is equal to the resolved part of W at right angles to the plane (or $R = W \operatorname{Cos} a$), and it represents the pressure between the surfaces. P is the resolved part of W, parallel to the plane (or $F = W \operatorname{Sin} a$).

and $\frac{F}{R}$ is the co-efficient of friction.

$$\therefore \frac{F}{R} = \frac{W \operatorname{Sin} a}{W \operatorname{Cos} a} = \operatorname{Tan} a = \frac{h}{b} = \frac{\text{height of plane}}{\text{base of plane}}$$

The angle a, to which the plane must be inclined before the free body will slip over the fixed one, has been termed the "*angle of repose*" or "*angle of friction*."

Therefore, the tangent of the angle of repose is equal to the co-efficient of friction.

But $\frac{P}{W}$ was proved by the previous experiment to be also equal to the *co-efficient of friction*,

$$\therefore \frac{P}{W} = \frac{F}{R} = \frac{h}{b} = \operatorname{Tan} a = \mu$$

Or, $$P = \mu W \quad \text{and} \quad F = \mu R$$

* Sir Robert Stawell Ball, when Professor of Mechanism at the Royal College of Science, Ireland, tried a careful experiment in the above way with a smooth horizontal surface of pine 72″ × 11″, and a slide, also of pine, 9″ × 9″ grain crosswise. He loaded and started the slide, and applied a force sufficient to maintain it in uniform motion, and he found that on increasing the load from 14 to 112 lbs., by increments of 14 lbs., the co-efficient of friction diminished from ·336 to ·262. From these experiments he constructed the empirical formula for this case that $F = \cdot 9 + \cdot 266\ R$, where F is the frictional resistance and R the reaction from the surface or net load.

where μ is the Greek letter universally adopted to represent co-efficients of friction.

The accompanying figure is a diagram of the "*angles of repose*" for various common materials, together with the numerical values of μ, or their co-efficients of friction.

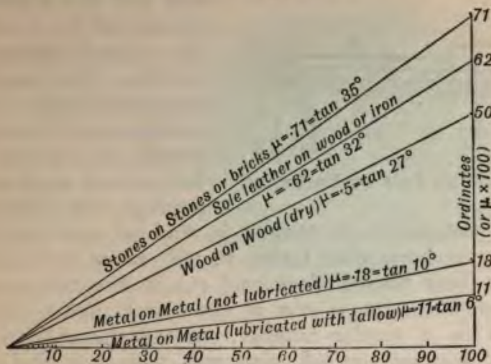


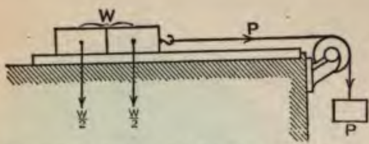
DIAGRAM OF ANGLES OF REPOSE.

Limiting Angle of Resistance, or Sliding Angle.—A third way of proving the first law of friction is to place the bodies so that the sliding surface is perfectly level. Then begin by pressing the upper body through the intervention of a compression spring-balance fitted with a sharp point, so that it will not slip off, and with a clinometer to indicate the angle through which it is tilted away from the perpendicular. Now gradually incline your pressure to the perpendicular, until you arrive at such an angle as will just cause the upper body to slide over the under one. This angle is termed the "*sliding angle*," or "*limiting angle of resistance*," because it is the limit, or maximum angle which the reaction from the surface can make with the perpendicular to the surfaces, for the reaction must act in the directly opposite direction to the pressing force.* Again, apply the spring-balance, but with double the registered pressure, and you can just incline this force to the same angle as before. If, however, you press with ten times the former force, you would probably be able to act at a greater angle than before. It will be seen from this experiment that

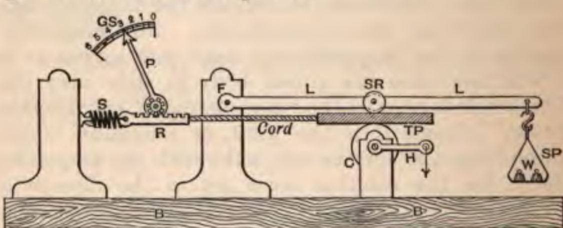
The Limiting Angle of Resistance = The Angle of Repose.

* *Here the weight of the upper body is supposed to be negligible in comparison with the inclined pressure upon it.*

Demonstration of the Second Law of Friction.—EXPERIMENT III.—Take a block of planed yellow pine, and cut it into two equal pieces at right angles to the planed surface. Place one piece on the horizontal strip of yellow pine (used in previously demonstrating the first law), with the planed side next to it, and put the other piece on the top of it, as shown by the second figure in this lecture. Now ascertain the horizontal pull, P, required to overcome friction. Then attach the top piece to the bottom one, as shown by the accompanying figure, so that the area of the surface in contact is doubled, and you will find that the same horizontal force, P, will cause it to move. If you take a long planed block and cut it into ten equal pieces, each of the same size as one of the above pieces, and try the experiment in a similar manner, you will be able to increase the area of contact tenfold, and you will then find that the ratio $\frac{P}{W}$ is not exactly the same with the surface of one block in contact with the strip, as when the surface of the whole ten came into action at once. The result of increasing the area in contact may also be tried by placing the blocks on the inclined plane, and observing the angle to which the plane is tilted when they begin to slide down the plane.



PROOF OF SECOND LAW OF FRICTION.



APPARATUS FOR DEMONSTRATING THE FIRST AND THIRD LAWS OF FRICTION.

INDEX TO PARTS.

L represents	Lever.	H represents	Handle.
F ”	Fulcrum.	R ”	Rack.
SR ”	Small roller.	S ”	Spiral spring.
SP ”	*Scale-pan.*	P ”	Pointer.
TP ”	*Test-piece.*	GS ”	Graduated scale.
C ”	*Cylinder.*	B ”	Base of apparatus.

Demonstration of the Third Law of Friction.—The preceding figure represents the apparatus belonging to the Applied Mechanics Department of the Royal College of Science, South Kensington (as described by Prof. Goodeve in his "Manual of Applied Mechanics"), for demonstrating the first and third laws of friction.

If the weights, W, be removed from the scale-pan SP, then there will be but a slight pressure between the lower surface of the test-piece TP, and the roller cylinder C. Consequently, on turning the handle H in the direction of the arrow, there will be a slight pull on the cord, causing the pointer P to move a degree or two over the graduated scale GS. The pointer should therefore be set back to zero.

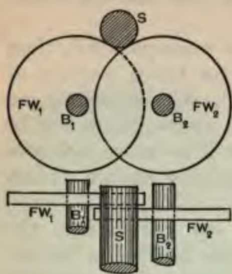
EXPERIMENT IV.—Put a weight, W, of say 5 lbs., into the scale-pan, and turn the cylinder slowly by the handle as before. The pointer deflects so many degrees. Increase the weight W to 10 lbs., and the pointer instantly indicates twice the amount of friction; put in 15 lbs., and it shows treble the friction; thus demonstrating the first law. Then turn the handle faster and faster, and the pointer remains fairly stationary, thus proving within certain limits that friction is independent of the velocity.*

Lubrication.—Lubricants, such as tallow, grease, soft soap, and many kinds of oils, are used to reduce friction. Both skill and knowledge are required to decide upon the best kind of lubricant and the proper amount for different cases. Lubrication and lubricants should receive greater attention from the engineer, for the satisfactory working and length of life of most machines depend so largely upon effective lubrication. Where very heavy pressures and high speeds are experienced as in some cases of electrical machinery, it pays to use the very best kind of oil, and to distribute it to all the bearings from one common centre under pressure by means of a force-pump. It thereby flows in a continuous stream through the bearings to a filtering tank, from which it is again and again pumped on its soothing mission for months on end, without change or great loss in quantity. This is a very different state of matters from the "travelling oil-can" system, where the amount applied may vary, and the times of application may be erratic, according to the opinion of the attendant.

Anti-Friction Wheels.—In the case of delicate machinery, such as in Atwood's machine for ascertaining by experiment the acceleration of gravity, and in Lord Kelvin's mouse-mill for driving the paper rollers of his Syphon Recorder, when receiving

* *See Molesworth's* Pocketbook of Engineering Formulæ, and the *Transactions of the Institution of* Mechanical Engineers, for results of friction *experiments with shafts* run at different speeds.

telegraphic signals from long submarine cables, anti-friction wheels are used for the purpose of reducing the friction to a minimum. The accompanying figures illustrate one pair of anti-friction wheels. The spindle S, which carries the driving-wheel, instead of resting on two ordinary bearings, is supported by two wheels at *each* end, so that a rolling contact is produced between it and the wheels. This form of contact implies far less friction to begin with, than a sliding or scraping contact. Besides, the small amount of force required to overcome the friction between the spindle and the rims of these wheels, has a great advantage or leverage given to it, in as far as, it acts with an arm equal to the radius of the wheels FW_1 and FW_2. This enables it to turn them with great ease at a slow rate in the very small bearings B_1 and B_2.



ANTI-FRICTION WHEELS.

In merely overcoming friction at a bearing, there is a considerable advantage in using large pulleys; for, the force necessary at the periphery of the pulley to overcome the friction at the bearing, is inversely proportional to the radius or diameter of the pulley. (See Lecture XI. fig. 1).

Ball Bearings.—Another example of the effect of rolling contact reducing friction is found in the use of ball bearings, which are now so common in all kinds of cycles and in high-class foot-driven lathes.*

When it is necessary to move heavy beams, guns, &c., a common practice is to place them on rollers or on two channel iron girders with round cannon-shot between them, when a comparatively small force, properly applied, will have the desired effect.

We will have to return to this subject in the Advanced Course when dealing with the friction between shafts and their bearings, and the various means that have been adopted for minimising the same. In the meantime, we will complete this Lecture with an example of work done on an incline when friction is included.

Work done on Inclines, including Friction.—The method of calculating the work expended in moving a body along a *smooth* inclined plane was fully dealt with in Lecture IX.; consequently, the student is prepared, after what has been said about friction in

* Refer to the index for the page where the illustration is to be found of *the ball bearings* for the screw-cutting lathe, or to Lecture XVI.

this Lecture, to consider the case of pulling a body up or down a plane when the co-efficient of friction between the body and the plane is known.

The *total work* expended is evidently divisible into two distinct portions—

(1) The work done *with* or *against the action of gravity*, according as the body is moved down or up the inclined plane $= W \times h$ (where h is the height of the plane).

(2) The work done *against friction* $= F \times l$ (where l is the length of plane passed over).

The work to be done against friction is the same whether the body is urged up or down the incline; for it is equal to the co-efficient of friction × the reaction from the plane × the distance through which it is moved.

Or, $\qquad F \times l = \mu \times R \times l$

But by Lecture IX. $\quad R \times l = W \times b$; $\therefore F \times l = \mu \times W \times b$

Or, the work done against friction in moving a body along the inclined distance l, is equal to the work done in moving the same body along a horizontal distance b, equal to the base of the incline.

If the work to be done in overcoming friction, is *equal* to the work capable of being done on the body by *gravity*, the body will be in equilibrium, and the inclination of the plane is equal to the angle of repose.

If the work to be done in overcoming friction is *less than* the work which gravity can do on the body, the body will slide down the incline, or, in technical language, the machine will overhaul.

EXAMPLE II.—What is the co-efficient of friction, and how is it ascertained? There is an inclined plane of 1 foot vertical to 10 feet horizontal; what work is done in moving 700 lbs. 5 feet along the plane, the co-efficient of friction being ·08? (S. and A. Exam. 1892.)

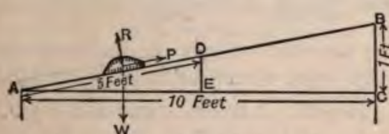


FIGURE FOR EXAMPLE II.

ANSWER.—The co-efficient of friction for two bodies in contact *is the passive resistance* (*opposing the motion of the one over the other*) *divided by the reaction or normal pressure between the surfaces in contact*—

i.e., $\qquad$ Co-efficient of friction $= \dfrac{\text{Friction}}{\text{Reaction}} = \dfrac{F}{R} = \mu$

For methods of *ascertaining* co-efficients of friction, see the text in this *Lecture*.

Total work done = work done against gravity + work done against friction.

Referring to the accompanying figure, we see that—

(1) Work done against gravity $= W \times DE$

(2) Work done against friction $= F \times AD$

$$\therefore \text{ Total work done } = W \times DE + F \times AD$$

We have therefore only to interpolate the numerical values corresponding to these letters in order to arrive at the result. From the question W = 700 lbs. From the figure we see that DE is parallel to BC; consequently by Euclid the $\triangle^s$, ADE, and ABC are similar in every respect; and therefore

$$DE : BC :: AD : AB; \text{ or, } DE = \frac{BC \times AD}{AB}$$

But, also by Euclid, $AB = \sqrt{AC^2 + BC^2} = \sqrt{10^2 + 1^2} = 10{\cdot}05$ ft. (nearly)

Consequently, $DE = \dfrac{BC \times AD}{AB} = \dfrac{1 \times 5}{10{\cdot}05} = {\cdot}497$ ft.

And, $F = \mu R$

From the question we are told that $\mu = {\cdot}08$, and we learn from Lecture IX. that

$$R : W :: AC : AB; \text{ or, } R = \frac{W \times AC}{AB} = \frac{700 \times 10}{10{\cdot}05} = 696{\cdot}5 \text{ lbs.}$$

$$\therefore F = \mu R = {\cdot}08 \times 696{\cdot}5 = 55{\cdot}72 \text{ lbs.}$$

Hence Total Work $= W \times DE + F \times AD$

" " " $= 700 \times {\cdot}497' + 55{\cdot}72 \times 5'$

" " " $= 347{\cdot}9$ ft.-lbs. $+ 278{\cdot}6$ ft.-lbs.

" " " $=$ **626·5 ft.-lbs.**

NOTE.—For *the work done against friction* quite a simple way would have been to have taken the formula deduced on the previous page—

viz.: $F \times l = \mu \times W \times b = \mu \times W \times AE = {\cdot}08 \times 700 \times 4{\cdot}97 =$ **278·6** ft.-lbs.

For, $\dfrac{AE}{AC} = \dfrac{DE}{BC}$ $\therefore AE = \dfrac{AC \times DE}{BC} = \dfrac{10 \times {\cdot}497}{1} = 4{\cdot}97$.

APPROXIMATE ANSWER.—Since the inclination of the plane is so very small in this case, we might have assumed that

$$R = W; \; AB = AC, \text{ and } DE = \tfrac{1}{2}BC$$

Then,

(1) Work done against gravity $= W \times DE = 700 \times \frac{1}{2}' =$ **350 ft.-lbs.**

(2) Work done against friction $= F \times AD = {\cdot}08 \times 700 \times 5 =$ **280 ft.-lbs.**

$\therefore$ Total work $= W \times DE + F \times AD = 350 + 280 =$ **630 ft.-lbs.**

LECTURE X.—QUESTIONS.

1. What is friction, and how does it act? What is developed when force overcomes friction? How do you measure the result?

2. Explain by sketches and concise description how the laws of friction may be tested experimentally. What is meant by the "co-efficient of friction," "angle of repose," "angle of friction," and "sliding angle" or "limiting angle of resistance"?

3. How is the co-efficient of friction between two surfaces ascertained approximately by experiment? When two rough surfaces are pressed together, how much may the line of pressure be inclined to the common perpendicular to the surfaces in contact before motion ensues? (S. and A. Exam. 1889.)

4. What is the co-efficient of friction when the angle of repose is—(*a*) $4° 42'$; (*b*) $11° 18'$; (*c*) $16° 42'$; (*d*) $21° 48'$; (*e*) $26° 36'$: (*f*) $30°$; (*g*) $45°$? Draw the angles to scale. *Ans.* (*a*) ·1; (*b*) ·2; (*c*) ·3; (*d*) ·4; (*e*) ·5; (*f*) ·5774: (*g*) 1.

5. An inclined plane is 100 feet long and 20 feet high. A body weighing 100 lbs. is pulled up from the bottom to the top, and then down again. If the co-efficient of friction between the body and the plane is ·5, what work was expended in each case? What would require to be the co-efficient of friction in order that the body might just slide down of its own accord? *Ans.* 6,900 ft.-lbs.; 2,900 ft.-lbs.; $\mu = \frac{h}{b} = \frac{\sqrt{6'}}{12} = \cdot 204$.

6. What is the co-efficient of friction, and how is it ascertained? There is an inclined plane of 1 foot vertical to 5 feet horizontal; what work is done in moving 100 lbs. through 100 feet along the plane, the co-efficient of friction being ·1? *Ans.* 2,980 ft.-lbs.

7. An incline is 80 feet long, with a rise of 20 feet. A body weighing 100 lbs. is drawn 40 feet along the incline; what work is expended if the co-efficient of friction is ·6? *Ans.* 3,320·8 ft.-lbs.

8. A weight of 5 cwts. resting on a horizontal plane requires a horizontal force of 100 lbs. to move it against friction. What is the co-efficient of friction? *Ans.* ·18.

9. A plank of oak lies on a floor with a rope attached to it. When the rope is pulled horizontally with a force of 70 lbs. it just moves, but when pulled at an angle of 30° to the floor a force of 60 lbs. moves it. What is the weight of the plank and the co-efficient of friction between it and the floor? *Ans.* 116·6 lbs.; ·6.

10. Suppose a locomotive weighs 30 tons, and that the share of this weight borne by the driving wheel is 10 tons. Then, if the co-efficient of friction between the wheels and the rails be ·2, what load will the engine draw on the level if the required co-efficient of traction be 10 lbs. per ton of train load? What load will this engine draw at the same rate up an incline of 1 in 20? *Ans.* 448 tons (including engine); 36·72 tons (including engine).

11. State the laws of friction, and explain the contrivance known as *friction wheels*. What is the advantage of ball bearings for bicycles? Sketch in section such a bearing. (S. and A. Exam. 1891.)

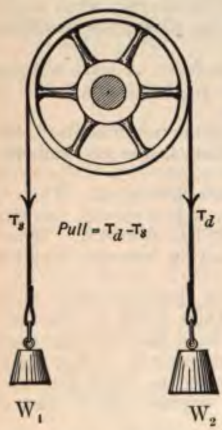
12. What are lubricants, and for what purposes are they used in machinery? What kind of lubricant would you use for the moving parts of a very high-speed engine and direct-driven dynamo, and how would you apply it so as to be able to use it over and over again?

LECTURE XI.

CONTENTS.—Difference of Tension in the Leading and Following Parts of a Driving Belt—Brake Horse-Power transmitted by Belts—Examples I. II.—Velocity Ratios in Belt Gearing—Examples III. IV.—Open and Crossed Belts—Fast and Loose Pulleys—Belt Gearing Reversing Motions—Stepped Speed Cones with Starting and Stopping Gear—Driving and Following Pulleys in Different Planes—Shape of Pulley Face—Questions.

WE shall devote this Lecture to the transmission of power by belting and to belt-gearing.

Difference of Tension in the Leading and Following Parts of a Driving-Belt.—In Lecture VI., when discussing the case of the simple pulley, we assumed that the belt or rope passing over the pulley was perfectly flexible, and that there was no friction at the axle of the pulley. Consequently, we found that equal weights would balance each other, or that the tension of the two sides of the belt were equal. A little consideration of the subject will show that when one pulley is driven from another one by an endless belt or rope, the tension on the driving side must be greater than that on the following side.



DIFFERENCE OF TENSION DUE TO FRICTION.

1. Take the case of an ordinary vertical pulley with its axle or shaft resting in two bearings (one on each side of the pulley), with a belt or rope passed over it, and with weights attached to the free ends of the same. Here we must have a certain amount of friction between the axle and its bearings, which can only be overcome by a force applied to the circumference of the pulley.

Let . $F_1 = \left\{ \text{Force required to overcome friction at the circumference of the axle or shaft.} \right.$

Let . . $r_1 =$ Radius of the axle.

" . . . $F_2 = \left\{ \begin{array}{l} \text{Force required to overcome the friction of the axle} \\ \text{when acting at the circumference of the pulley.} \end{array} \right.$

" . . . $r_2 =$ Radius of the pulley to centre of belt.

Then, $F_1 \times r_1 = F_2 \times r_2$.

Let . . $W_1 = \left\{ \begin{array}{l} \text{Weight attached to the left-hand side of the belt,} \\ \text{and which therefore produces a tension on the} \\ \text{slack side} = T_s. \end{array} \right.$

" . . . $W_2 = \left\{ \begin{array}{l} \text{Least weight that can be attached to the right} \\ \text{hand side of the belt, and which therefore pro-} \\ \text{duces a tension on the driving side} = T_d. \end{array} \right.$

Then taking moments about the centre of the axle, we have—

$$W_1 \times r_2 + F_2 \times r_2 = W_2 \times r_2$$

Or, . . $$T_s \times r_2 + F_2 \times r_2 = T_d \times r_2$$

Dividing both sides of the equation by r_2 we get

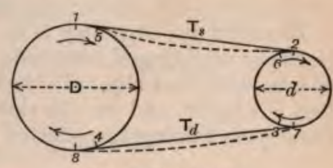
$$T_s + F_2 = T_d$$

$$\therefore \; F_2 = T_d - T_s$$

Or, expressed in words, the force F_2, acting at the circumference of the pulley (which is required to overcome the friction of the axle) is equal to the tension T_d on the driving or forward side of the pulley, *minus* the tension T_s on the slack or following side.

In order that the periphery of the pulley may move at the same rate as the under face of the belt, we must have sufficient tension on each part of the latter, and the co-efficient of friction between them must not have less than a certain value. Too great adhesion between them would result in a loss of work, for in that case an extra force would have to be applied solely for the purpose of pulling the belt from the pulley.

2. Take the case of one vertical pulley of diameter D, driving another vertical pulley of diameter d by means of an endless belt, rope, or chain in the direction of the arrows shown on the accompanying figure. Whenever the pulley D is moved, the tension on the driving side T_d tends to stretch the belt on that side, and this tension increases until the pulley d begins to move; whereas the tension on the following or slack side, T_s, is gradually diminished until the difference of the tensions $(T_d - T_s)$ produces a uniform velocity of the belt. Of course the tension on the slack side must be sufficient to pre-



BELT DRIVING.

vent the slipping of the belt on either of the pulleys if the periphery of the driven pulley is to keep pace with the periphery of the driving one. In order that there may be a minimum chance of the belt slipping, its *slack side* should always run *from the top side of the driving pulley*. By so arranging the drive, the sag of the belt on the slack side will cause it to encompass a greater length of the circumferences of both pulleys. The motion of the belt will be easier, and the wear and tear of the bearings will be less, because there will be less total stress (T_d+T_s) tending to draw the pulleys together for the transmission of a certain horse-power, than if the slack side left the under side of the driving-pulley. Referring to the previous figure, if the slack side leaves the top side of the pulley D, it grips the same from position 4, round the back of the pulley to 5, and the pulley d from 6 round to 3; whereas, if D were rotated in the opposite direction, we should have the slack side entering on it at 1, and only gripping it as far as position 8; entering on d at 7, and only gripping it to position 2, thus having far less grip on the pulleys and thereby encouraging the natural tendency of the belt to slip on the pulleys.*

Brake Horse-power transmitted by Belts.

Let . . $V =$ Velocity of belt in feet per minute.

" . . $P = (T_d - T_s)$ the net pull causing motion in lbs.

Then, $\text{B.H.P.} = \dfrac{VP}{33{,}000} = \dfrac{V(T_d - T_s)}{33{,}000}.$

Let . . $D =$ Diameter of driving pulley in feet $= 2r$.

Then . $\pi D =$ Circumference of driving pulley in feet $= 2\pi r$.

Let . $n =$ Number of revolutions of pulley per minute.

Then . $V = \pi D n = 2\pi r n =$ velocity of belt (with no slip).

And, the . . $\text{B.H.P.} = \dfrac{\pi D n P}{33{,}000} = \dfrac{2\pi r n P}{33{,}000}$

Example I.—A pulley 6′ in diameter is driven at 100 revolutions per minute and transmits motion to another pulley by means of a belt without slip. If the tension on the driving side of the belt is 120 lbs. and on the slack side 20 lbs., what is the brake horse-power being transmitted?

* The previous figure should have been drawn with the full and dotted lines at T_s, reversed, but the student will easily follow the explanation.

ANSWER.—Here $r = 3'$; $n = 100$; $P = (T_d - T_s) = (120 - 20) = 100$lbs.

$$\therefore \text{B.H.P.} = \frac{2\pi r n P}{33{,}000} = 2 \times \frac{\frac{22}{7} \times 3 \times 100 \times 100}{33{,}000} = 5{\cdot}71$$

EXAMPLE II.—What must be the number of revolutions per minute of a driving pulley 6′ in diameter, in order that it may transmit 5·71 B.H.P. by a belt to another pulley, if the net pull on the belt is 100 lbs.?

ANSWER.—Here we have the same data to go upon as in Example I., except that we are given the B.H.P. instead of the revolutions per minute. Then, transposing every quantity except n (the revolutions per minute) to one side of the above equation, we have

$$n = \frac{(\text{B.H.P.}) \times 33{,}000}{2\pi r P} = \frac{5{\cdot}71 \times 33{,}000}{2 \times \frac{22}{7} \times 3 \times 100} = 100 \text{ revolutions.}$$

In precisely the same way, if you were given the power to be transmitted, the revolutions per minute, the difference of tension on the two sides of the belt, and you were asked for the diameter of the pulley, the formula would appear thus—

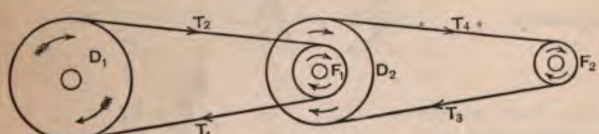
$$D = \frac{(\text{B.H.P.}) \times 33{,}000}{\pi n P}$$

If it was the difference of tension in the belt that was asked for, then—

$$(T_d - T_s) = P = \frac{(\text{B.H.P.}) \times 33{,}000}{\pi D n}$$

You would (after arranging the formula in this way, so as to keep the unknown quantity on one side of the equation) simply have to interpolate the numerical values corresponding to the different symbols, and then cancel out the figures in numerator and denominator, in order to reduce the long multiplication and division to a minimum, and thereby arrive at the result as quickly as possible.

Velocity Ratios in Belt Gearing.—Let two or more pulleys be connected by belting in the manner shown by the accompanying figure. Then, if there is no slipping of the belts, the circum-



VELOCITY RATIOS IN BELT GEARING.

ferential speeds of the pulleys will be the same as the velocity of the belts passing round them.

Let D_1, D_2 = Diameters of the drivers.
„ F_1, F_2 = Diameters of the followers.
„ N_{D_1}, N_{D_2} = Number of revolutions per minute of the drivers.
„ N_{F_1}, N_{F_2} = Number of revolutions per minute of followers.

Then, taking the first pair of pulleys, D_1 and F_1 we have—
Circumferential speed of driver 1 = *Circumferential speed of follower* 1.

i.e. . . . . . $\pi D_1 N_{D_1} = \pi F_1 N_{F_1}$

(÷ both sides by π)

$$D_1 N_{D_1} = F_1 N_{F_1} \quad \therefore N_{D_1} = \frac{F_1 N_{F_1}}{D_1}$$

Or, $\left.\begin{array}{l}\textit{The product of the diameter of}\\ \textit{the driver and its number of}\\ \textit{revolutions per minute.}\end{array}\right\} = \left\{\begin{array}{l}\textit{The product of the dia-}\\ \textit{meter of the follower}\\ \textit{and its number of re-}\\ \textit{volutions per minute.}\end{array}\right.$

Or, ... $\dfrac{D_1}{F_1} = \dfrac{N_{F_1}}{N_{D_1}}$ . . . (1)

i.e., The ratio of the diameters of the pulleys is in the inverse ratio of their speeds or revolutions per minute.

Treating the motion of the second set of pulleys in *exactly* the same way, we have—

Circumferential speed, of D_2 = circumferential speed, of F_2

$$\therefore \pi D_2 N_{D_2} = \pi F_2 N_{F_2}$$

(÷ both sides by π)

$$D_2 N_{D_2} = F_2 N_{F_2}$$

Or, ... $\dfrac{D_2}{F_2} = \dfrac{N_{F_2}}{N_{D_2}}$ . . . . . (2)

(But the revolutions of F_1 and of D_2 are the same) $\therefore N_{F_1} = N_{D_2}$

Or, . . . $D_2 N_{F_1} = F_2 N_{F_2} \quad \therefore N_{F_2} = \dfrac{D_2 N_{F_1}}{F_2}$

Consequently . . $\dfrac{N_{D_1}}{N_{F_2}} = \dfrac{\dfrac{F_1 N_{F_1}}{D_1}}{\dfrac{D_2 N_{F_1}}{F_2}}$

$\left\{\begin{array}{l}\text{(Dividing both numerator}\\ \text{and denominator by } N_{F_1})\end{array}\right\}$ $\dfrac{N_D}{N_{F_2}{}^1} = \dfrac{\dfrac{F_1}{D_1}}{\dfrac{D_2}{F_2}} = \dfrac{F_1 \times F_2}{D_1 \times D_2}$

Or, we might have arrived at the same result by multiplying equations (1) and (2) together. Thus—

$$\frac{D_1}{F_1} \times \frac{D_2}{F_2} = \frac{N_{F_1}}{N_{D_1}} \times \frac{N_{F_2}}{N_{D_2}}; \text{ but } N_{F_1} = N_{D_2} \therefore N_{F_2} = N_{D_1}\left(\frac{D_1 \times D_2}{F_1 \times F_2}\right)$$

Or, $\dfrac{\textit{Speed of first driver}}{\textit{Speed of last follower}} = \dfrac{\textit{Product of diameters of followers}}{\textit{Product of diameters of drivers}}$

Or, . $N_{D_1} \times D_1 \times D_2 = N_{F_2} \times F_1 \times F_2$

i.e., $\left.\begin{array}{l}\textit{Speed of first driver} \times \textit{dia-}\\ \textit{meters of the drivers}\end{array}\right\} = \left\{\begin{array}{l}\textit{Speed of last follower} \times \textit{diame-}\\ \textit{ters of the followers.}\end{array}\right.$

In the same way we may treat any number of drivers and followers by this general formula—viz.,

$\left.\begin{array}{l}\textit{Speed or number of revolutions}\\ \textit{per minute of the first driver}\\ \times \textit{the successive diameters of}\\ \textit{the drivers}\end{array}\right\} = \left\{\begin{array}{l}\textit{Speed of the last follower} \times \textit{the}\\ \textit{successive diameters of the}\\ \textit{followers.}\end{array}\right.$

Precisely the same rule holds good for discs driven by contact friction and for wheel gearing, as you will find from the next lecture; but in friction gearing and wheel gearing the driver and the follower move in different directions, whereas in belt gearing they move in the same or in the opposite direction, according as the driving belts are "open" or "crossed."

EXAMPLE III.—Referring to the previous figure, suppose that a driving pulley, D_1, is connected by a belt to a follower, F_1, and drives the latter at 100 revolutions per minute. If the diameter of the driver is 6′ and of the follower 3′, what will be the number of revolutions *per minute* of the follower?

By the previous formula for two pulleys,

$$D_1 \times N_{D_1} = F_1 \times N_{F_1}$$

$$\therefore N_{F_1} = \frac{D_1 \times N_{D_1}}{F_1} = \frac{6' \times 100}{3'} = \mathbf{200} \text{ revolutions.}$$

EXAMPLE IV.—Referring to the previous figure, suppose that a driving pulley D_1 (4′ diameter), is geared to a follower, F_1 (2′ in diameter), and that a second driver D_2 (4′ diameter), fixed to the same shaft as F_1, is geared to a second follower F_2 (1′ diameter). If D_1 makes 60 revolutions per minute, what is the speed of F_2?

By the previous formula for four pulleys,

$$N_{D_1} \times D_1 \times D_2 = N_{F_2} \times F_1 \times F_2$$

$$\therefore N_{F_2} = \frac{N_{D_1} \times D_1 \times D_2}{F_1 \times F_2}$$

$$N_{F_2} = \frac{60 \times 4 \times 4'}{2' \times 1'} = \mathbf{480} \text{ revolutions.}$$

Open and Crossed Belts.—By referring to the next figure, the student will observe that the left-hand end view shows what

is termed an open belt, OB, and that the right hand end view shows a crossed belt, CB. In the case of open belts, the driver and the follower rotate in the *same* direction (as may be seen from the second and third figures in this Lecture); whereas, with crossed belt driving, the follower revolves in the *opposite* direction to that of the driver, just as it does when direct friction or wheel-gearing is used.

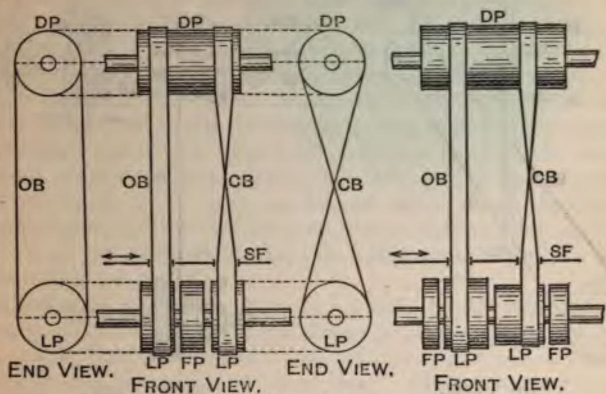
Fast and Loose Pulleys.—As will be seen from the two front views in the next set of illustrations, the open and the crossed belts are shown passing from the broad driving-pulleys DP, to the broad loose pulleys LP. Loose pulleys are generally bushed with gun-metal, and then bored out so as to fit their shafts easily. This permits them to rotate without turning the shaft upon which they bear. The pulleys, FP, are keyed hard on to the shafts, so that when the belt is forced over upon them by means of the shifting forks, SF, the machines connected with the same are set agoing. This simple combination of fast and loose pulleys therefore enables a machine to be stopped or started at pleasure, without interfering with the motion of the driving pulley and the belt. In ordinary cases where there is only one driving belt required, the loose pulley is of the same breadth as the fixed pulley.*

Belt-Gearing Reversing Motions.—In many kinds of machine tools it is desirable to be able to drive the tool first in one direction and then in the opposite direction, as well as to start or stop it. This is frequently effected by a combination of open and crossed belts with fast and loose pulleys, as illustrated by the accompanying figure.

From what has just been said about open and crossed belts, as well as fast and loose pulleys, the student will have no difficulty in understanding this arrangement of reversing gear. If applied to a machine for planing metals, the shaft which is keyed to the fixed pulley FP would be connected either through wheel gearing and a rack, or through a central screw, to the travelling table of machine upon which the job to be acted upon is secured. Whenever the table had been moved backwards to the end of the required stroke by the crossed belt, the shifting fork SF would be pushed forward by an outstanding arm or kicker attached to

* See the set of figures *after* the next, where B_1 is the driving belt, engaging the fixed pulley, FP; and where LP is the loose pulley, to which the belt may be shifted by means of the shifting-fork, SF, whenever it is desirable to stop the speed cones, SC_1, SC_2, and the machine to which they are connected. In the first front view of the *next* set of figures both of the loose pulleys, LP, are drawn too narrow. They should have been represented *half as* wide again, in order to prevent the belts slipping over the *outside edge,* when the other belt is shifted on to the fixed pulley situated *between them.*

the side of the table at such a position as would cause the crossed belt CB to be shifted from the central fixed pulley to its loose one, and at the same time bring over the open belt from its loose pulley to the central fixed one. Whenever the planing tool had finished its cut on the metal, the shifting fork would be pulled backward by another similar outstanding arm or kicker (also attached to the travelling table of the planing machine, at a position just beyond the end of the required stroke for the particular job under operation), thereby shifting the open belt OB from FP, to its loose pulley, LP, and at the same time pulling over the crossed belt, CB,



Forward and Return at same Speed. Quick Return.

BELT GEARING REVERSING MOTIONS.

INDEX TO PARTS.

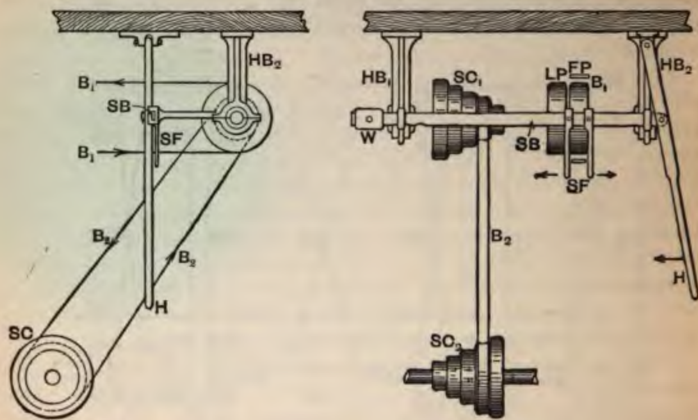
DP	represents	Driving pulleys.	OB	represents	Open belts.
FP	"	Fixed pulleys.	CB	"	Crossed belts.
LP	"	Loose pulleys.	SF	"	Shifting forks.

from its loose pulley to the central fixed pulley, thus causing the table to make the return stroke.

The left-hand front view, with its accompanying end views, show the necessary arrangements when the forward and backward velocities of the table are equal. The right-hand front view illustrates the case wherein the backward or non-planing motion is intentionally made quicker than the forward or cutting stroke, so as to save time, by having the back motion fixed pulley, FP, and its corresponding loose one, LP, made smaller than the forward set. The end views for this latter case would be similar to the former one, with the exception that the crossed belt would engage a smaller

pulley of the same size as shown by the front view. This latter arrangement can evidently be employed to obtain a fast or a slow motion *in the same direction*, by simply having both belts open or both crossed.

Stepped Speed Cones with Starting and Stopping Gear.—In many machines, such as lathes, planers and other machine tools, it is very desirable not only to be able to start and stop them, but also to alter their speed so as to suit different classes of



END VIEW. SIDE VIEW.

STEPPED SPEED CONES WITH STARTING AND STOPPING GEAR.

INDEX TO PARTS.

HB_1, HB_2	represent	Hangingbrackets for supporting shaft, &c.	B_1, B_2	represent	Belts.
			H	,,	Handle.
			SB	,,	Sliding bar.
SC_1, SC_2	,,	Speed cones.	SF	,,	Shifting fork.
FP	,,	Fast pulley.	W	,,	Weight to fix SB in positions ← →.
LP	,,	Loose pulley.			

work, without affecting the motion of the prime motor or that of the shop driving-shaft. These objects are generally attained by a combination of fast and loose pulleys with what are termed "stepped speed cones." The accompanying side and end views illustrate the arrangement as usually carried out in engineering works. When the starting-handle, H, is turned to the right hand, it pulls over the sliding-bar, SB, with its shifting-fork, SF, which moves the belt, B_1, from the loose pulley, LP, tc the fixed one, FP; thus setting the speed cones, SC_1, SC_2, and thereby the

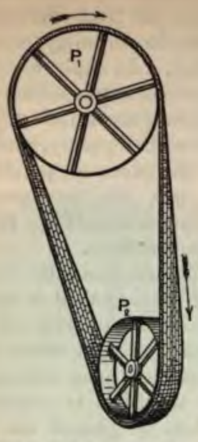
machine in motion. When the handle is turned to the left, it pushes the sliding-bar and shifting-fork also in that direction, thus moving the belt from the fixed to the loose pulley, which allow the cones and machine to come to rest. In each case the weight, W, causes a notch in the sliding-bar to engage with its left-hand supporting bracket, thereby preventing the shifting fork from pushing the driving belt too far, or off either pulley, and at the same time ensuring that it remains in the desired position. Both supporting brackets for the sliding-bar, SB, are merely right-angle extensions from the hanging brackets, HB_1, HB_2, which carry the upper shaft with its cone and pulleys.

The upper and lower speed cones, SC_1, SC_2, are generally made of the same size and shape, but they are always keyed to their respective shafts in opposite directions. Consequently, if it should be desirable to run the machine fast for light work, the belt, B_2, is shifted on to the largest pulley of the upper cone and the smallest one of the lower cone. If the machine is required to move slowly for heavy cuts, then the belt is placed on the smallest upper pulley and the largest lower one. Any desired intermediate speed between these extremes is obtained by adjusting the belt on one or other of the remaining sets of pulleys of the upper and lower cones.

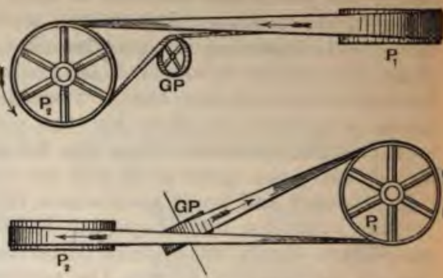
The student can easily prove to himself (by drawing down the arrangement to scale) that such stepped speed cones, if connected by a *crossed* belt on one pair of its pulleys, will produce the same tension in the belt with any other pair.* With open belt-driving the tightness of the belt will not be the same when on one pair of the pulleys as when on another; but the difference is so small that it can generally be disregarded in practice without having recourse to tightening or slackening the same.

Driving and Following Pulleys in Different Planes.—It is often necessary to drive a follower placed in a different plane from the driver. The accompanying set of illustrations show very clearly how this is effected. The important precaution to be observed is, that the leading or on-going part of the belt *must* enter upon the follower in a *fair or direct line with its plane of rotation.* If this rule be attended to, then power may be transmitted between two non-parallel shafts, as shown by the first figure, even if their centre lines are in planes at right angles to each other—*i.e.*, when the belt is working with quarter-twist. When two shafts are in planes at right angles to each other, and

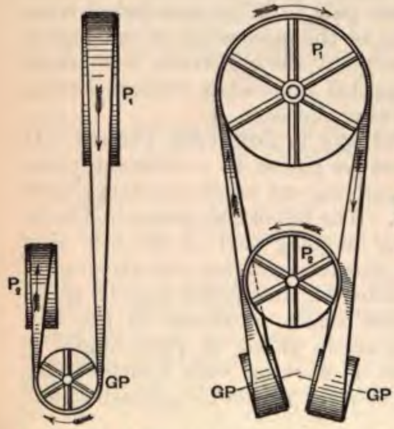
* The algebraical proof of this will be considered in our "Advanced Book on Applied Mechanics." The student should refer to the general view and to the detail drawings of the stepped speed cones in the foot-driven screw-cutting lathe illustrated in Lecture XVI.



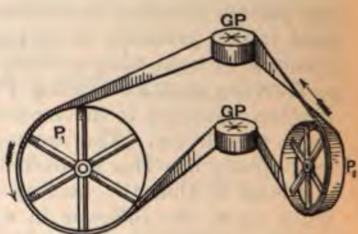
Tullis's Thick-sided Leather Chain Belt, Working Quarter-twist, and Transmitting Power between two shafts which are not parallel. No Guide Pulleys are required for this drive.



Flat Belt Working Quarter-twist and Transmitting Power between two right-angled shafts, with leading Guide Pulley (GP) to remove the twist from the Belt before it enters upon the Follower, and to give the belt more grip on the pulleys.



Flat Belt Transmitting Power between two parallel shafts not in the same plane aid of guide pulleys (GP).



Flat Belt Transmitting Power over Guide Pulleys between two non-parallel shafts in the same plane.

it is found desirable to remove the twist from the belt before it enters upon the follower, then a guide-pulley, GP, must be used as shown by the second figure. When the shafts are parallel, but not in the same plane, then the power must be transmitted by aid of two guide-pulleys, as seen from an inspection of the third figure. Or, should the shafts not be parallel, but in the same plane, two guide-pulleys are necessary, as in the fourth figure. Guide-pulleys, if supported by spindles running in adjustable bearings or brackets, may be made serviceable as tightening-pulleys for the purpose of taking up the slack of the belt, and thus giving the necessary grip for transmitting more power with a steadier drive than can be obtained without them.

Shape of Pulley Face.—The student will have observed that the faces or rims of the fast and loose pulleys, as well as those of the stepped cones in the previous set of figures, are slightly curved. This convex curvature, or double coning, is purposely done in order to ensure that the belt may ride easily and fairly in the centre line of the pulley face without inclining to either side. A flat band, if placed on the smaller end of a revolving straight conical pulley, will naturally tend to rise to the larger end of the cone. Consequently, if each half of the face of a pulley is coned (or, which amounts to the same thing, if the rim of the pulley be curved so as to have its largest diameter in the middle of its face), each half of the breadth of the belt will have an equal tendency towards the middle of the pulley's rim. When very fast driving and sudden severe stresses are brought to bear upon a machine, as in the case of circular saws, morticing machines, and emery-wheel grinders, it is found necessary to fit the pulleys with side flanges, in addition to curving their rims, in order to prevent the belts from sliding off the pulley's face to one side or to the other.

LECTURE XI.—QUESTIONS.

1. In machinery, where one pulley drives another by means of an endless belt, there is a difference of tension in the two parts of the belt. Why is this? The pulley on an engine shaft is 5 feet in diameter, and it makes 100 revolutions per minute. The motion is transmitted from this pulley to the main shaft by a belt running on a pulley, and the *difference in tension* between the tight and slack sides of the belt is 115 lbs. What is the work done per minute in overcoming the resistance to motion of the main shaft? (S. and A. Exam. 1891.) *Ans.* 180,550 ft.-lbs.

2. Deduce from the "principle of work" a formula for the brake horse-power transmitted by a belt. The pull on the driving side of a belt is 200 lbs. and on the following side 100 lbs., whilst the belt has a velocity of 990 ft. per minute. Find the number of units of work performed in two minutes and the B.H.P. transmitted. *Ans.* 198,000 ft.-lbs., 3 B.H.P.

3. State and prove the rule for estimating the relative speeds of two pulleys connected by a belt. Also, the velocity ratio between the first driver and the last follower in belt gearing, where there are two or more drivers and a corresponding number of followers. [A main shaft carrying a pulley of 12 inches diameter and running at 60 revolutions per minute, communicates motion by a belt to a pulley of 12 inches diameter, fixed to a countershaft. A second pulley on the countershaft, of $8\frac{3}{4}$ inches diameter, carries on the motion to a revolving spindle which is keyed to a pulley of $4\frac{1}{4}$ inches diameter. Sketch the arrangement and find the number of revolutions per minute made by this last pulley. (S. and A. Exam. 1891.)] *Ans.* 123·53.

4. Two pulleys are connected by a driving belt, and the sum of their diameters is 30 inches; one pulley makes 2 revolutions while the other makes 3 revolutions; find their respective diameters. *Ans.* 18", 12".

5. An engine works normally at 106 revolutions per minute. At that speed it was found that it drove by belting a dynamo at 420 revolutions per minute, but to show off the electric lights at their normal candle power the dynamo had to be run at 460 revolutions per minute. At what speed was the engine being driven? *Ans.* 116 revolutions per minute.

6. A pulley of 3 feet radius rotates at 100 revolutions per minute and transmits motion to another pulley of 36 inches diameter. If there is 10 per cent. slip on the belt what will be the speed of the follower? What will be the net driving pull on the belt if 5 B.H.P. is transmitted by it? *Ans.* 180 revolutions per minute; $97\cdot\dot{2}$ lbs.

7. Sketch an arrangement of pulleys and bands for obtaining a reversing motion from a shaft driven at a constant rate in one direction, and describe the action of the combination.

8. Sketch a combination of fast and loose pulleys as used for setting in motion, or stopping machinery. Explain the construction adopted for retaining a flat belt upon a pulley, pointing out where the fork is to be applied, and why.

9. Sketch and describe a good form of slow forward and quick return for a shaping machine.

10. Sketch and describe an arrangement for driving the table of a planing machine by means of a screw, so that the table may travel 50 per cent. faster in the return than in the forward or cutting stroke. (S. and A. *Exam. 1888.)*

11. What is the object of using guide-pulleys in machinery? Mention instances of their use, and show how the directions of their axes are ascertained.

12. Describe, with a sketch, the mode of reversing the motion of the table in planing machine, a screw being employed to drive the table.

13. What is the object of using speed pulleys? Show their application in a foot lathe, and the manner in which the motion of the workman's foot is converted into the circular motion of the mandril. Sketch the arrangement. The diameter of the largest pulley on the crank shaft being 2 feet, and that of the smallest pulley on the mandril being 3 inches; find the number of rotations of the mandril for each complete rotation of the crank shaft when these pulleys are connected by a band. *Ans.* 8 revolutions. [N.B.—Refer to the figures in Lecture XVI. of the foot-driving gear for the screw-cutting lathe, before answering this question.]

LECTURE XII.

CONTENTS.—Velocity Ratio of Two Friction Circular Discs—Pitch Surfaces and Pitch Circles—Pitch of Teeth in Wheel Gearing—Rack and Pinion Velocity Ration in Wheel Gearing—Example I.—Principle of Work applied to Wheel Gearing—Examples II. III.—Questions.

Velocity Ratio of Two Circular Friction Discs.—If two truly centred circular discs or cylindrical rollers, having their shafts parallel to each other and free to turn in fixed bearings, be brought into firm contact; then, if one of them be driven round, and if there be no slipping, the other one will rotate in the *opposite* direction with the same circumferential speed or surface velocity (see the next figure).

Consequently, *their velocity ratio will be in the inverse ratio to their diameters.*

This may be proved in exactly the same way as we found the velocity ratio of two pulleys driven by a belt in Lecture XI.

Let D_1 = Diameter of the driving disc.
,, F_1 = Diameter of the following disc.
,, N_{D_1} = Number of revolutions per minute of D_1.
,, N_{F_1} = Number of revolutions per minute of F_1.

Then, *The surface velocity of* D_1 = *Surface velocity of* F_1

i.e. . . . . . $\pi D_1 N_{D_1} = \pi F_1 N_{F_1}$

Or, ... $D_1 N_{D_1} = F_1 N_{F_1}$

i.e. The Driver's diameter × its speed = Follower's diamr. × its speed.

$$\therefore \frac{N_{D_1}}{N_{F_1}} = \frac{F_1}{D_1}$$

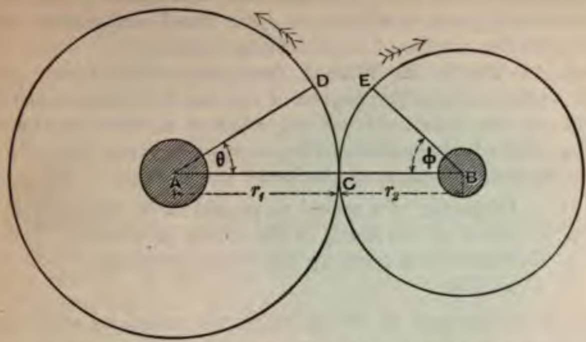
i.e. . . $$\frac{\text{Speed of the Driver}}{\text{Speed of the Follower}} = \frac{\text{Diameter of Follower}}{\text{Diameter of Driver}}$$

This velocity ratio may also be proved in the following way :—

Let the two circles centred at A and B represent a cross section of the two friction discs in contact at C; and let them *move by rolling* contact through the angles θ and ϕ respectively *in the same time.*

Since the magnitude of an angle in circular measure is

always = *the length of the arc subtended by the angle at the centre of the circle ÷ the radius of the circle.*



VELOCITY RATIO OF TWO CIRCULAR DISCS.

Then, $\angle \text{DAC} = \theta = \frac{\text{arc DC}}{r_1}$; and $\angle \text{EBC} = \phi = \frac{\text{arc EC}}{r_2}$

But, the arc DC = the arc EC since there is no slipping. Consequently,

$$\frac{\text{The angular velocity of circle A}}{\text{The angular velocity of circle B}} = \frac{\theta}{\phi} = \frac{\frac{\text{DC}}{r_1}}{\frac{\text{EC}}{r_2}} = \frac{r_2}{r_1}$$

Or,*

$$\frac{\text{The angular velocity or speed of driver, A}}{\text{The angular velocity or speed of follower B}} = \frac{\text{Radius of follower B}}{\text{Radius of driver A}}$$

Pitch Surfaces and Pitch Circles.—In the case of the two discs or rollers just considered, their cylindrical surfaces are termed the *pitch surfaces;* and the two circles in the previous figure (which is simply a representation of their cross section, or section in the plane of their rotation) are called the *pitch circles.*

* The *angular velocity* of a rotating disc is the *angle* described by its radius in unit time.

The relation between angular velocity and linear velocity may be shown thus:—Let ω = the angular velocity; whilst v = the linear velocity of a point at radius r from the centre of motion when the disc makes n revolutions in unit time;

$$\text{Then } \omega \times r = v\text{ ; or, } \omega = \frac{v}{r}\text{; but } v = 2\pi r n,$$

$$\therefore \omega = \frac{2\pi r n}{r} = 2\pi n.$$

When the resistance to motion of the follower is great, the discs have to be provided with teeth in order to prevent slipping.

Consequently, the *pitch surfaces* and the *pitch circles* of such toothed rollers, toothed wheels, or spur wheel and pinion, are the surfaces and the circles of their rolling contact.*

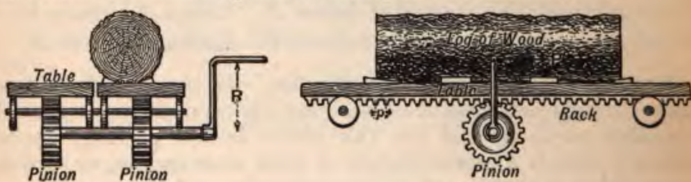
Pitch of Teeth in Wheel Gearing.—The linear or the circular distance from the centre of one tooth to the centre of the next one, or the distance from one edge of a tooth to the corresponding edge of its neighbouring one, *as measured on the pitch circle,* is termed the pitch of the teeth of a wheel.

Let D = Diameter of a wheel or pinion at its pitch circle.
p = Pitch of the teeth in the wheel or pinion.
n = Number of teeth in the wheel or pinion.

Then $D = p \times n$

For the circumference of the pitch circle must be equal to the pitch between any two neighbouring teeth × the number of teeth in the wheel or pinion ; since the pitch between each pair of teeth must be the same all round the pitch circle, otherwise the wheel would not gear properly with any other wheel or pinion of the same pitch.

Rack and Pinion.—If a straight bar of iron be furnished with teeth on one side it is called a *rack*. It may therefore be considered as a wheel of infinite radius. When a rack has a pinion of the same pitch geared with it, the two form the useful combination termed the *rack and pinion*. It is employed for moving to and fro the tables of planing machines and large saw benches, as well as for elevating and lowering sluices in dams, &c.



END VIEW. SIDE VIEW.

RACK AND PINION APPLIED TO A SAW-MILL TABLE.

The accompanying illustrations show the second of these applications, where two parallel racks are fitted to the under side of

* When a large toothed wheel gears with a small one, the larger is termed a spur-wheel and the smaller a pinion. It is not possible in the space allotted to this elementary manual to enter into the best forms of the teeth of different kinds of wheel gearing. We will have to take up this subject in our "Advanced Text Book."

two movable tables or platforms. Upon the upper side of one of the tables is laid a log of wood adjusted in the desired position by wedges. The tables are each carried and guided by four rollers turning on fixed spindles. To the projecting end of the pinion shaft there is fitted a lever handle, so that by merely turning this handle in one direction, the racks, tables, and log of wood are pushed forward upon the projecting circular saw which revolves between the platforms, and if turned in the opposite direction they are drawn backwards. The pinions with their shaft and handle, have no linear motion, for the shaft is simply free to rotate in fixed bearings.

The *rack and pinion* with their handle constitute a modification of the wheel and axle, or lever and winch barrel, where the resistance offered by the rack and its load is overcome by a force applied to the handle. Every revolution of the handle turns the pinion, and consequently moves the rack through a linear distance equal to the circumference of the pinion's pitch circle. The principles of moments and of work can therefore be applied to this machine in exactly the same way as we applied them to the wheel and axle and the winch.

If P = Pull acting on the handles,
R = Radius of handle,
r = Radius of pinion's pitch circle,
W = Weight or resistance overcome;

Then . . $$P \times 2\pi R = W \times 2\pi r$$
$$P \times R = W \times r$$

Theoretical advantage . . $$= \frac{W}{P} = \frac{R}{r} = \frac{\text{P's velocity}}{\text{W's velocity}}$$

Velocity Ratio in Wheel Gearing.—From what has been said about belt gearing, pitch surfaces, pitch circles, and pitch of teeth, it must be at once apparent to the student that the same rule which was worked out in Lecture XI., in connection with belt gearing, will equally apply to the case of wheel gearing, where there are an equal number of drivers and followers. In the accompanying figure, where there are three drivers and three followers,

Let D_1, D_2, D_3 = Diameters of the drivers.
" F_1, F_2, F_3 = Diameters of the followers.
" N_{D_1}, N_{F_3} = Number of revolutions in the same time of the *first* driver and the *last* follower.

Then, following the same reasoning as was expounded in Lecture XI. for the velocity ratio of belt gearing, we have

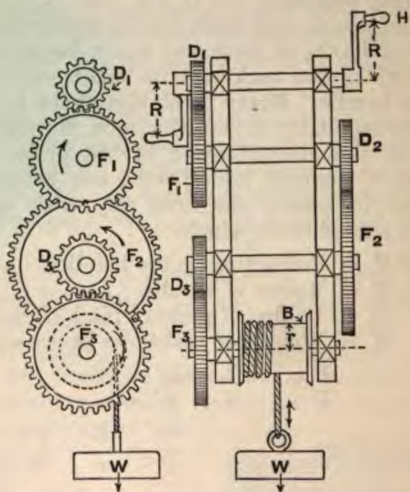
$$\left.\begin{array}{l}\text{The speed of the } \textit{first} \text{ driver } \times \\ \text{the successive diameters of} \\ \text{the drivers}\end{array}\right\} = \left\{\begin{array}{l}\text{The speed of the } \textit{last} \text{ follower} \\ \times \text{ the successive diameters} \\ \text{of the followers.}\end{array}\right.$$

i.e., $N_{D_1} \times D_1 \times D_2 \times D_3 = N_{F_3} \times F_1 \times F_2 \times F_3$

Or, ... $\dfrac{N_{D_1}}{N_{F_3}} = \dfrac{F_1 \times F_2 \times F_3}{D_1 \times D_2 \times D_3}$

$$\frac{\text{The speed of } \textit{first} \text{ driver}}{\text{The speed of } \textit{last} \text{ follower}} = \frac{\text{Product of the diameters of the followers.}}{\text{Product of the diameters of the drivers.}}$$

In the above equation we may substitute the radii, or the cir-



SIDE VIEW. PLAN.

WHEEL GEARING IN A TRIPLE PURCHASE WINCH

cumferences, or the number of teeth in the drivers and in the followers respectively, for their diameters; consequently,

Let r_{D_1}, r_{D_2}, r_{D_3} = Radii of the respective drivers.
,, C_{D_1}, C_{D_2}, C_{D_3} = Circumferences ,,
,, n_{D_1}, n_{D_2}, n_{D_3} = Number of teeth in ,,
,, r_{F_1}, r_{F_2}, r_{F_3} = Radii of the respective followers.
,, C_{F_1}, C_{F_2}, C_{F_3} = Circumferences ,,
,, n_{F_1}, n_{F_2}, n_{F_3} = Number of teeth in ,,

Then, $N_{D_1} \times r_{D_1} \times r_{D_2} \times r_{D_3} = N_{F_3} \times r_{F_1} \times r_{F_2} \times r_{F_3}$

Or, $N_{D_1} \times C_{D_1} \times C_{D_2} \times C_{D_3} = N_{F_3} \times C_{F_1} \times C_{F_2} \times C_{F_3}$

Or, $N_{D_1} \times n_{D_1} \times n_{D_2} \times n_{D_3} = N_{F_3} \times n_{F_1} \times n_{F_2} \times n_{F_3}$

EXAMPLE I.—Three drivers of 10, 20, and 30 teeth each, gear respectively with three followers of 40, 80, and 120 teeth each. Ascertain the velocity ratio between the *first* driver and the *last* follower.

By the above formula—

$$N_{D_1} \times n_{D_1} \times n_{D_2} \times n_{D_3} = N_{F_3} \times n_{F_1} \times n_{F_2} \times n_{F_3};$$

$$\therefore \frac{N_{D_1}}{N_{F_3}} = \frac{n_{F_1} \times n_{F_2} \times n_{F_3}}{n_{D_1} \times n_{D_2} \times n_{D_3}}$$

Interpolating the corresponding numerical values for the letters, we get

$$\frac{N_{D_1}}{N_{F_3}} = \frac{40 \times 80 \times 120}{10 \times 20 \times 30} = \frac{4 \times 4 \times 4}{1} = \frac{\mathbf{64}}{\mathbf{1}}$$

Principle of Work applied to Wheel-gearing.—Referring to the previous figure, it is perfectly evident from the former applications to other machines of the "principle of work," that, *neglecting friction*, the force applied (to the handles of the machine) × the distance through which it acts, will be equal to the weight raised × the distance through which it is elevated.

Let P = Push applied to the handles in lbs.
" R = Radius or leverage at which P acts.
" W = Weight raised by the rope on the barrel B.
" r = Radius or leverage with which W acts.
" D_1, D_2, D_3 = Diameters of the driving wheels.
" F_1, F_2, F_3 = Diameters of the following wheels.
" N_{D_1} = Number of revolutions of the *first* driver, D_1, or of the handles, H.
" N_{F_3} = Number of revolutions *in the same time* of the *last* follower, F_3, or of the barrel, B.

Then, by the principle of work and *neglecting friction*—

$$P \times \text{its distance}^* = W \times \text{its distance}.$$

i.e., $P \times 2\pi R \times N_{D_1} = W \times 2\pi r \times N_{F_3}$

($\div$ both sides of the equation by 2π)

$$\therefore P \times R \times N_{D_1} = W \times r \times N_{F_3}$$

Or, $$\frac{P \times R}{W \times r} = \frac{N_{F_3}}{N_{D_1}}; \text{ or } \frac{P}{W} = \frac{N_{F_3} \times r}{N_{D_1} \times R}$$

* It is evident that in order to obtain the distance through which P acts, we must multiply the circumference of the circle described by the handles by the number of revolutions they make; and in the same way the circumference of the barrel must be multiplied by the revolutions *which it makes in the same time*, in order to get W's distance.

But by the previous equation for velocity ratios,

$$\frac{N_{F_3}}{N_{D_1}} = \frac{D_1 \times D_2 \times D_3}{F_1 \times F_2 \times F_3}$$

i.e., . . . $$\frac{P \times R}{W \times r} = \frac{D_1 \times D_2 \times D_3}{F_1 \times F_2 \times F_3}$$

Or, $P \times R \times F_1 \times F_2 \times F_3 = W \times r \times D_1 \times D_2 \times D_3$

Hence the general rule for work done in wheel-gearing P × *its leverage* × *all the diameters* (or radii, or circumferences, or number of teeth) *of the followers* = W × *its leverage* × *all the diameters* (or radii, or circumferences, or number of teeth) *of the drivers.*

EXAMPLE II.—If four men exert a constant force of 15 lbs. each on the handles of a compound crab or winch (such as that illustrated by the previous figure), and if the leverage of the handles is 15″, whilst the weight to be raised acts on the barrel or drum at a leverage of 5″, what load will they lift if the respective diameters of the drivers are 12″, 20″, and 20″; and of the followers, 36″, 80″ and 100″, neglecting friction?

Answer.—In this case, $P = 4 \times 15 = 60$ lbs.; $R = 15''$; $r = 5''$; $D_1 = 12''$; $D_2 = 20''$; $D_3 = 20''$; $F_1 = 36''$; $F_2 = 80''$, and $F_3 = 100''$.

By the above formula and by interpolating the corresponding numerical values we have—

$$P \times R \times F_1 \times F_2 \times F_3 = W \times r \times D_1 \times D_2 \times D_3$$

$$60 \times 15'' \times 36'' \times 80'' \times 100'' = W \times 5'' \times 12'' \times 20'' \times 20''$$

$$\therefore W = \frac{60 \times \overset{3}{\cancel{15}} \times \overset{3}{\cancel{36}} \times \overset{4}{\cancel{80}} \times \overset{5}{\cancel{100}}}{\cancel{5} \times \cancel{12} \times \cancel{20} \times \cancel{20}}$$

Or, ... $W = 60 \times 3 \times 3 \times 4 \times 5 = \mathbf{10{,}800\ lbs.}$

EXAMPLE III.—If 40 % of the force applied to the handles be absorbed in overcoming internal friction in the above example of a winch, what weight can then be raised by the four men, each acting, as before, with a constant force of 15 lbs.?

ANSWER.—If 40 % of the applied force be lost in overcoming friction, then only 60 % is left for effective work, or the efficiency or *modulus* of the machine is said to be 0·6.*

Consequently, $100 : 60 :: 10{,}800 \text{ lbs.} : x \text{ lbs.}$

$$\therefore x = \frac{60 \times 10{,}800}{100} = \mathbf{6480\ lbs.}$$

* The term *modulus of a machine* is only another expression for the more ppropriate phrase, *efficiency of a machine.*

LECTURE XII.—QUESTIONS.

1. When two circular discs with fixed centres are in firm contact and roll uniformly together, state and prove the rule for estimating their relative speeds of rotation.

2. Define the pitch circle of a toothed wheel. When two pitch circles, A and B, of diameters 2 and 3 respectively, roll together, prove that the angular velocity of A is to that of B as 3 to 2. Three spur wheels, A, B, C, with parallel axes, are in gear. A has 8 teeth, B has 32 teeth, and C has 42 teeth. How many turns will A make upon its axis while C goes round 8 times? Why is B termed an *idle* wheel? (S. and A. Exam. 1888.) *Ans.* 42 turns.

3. What is the *pitch* of a tooth in a spur wheel? Two parallel shafts, whose axes are to be as nearly as possible 2 feet 6 inches apart, are to be connected by a pair of spur wheels, so that while the driver runs at 100 revolutions per minute, the follower is required to run at only 25 revolutions per minute. Sketch the arrangement, and mark on each wheel its diameter and the number of teeth, supposing the pitch of a tooth to be $1\frac{1}{4}$ inch. (S. and A. Exam. 1890.)

Ans. The driver is 48 inches diameter with 120 teeth.
The follower is 12 „ „ 30 „

4. Define the "pitch surface" and the "pitch circle" of a toothed wheel. Two parallel axes are at a distance of 10 inches, and they are to rotate with velocities as the numbers 2 and 3 respectively. What should be the diameters of the pitch circles of a pair of wheels which would give the required motion, and what might be the numbers of teeth on the wheels? *Ans.* 12 inches and 8 inches.

5. Sketch and describe the "*rack and pinion*," and give instances from personal observation of its application. A pinion of 3·2″ diameter has teeth of 1″ pitch, and gears with a straight rack applied to a sluice gate. If the weight of the sluice and rack be 100 lbs. and the lever handle describes a circle of 40·2″ in each turn, what force must be applied to the handle to lift the gate? How many feet will the sluice be lifted by six turns of the handle. *Ans.* 25 lbs.; 5 ft.

6. Sketch the arrangement known as the rack and pinion. Apply the "principle of moments" and the "principle of work" to find the relation between the force applied and the weight raised by aid of this machine. A pinion has sixteen teeth of $\frac{7}{8}$-inch pitch in gear with a rack. If the pinion makes $3\frac{1}{2}$ turns, through what distance has the rack been moved? If the pinion is turned by a handle 14 inches long, and with a force of 35 lbs. applied to the handle, find the force with which the rack is urged forward. *Ans.* 49 inches; 220 lbs.

7. Deduce the formula for the velocity ratio in wheel gearing where there are three drivers and three followers, and state the rule derived therefrom in general terms. Three drivers of 20, 30 and 40 teeth respectively gear with three followers of 40, 60 and 80 teeth. If the first driver makes 160 revolutions, how many revolutions will the last follower make? *Ans.* 10.

8. In the previous question, if the handles attached to the first driver have each a radius of 15″, and the drum connected to the last follower be 15″ diameter, what force must be applied to the handles in order that they may lift 1120 lbs., supposing that the efficiency of the machine is 70 per cent.? *Ans.* 100 lbs.

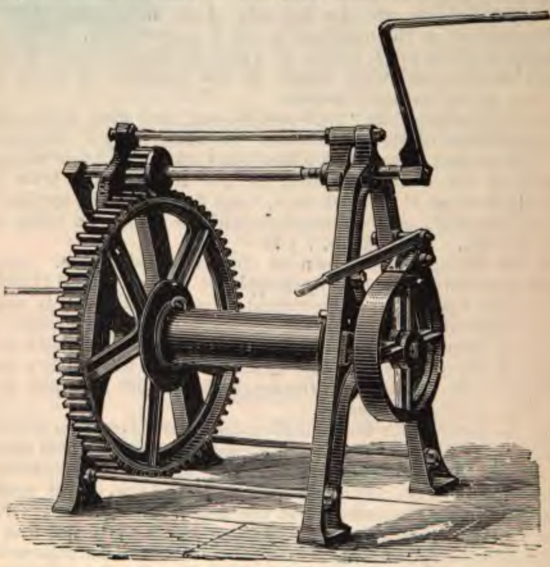
9. The hour and minute hands of a clock are on the same arbor or axis, and the hour hand takes its motion from the minute hand. Devise some train of wheels for connecting the two hands.

LECTURE XIII.

CONTENTS.—Single-purchase Winch or Crab—Example I.—Double-purchase Winch or Crab—Example II.—Wheel Gearing in Jib-Cranes—Questions.

In this Lecture we will apply the principles and formulæ discussed in the previous one to a few practical applications of gearing in machines for lifting weights.

Single-purchase Winch or Crab.—The comparatively small working advantage of the simple hand-driven wheel and axle or



SINGLE-PURCHASE WINCH OR CRAB.
By Messrs. Loudon Bros., Glasgow.

handle and winch barrel (illustrated in Lecture V.) renders it unfit for lifting greater weights than one or two hundredweight. Consequently, whenever heavier loads have to be raised by manual

labour, one of the most useful machines that can be employed is the single-purchase crab. As will be seen from the accompanying perspective view, this machine consists of a pair of lever handles fitted to the squared ends of a round shaft carrying a pinion. This pinion gears with a spur-wheel keyed to a lower shaft, upon which is also fixed a drum or barrel. To a hook or eye on the inside neck of the left-hand flange of this barrel the rope or chain (to be connected to the load) is attached. Therefore, the turning of the handles causes the barrel to rotate and wind the rope upon it, thereby elevating the load. Both shafts turn in bearings bored in the cast-iron end standards or **A** frames. These frames are bound tightly together and kept at a fixed distance apart by three wrought-iron collared stays, secured on the outside by screw nuts. To the outside right-hand end of the barrel shaft there is keyed a friction pulley acted on by a steel brake-strap, for the purpose of enabling the labourers to lower a load gently or quickly without enduring the stress and danger of pulling back on the handles. In fact, after applying the brake-strap by its outstanding handle, they can lift the claw pawl which is hinged on the top stay (and which keeps the pinion in gear with the spur-wheel when in the position shown on the figure) and by pulling the upper shaft to the right, disengage the pinion from its wheel. Then, by adjusting the pawl into the other groove of this shaft, they are free to lower the load by the brake without having the handles flying round. Between the right hand flange of the barrel and its neighbouring **A** frame there is a ratchet-wheel (not seen on the figure). This ratchet-wheel is generally cast along with the barrel. Its pawl, which is hinged to the inner side of the standard, can therefore be dropped down so as to engage with a tooth of the stop-wheel, whenever it is necessary to cease heaving up a heavy weight; thereby preventing the machine overhauling, and giving the labourers freedom to leave the handles and attend to other duties.

EXAMPLE I.—In a single-purchase crab the lever handles are each 16″ long, the diameter of the barrel is 8″; the pinion or driver has 12 teeth, and the wheel or follower 60 teeth. If two men apply a constant force of 20 lbs. each to the handles, and are just able to raise a weight of 400 lbs. to a height of 20 feet in two minutes, find—(1) the theoretical advantage; (2) the working advantage; (3) the work put in for every foot the weight is lifted; (4) the work got out for every foot the weight is lifted; (5) the efficiency; (6) the percentage efficiency of the machine; (7) the H.P. developed by the two men.

Answer.—*Referring* to the notation in last Lecture, we have $P = 2 \times 20$ lbs. $= 40$ lbs.; $R = 16''$; $r = 4''$; $n_D = 12$ teeth; $n_F = 60$

teeth ; W_T = the theoretical weight that would be raised if there were no friction ; $W_A = 400$ lbs. (the actual weight raised) ; $h = 20$ feet.

(1) *Theoretical advantage* $= \dfrac{W_T}{P}$

By the principle of work (*neglecting friction.*)

$$P \times \text{by its distance}^* = W_T \times \text{its distance.}^*$$

$$P \times 2\pi R \times n_F = W_T \times 2\pi r \times n_D$$
$$P \times R \times n_F = W_T \times r \times n_D$$
$$\therefore \quad W_T = \frac{P \times R \times n_F}{r \times n_D}$$

(Interpolating the above numerical values we get)

$$W_T = \frac{40 \times \overset{4}{\cancel{16}} \times \overset{5}{\cancel{60}}}{\cancel{4} \times \cancel{12}} = 40 \times 4 \times 5 = \mathbf{800\ lbs.}$$

Consequently, . . $\dfrac{W_T}{P} = \dfrac{800}{40} = \dfrac{\mathbf{20}}{\mathbf{1}}$

(2) *Working advantage* $= \dfrac{W_A}{P} = \dfrac{600 \text{ lbs.}}{40 \text{ lbs.}} = \dfrac{\mathbf{15}}{\mathbf{1}}$

(3) *Work put in for every foot* W_A *is raised.* From equation (1) we see that for every foot W_A is raised P must have gone through 20 feet, since the velocity ratio is $\dfrac{20}{1}$

$$\therefore\ P \times 20 = 40 \text{ lbs.} \times 20 = \mathbf{800\ ft.\text{-}lbs.}$$

(4) *Work got out for every foot* W_A *is raised*

$$= W_A \times 1' = 600 \text{ lbs.} \times 1' = \mathbf{600\ ft.\text{-}lbs.}$$

(5) *The efficiency* $= \dfrac{\text{Work got out}}{\text{Work put in}} = \dfrac{600 \text{ ft.-lbs.}}{800 \text{ ft.-lbs.}} = \cdot 75$

(6) *The percentage efficiency* $= \cdot 75 \times 100 = 75\%$

(7) *The H.P. developed by the two men* $= \dfrac{\text{Work put in per minute}}{33{,}000}$

$\therefore$ H.P. $= \dfrac{8000 \text{ ft.-lbs.}}{33000 \text{ ft.-lbs.}} = \dfrac{\mathbf{1}}{\mathbf{4}}$ full, or $\dfrac{1}{8}$ of a horse-power per man.

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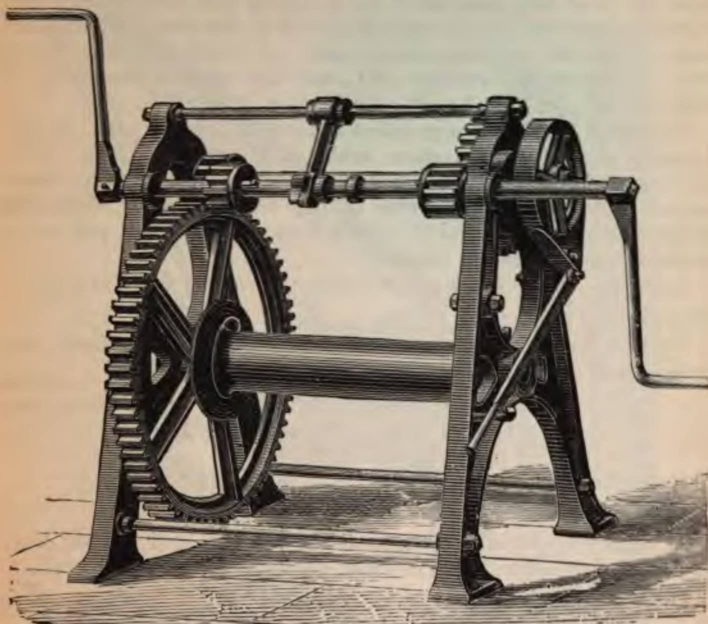
* It is evident that—

$$\frac{P \times 1 \text{ turn of handles}}{W_T \times 1 \text{ turn of barrel}} = \frac{\text{Number of teeth in the driver}}{\text{Number of teeth in the follower.}}$$

Or, . . $P \times 2\pi R : W_T \times 2\pi r :: n_D : n_F$

$$\therefore\ P \times 2\pi R \times n_F = W_T \times 2\pi r \times n_D$$

Double-purchase Winch or Crab.—It will be observed, from an inspection of the accompanying photographic view of a "Double-purchase Crab," that the chief difference between it and the single-purchase one is, that it has another pinion and wheel, with a view of increasing the velocity ratio or working advantage, and thus enabling the same manual force to lift a greater load, although by taking a longer time. It is also larger, heavier, and stronger.



DOUBLE-PURCHASE WINCH OR CRAB.

By Messrs. Loudon Bros., Glasgow.

As will be seen from the figure, it may be used as a single-purchase winch by simply lifting the claw-pawl hinged on the top stay, and pushing the handle shaft forward until its left-hand pinion gears with the large spur wheel, and then letting the pawl drop on to bearing to the right hand of the two collars on this shaft. By so doing, the right-hand pinion or first driver (when in double-purchase gear) is freed from the first follower, and both are inactive *during the time it is* used in single purchase, but the second

driver is still in gear and is turned round by the spur wheel. The brake strap pulley is keyed to the second shaft (carrying the first follower and second driver), and can be used for lowering the load without the handles coming into action (as described in the previous case) by placing the claw-pawl *between* the two collars in the first motion shaft. When the pawl is in this position, both of the pinions on this shaft are out of gear. The machine may be locked and the load left suspended by dropping the ratchet into the ratchet-wheel cast on the right-hand end of the barrel in the same way as with the single-purchase crab. A triple-purchase winch was illustrated in Lecture XII., and the student should again refer to the plan and the side elevation of its gearing.

EXAMPLE II.—Four men exert a force of 20 lbs. each, on the handles of a double-purchase crab, which are 15″ long. The driving pinions have 12 teeth each, the followers 24 and 48 teeth respectively, and the diameter of the barrel is 10″. Find the weight that can be raised if 25 per cent. of the work put in be absorbed in overcoming friction.

ANSWER.—Here $P = 4 \times 20 = 80$ lbs.; $R = 15''$; $n_{D_1} = 12$; $n_{D_2} = 12$; $n_{F_1} = 24$; $n_{F_2} = 48$; $r = 5''$.

By the formula deduced in the previous lecture from the principle of work (neglecting friction),

$$P \times R \times n_{F_1} \times n_{F_2} = W_T \times r \times n_{D_1} \times n_{D_2}$$

$$80 \times \underset{3}{\cancel{15}} \times \underset{2}{\cancel{24}} \times \underset{4}{\cancel{48}} = W_T \times \cancel{5} \times \cancel{12} \times \cancel{12}$$

Cancelling,

$$80 \times 3 \times 2 \times 4 = W_T = 1920 \text{ lbs.}$$

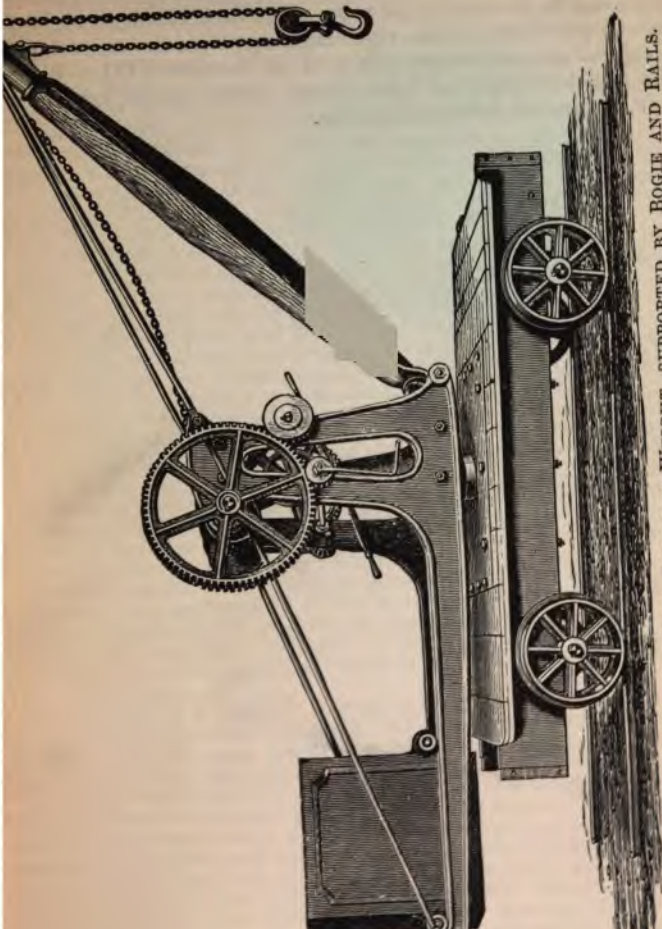
If 1920 lbs. of work be expended by the men and 25 per cent. of this be lost work, there remains 75 per cent. as useful work.

Or, . . $100 : 75 :: 1920 \text{ lbs.} : W_A$.

Weight actually raised $= \mathbf{W_A = 1440}$ **lbs.**

Wheel Gearing in Jib Cranes.—In Lecture VIII. the side view of a jib crane was given for the purpose of exemplifying the stresses on the jib, tie-rods, and central pillar. We now illustrate a swing jib crane on a bogie and rails, to show that the framework and lifting gear are simply those of an inverted double-purchase crab with the toothed wheels placed outside the standards instead of inside as in the ordinary winch. The snatch block pulley (previously referred to in Lecture VII.), to the hook of which the load is attached, doubles the theoretical purchase or *advantage* of the winch gearing, and therefore one, two or more men can lift nearly double the weight by aid of this simple addition to the machine. Large cranes of this description are

ed with slewing or horizontal turning gear, to enable the load
en lifted to be swung round before depositing it in a truck,



SLEWING JIB CRANE WITH BACK BALANCE WEIGHT, SUPPORTED BY BOGIE AND RAILS.
(By Messrs. P. & W. MacLellan, Glasgow.)

hold of a ship, or on a machine tool. This latter gear consists of
a horizontal wheel on the top of the vertical central cast-iron

supporting boss, with which is geared a bevel pinion, actuated by aid of a lever handle.

In order to prevent the whole machine being capsized by a heavy load, there is a back balance weight, and further the bogie wheels can be clamped to the rails. The back balance weight also tends to cancel the severe right angle stress on the central pillar which was specially taken notice of in Lecture VIII. We will defer the description of heavy steam power cranes, tripods and shear legs to our Advanced Course.

Lecture XIII.—Questions.

1. Where wheelwork is employed to modify motion, as in a crane, or in the double-geared headstock of a lathe, how is the change of motion calculated? Write down the formula employed.

2. Draw to scale a side elevation and end view of a single purchase crab, and describe the same by aid of an "index to parts." Apply the principle of work in solving the following question:—The lever handle of a crab is three times the diameter of the drum, and the wheelwork consists of a pinion of 16 teeth driving a wheel of 80 teeth; what weight will be lifted by a force of 30 lbs. acting at the end of the lever handle? *Ans.* 450 lbs.

3. Describe, with a freehand sketch, a single purchase lifting crab. The leverage of the handle of the crab is 16 in., and there is a pinion of 20 teeth driving a wheel of 100 teeth, the diameter of the barrel being 8 in. Assign the relative proportions of the working parts, and estimate the theoretical advantage. What weight would be raised by a man exerting a force of 15 lbs. on the lever handle, neglecting friction? *Ans.* 300 lbs.

4. A weight of 4 cwt. is raised by a rope which passes round a drum 3 feet in diameter, having on its shaft a toothed wheel also 3 feet in diameter. A pinion, 8 inches in diameter, and driven by a winch-handle 16 inches long, gears with the wheel. Find the force to be applied to the winch-handle in order to raise the weight. *Ans.* 112 lbs.

5. In a lifting crab the lever handle is 14 inches long, the diameter of the drum is 6 inches, and the wheel and pinion have 57 and 11 teeth respectively. Find the weight in pounds which could be raised by a force of 50 lbs. applied to the lever handle, friction being neglected. *Ans.* $1208\frac{2}{11}$ lbs.

6. In a crane there is a train of wheelwork, the first pinion being driven by a lever handle; and the last wheel being on the same axis as the chain barrel of the crane. The wheelwork consists of a pinion of 11 gearing with a wheel of 92, and of a pinion of 12 gearing with a wheel of 72, the diameter of the barrel being 18 inches and that of the circle described by the end of the lever handle being 36 inches; find the ratio of the pull to the weight raised, friction being neglected. *Ans.* 11 : 1104.

7. In a 30-ton crane the tension of the chain as it runs on the winding barrel is $7\frac{1}{2}$ tons, the barrel is 2 feet in effective diameter, and the spur wheel connected with it is 4 feet in diameter on the pitch line; what pressure will come upon the teeth of the spur wheel, supposing such pressure to act on the pitch line) friction is neglected)? (S. and A. Exam., 1889.) *Ans.* 3·75 tons.

8. The crank of an engine is 2′ long, and the diameter of the fly-wheel is 10′; also the fly-wheel has teeth on its rim, and drives a pinion 3′ in diameter. If the mean pressure on the crank pin be $7\frac{1}{2}$ tons, what is the mean driving pressure on the teeth of the pinion? *Ans.* 3 tons.

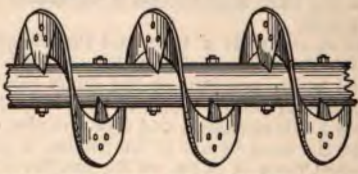
9. Draw to scale a side elevation, end view and plan of a double purchase crab, and describe the same by aid of an "index to parts." If four men each exert a constant force of 15 lbs. on the handles of such a crab; if the handles have a leverage of 16 inches whilst the barrel is 16 inches diameter, and if the drivers have 12 teeth each while the followers have 24 and 60 teeth respectively; find the weight which they could balance neglecting friction. If 30 per cent. of the work put in, be taken up in overcoming friction, what load can they lift? State (1) theoretical advantage; (2) working advantage; (3) work put in when lifting the load 1 foot; (4) the work got out; (5) the percentage efficiency; (6) the height through which they would lift the load in 1 minute if each man developed $\frac{1}{8}$ H.P. *Ans.* 1200 lbs.; 840 lbs.; (1) 20 : 1; (2) 14 : 1; (3) 1200 ft.-lbs.; (4) 840 ft.-lbs., (5) 70 per cent.; (6) 13·75 ft.

LECTURE XIV.

CONTENTS.—Screws—The Spiral, Helix, or Ideal Line of a Screw Thread—The Screw viewed as an Inclined Plane—Characteristics and Conditions to be Fulfilled by Screw Threads—Different Forms of Screw Threads—Whitworth's V-Threads—Whitworth's Tables of Standard V-Threads, Nuts and Bolt Heads—Seller's V-Thread—The Square Thread—The Rounded Thread—The Buttress Thread—Right and Left-hand Screws—The Screw Coupling for Railway Carriages—Single, Double and Treble Threaded Screws—Backlash in Wheel and Screw Gearings—Questions.

Screws.—Every one is more or less familiar with the form and uses of the screw nail for securing pieces of wood together, and of the bolt with its nut for fixing metal plates in position; but every one is not so familiar with the principle upon which screws are generated and act, or with the best shape to be given to a screw under different circumstances. We shall therefore endeavour in this Lecture to explain these points in an elementary manner, instancing a few examples of the practical applications of screws, but reserving for the following Lecture questions on the work done by screws and their efficiency.

The Spiral, Helix, or Ideal Line of a Screw Thread.—A very good idea of the form of a screw is obtained from the accompanying figure, which represents one means of elevating or trans-



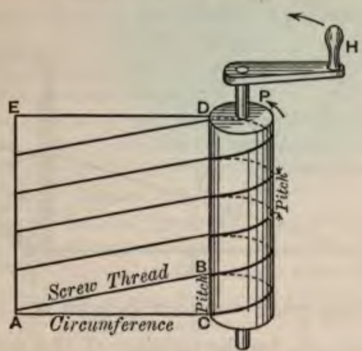
SPIRAL OR SCREW FOR MOVING GRAIN.

ferring grain, flour or other powdered substances from one part of a milling works to another. It consists of a steel band twisted around a cylindrical shaft in a continuous and uniformly pitched spiral. This shaft and screw are placed in a trough, tube or pipe. The grain or powdered substance is fed in at one end of the pipe, and by rotating the screw with a wheel or lever fixed

to one end of the shaft, the loose material is gradually pressed forward until it reaches the other end, from which it may be dropped into sacks or put through another process. It is evident from an inspection of the figure that as the screw is turned round by the lever, the particles of matter are forced *along the face of the continuous inclined plane* formed by the spiral steel band.

The principle upon which the screw acts is, therefore, a combination of the principles of the inclined plane and the lever.

To bring this view of the case still more forcibly before the student, take a cylinder and fix along the side thereof parallel to



FORMING A SCREW THREAD ON A CYLINDER.

its axis (by gum or drawing pins) a rectangle, ACDE, of paper or white cloth, having its sides, AC and DE exactly equal to the circumference of the cylinder. Then, when the envelope is wound round the cylinder by the turning of the handle, H (in the direction shown by the arrow at P), it exactly covers its cylindrical surface. On the outside of this rectangle when unfolded, draw any convenient number of parallel inclined black lines, AB, &c., equidistant from each other as shown by the figure, and again wrap it round the cylinder. These lines will be found to form a continuous spiral, helix, or screw-thread line from one end of the cylinder to the other. And the side AC of the right-angled triangle ACB forms the *circumference*, BC the *pitch*, AB the length of the *thread* (for one complete turn of the cylinder), and the angle BAC is the *inclination* or angle of the screw.

The Screw Viewed as an Inclined Plane.—Take another cylinder having an evenly pitched screw-thread line drawn upon it. *Cut a sheet of flexible cardboard into the form of a right-angled triangle with its* height BC or h equal to the pitch (or dis-

tance between two consecutive threads when measured parallel to the axis of the cylinder); AC or b equal to the *circumference* of the screw and wrap it round the cylinder, taking care to keep BC parallel to the axis. Then the hypothenuse AB or length l of the inclined plane will coincide with the contour of the screw-thread for one complete turn, and BAC or, a, is the *angle* of the thread to the plane at right angles to the axis of the cylinder.

Now conceive this screw-thread instead of being a mere line to be an inclined plane of known breadth, as in the case of the grain elevator.* Let the total weight of material being urged

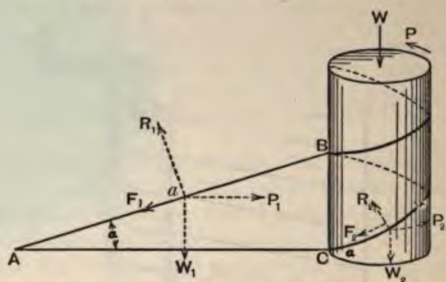


FIGURE TO PROVE THAT A SCREW THREAD IS AN INCLINED PLANE.

forward or upwards by the turning of the screw be W lbs., and let the resistance due to this load be uniformly distributed along the screw thread or inclined plane. Then, comparing the first and the third figures, it is evident that any small portion of the load having a weight W_2 lbs. will have a corresponding reaction R_2 lbs., and will require a part P_2 lbs. (of the total force, P, applied to turn the screw at the radius at which this portion is situated) to move it along the screw-plane against the frictional resistance F_2.

Imagine the work done to be transferred to the inclined plane, AB, then any portion of the load having a weight W_1 lbs. will have a corresponding reaction R_1 lbs., and will require a part P_1 lbs. (of the total force, P, applied parallel to the base to pull the whole load up the inclined plane) to move it along the plane against the frictional resistance F_1. Now, these forces act in identically the same way as the second case of the inclined plane, which was discussed in Lecture IX., consequently—

$$W_1 : P_1 : R_1 :: AC : CB : AB$$
$$W_2 : P_2 : R_2 :: AC : CB : AB$$
$$\therefore\ W : P : R :: b : h : l$$

* *Or, that the* screw-thread has a certain depth as measured radially *from the* axis of cylinder.

Or, $$\frac{P}{W}=\frac{C\,B}{A\,C}=\frac{\text{height}}{\text{base}}=\frac{h}{b}=\frac{\text{pitch of thread}}{\text{circumference of screw.}}$$

We therefore see that a screw may be treated as an inclined plane where the force turning the screw—*i.e.*, overcoming the resistance to motion—acts parallel to the base of the incline. The same reasoning may be applied to any screw turning in a nut or to a nut turning on a screw.

Characteristics of and Conditions to be Fulfilled by Screw Threads.—The essential characteristics of a screw-thread are its *pitch*, *depth*, and *form*.

The principal conditions to be fulfilled by a screw-thread are: (1) *efficiency*; (2) *strength*; (3) *durability*.

(1) The *efficiency* depends on the pitch and the friction, and hence on the pitch and form of thread.

(2) The *strength* depends upon the form or the shearing thickness and depth, or area of the cross section parallel to the axis.

(3) The *durability* depends chiefly on the depth—that is, upon the extent of bearing surface.

Different Forms of Screw Threads.—Sir Joseph Whitworth, the famous tool and gun manufacturer, was so impressed with the great inconvenience and loss of money which arose from the use of different pitches and forms of threads for screws and nuts, that he published the following tables giving the dimensions of what has now become known as the Whitworth standard. Prior to 1841, the year in which Whitworth proposed the adoption of standard sizes for screws, and for several years afterwards, different engineering works in this country not only used different pitches for screws of the same diameter, but it was no uncommon thing to find a want of uniformity in the same shop. Now, every one in Great Britain and her colonies uses the Whitworth standard sizes for **V**-threaded bolts and nuts of $\frac{1}{4}$-inch and upwards, and the British Association standard for smaller screws in electrical and philosophical instruments.

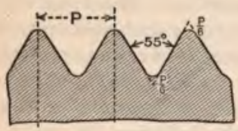
Whitworth's V Thread.*—The following figures of a Whitworth thread and nut, together with the tables, will serve to give full information regarding the number of threads per inch for different diameters of screw-bolts, nuts and bolt-heads, &c.

The angle between opposite sides of the threads and of the intervening spaces is 55°. One-sixth of the *depth* of the thread is rounded off at both the top and the bottom for the purpose of preventing a sharp nick at the bottom (which would weaken a

* For a description of Whitworth's screw-taps, plates, stocks, dies and combs, see "*Workshop Appliances*" by Professor Shelley. *And for a table of the B.A. Standard for Small Screws*, see Munro and Jamieson's *Electrical Rules and Tables, 9th ed.*, p. 70.

WHITWORTH'S STANDARD FOR SCREWS WITH ANGULAR THREADS.

No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.	No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.	No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.
48		0·100	12		0·600	4	2⅜	2·375
40	⅛	0·125	11	⅝	0·625	4	2½	2·500
32		0·150	11		0·650	4	2⅝	2·625
24		0·175	11		0·675	3½	2¾	2·750
24		0·200	11		0·700	3½	2⅞	2·875
24		0·225	10	¾	0·750	3½	3	3·000
20	¼	0·250	10		0·800	3¼	3¼	3·25
20		0·275	9	⅞	0·875	3¼	3½	3·50
18		0·300	9		0·900	3	3¾	3·75
18		0·325	8	1	1·000	3	4	4·00
18		0·350	7	1⅛	1·125	2⅞	4¼	4·25
16	⅜	0·375	7	1¼	1·250	2⅞	4½	4·50
16		0·400	6	1⅜	1·375	2¾	4¾	4·75
14		0·425	6	1½	1·500	2¾	5	5·00
14		0·450	5	1⅝	1·625	2⅝	5¼	5·25
14		0·475	5	1¾	1·750	2⅝	5½	5·50
12	½	0·500	4½	1⅞	1·875	2½	5¾	5·75
12		0·525	4½	2	2·000	2½	6	6·00
12		0·550	4½	2⅛	2·125			
12		0·575	4	2¼	2·250			



WHITWORTH VEE THREAD.

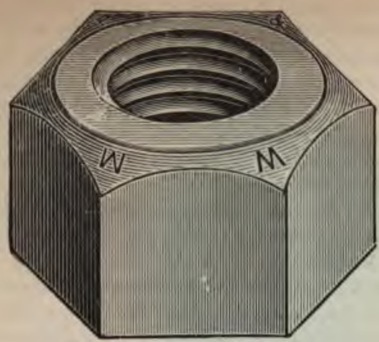
Angle of thread = 55°. One-sixth of depth is rounded off at top and bottom.

Number of threads to the inch in square threads = ½ number of those in angular threads.

Depth of threads = 0·64 pitch for angular = 0·475 pitch for square threads.

WHITWORTH'S GAS THREADS.

Diameter in Inches.	⅛	¼	⅜	½	¾	1	1¼	1½	1¾	2
No. of threads per inch	28	19	19	14	14	11	11	11	11	11



WHITWORTH SCREW NUT.

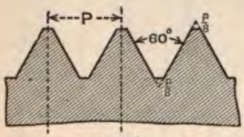
WHITWORTH'S STANDARD NUTS AND BOLT HEADS.

Diameter of Bolt, Inches.	Width across Flats.	Thickness of Nuts.	Thickness of Bolt-heads.	Diameter of Bolt at Bottom of Thread.	Diameter of Bolt, Inches.	Width across Flats.	Thickness of Nuts.	Thickness of Bolt-heads.	Diameter of Bolt at Bottom of Thread.
$\frac{1}{8}$	0·338	$\frac{1}{8}$	0·1093	0·0929	$1\frac{1}{8}$	1·8605	$1\frac{1}{8}$	0·9843	0·942
$\frac{3}{16}$	0·448	$\frac{3}{16}$	0·1640	0 1341	$1\frac{1}{4}$	2·0483	$1\frac{1}{4}$	1·0937	1·067
$\frac{1}{4}$	0·525	$\frac{1}{4}$	0·2187	0·1859	$1\frac{3}{8}$	2·2146	$1\frac{3}{8}$	1·2031	1·1615
$\frac{5}{16}$	0·6014	$\frac{5}{16}$	0·2734	0·2413	$1\frac{1}{2}$	2·4134	$1\frac{1}{2}$	1·3125	1·2865
$\frac{3}{8}$	0·7094	$\frac{3}{8}$	0·3281	0·2949	$1\frac{5}{8}$	2·5763	$1\frac{5}{8}$	1·4218	1·3688
$\frac{7}{16}$	0·8204	$\frac{7}{16}$	0·3828	0·346	$1\frac{3}{4}$	2·7578	$1\frac{3}{4}$	1·5312	1·4938
$\frac{1}{2}$	0·9191	$\frac{1}{2}$	0·4375	0·3932	$1\frac{7}{8}$	3·0183	$1\frac{7}{8}$	1·6406	1·5904
$\frac{9}{16}$	1·011	$\frac{9}{16}$	0·4921	0·4557	2	3·1491	2	1·75	1·7154
$\frac{5}{8}$	1·101	$\frac{5}{8}$	0·5468	0·5085	$2\frac{1}{8}$	3·337	$2\frac{1}{8}$	1·8593	1·8404
$\frac{11}{16}$	1·201	$\frac{11}{16}$	0·6015	0·571	$2\frac{1}{4}$	3·546	$2\frac{1}{4}$	1·9687	1·9298
$\frac{3}{4}$	1·3012	$\frac{3}{4}$	0·6562	0·6219	$2\frac{3}{8}$	3·75	$2\frac{3}{8}$	2·0781	2·0548
$\frac{13}{16}$	1·39	$\frac{13}{16}$	0·7109	0·6844	$2\frac{1}{2}$	3·894	$2\frac{1}{2}$	2·1875	2·1798
$\frac{7}{8}$	1·4788	$\frac{7}{8}$	0·7656	0·7327	$2\frac{5}{8}$	4·049	$2\frac{5}{8}$	2·2968	2·3048
$\frac{15}{16}$	1·5745	$\frac{15}{16}$	0·8203	0·7952	$2\frac{3}{4}$	4·181	$2\frac{3}{4}$	2·4062	2·384
1	1·6701	1	0·875	0·8399	3	4·531	3	2·625	2·634

bolt or a nut), as well as for ease in manufacturing them, since it would be practically impossible to maintain such perfectly sharp edges in the stocks and dies or in the combing tools with which such bolts and nuts are generally screwed. Besides, it would be most inconvenient to handle such sharp-pointed screws if they had edges tapering right off to 55°, and, moreover, it would serve no useful purpose, for such a thin edge cannot materially add to the strength of a screw-thread.

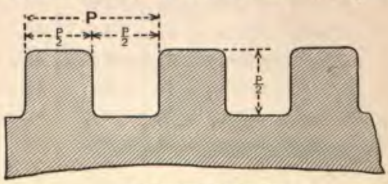
The Whitworth thread is stronger than any other, except that of the buttress one, since its thickness at the bottom of the thread is nearly equal to the pitch of the screw. The compression or grip is considerably greater than with the square thread, because the pitch is only half as much for the same size of bolt. The efficiency of the Whitworth **V**-thread as a means of transmitting motion is, however, small, since the reaction being at right angles to the face of the thread, a large part of the force employed in turning the screw is expended in tending to burst the enveloping nut. This very inefficiency, however, adds to its utility as a binder for all kinds of machinery, since a properly fitted nut when once screwed down, will not run back or overhaul, unless the pitch be very great and the threads be well oiled.

Seller's V-Thread.—In the United States of America, Seller's **V**-thread is used. It differs from the Whitworth **V**-thread in that the angle between the opposite sides of the thread and between the spaces is 60° instead of 55°, also the depth is reduced by a sharp flat top and bottom, equal to one-eighth of the pitch, instead of being rounded. This is rather a curious divergence from the usual American practice, where almost every other part of their excellent machine tools are beautifully rounded off by symmetrical curves.



SELLER'S V THREAD.

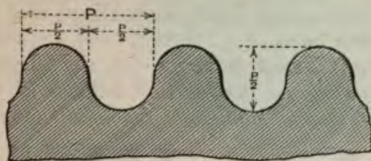
The Square Thread.—Since the bearing surface in this thread is very nearly at right angles to the direction of pressure and resistance it is much used for transmitting motion. Of the



SQUARE THREAD.

force applied to turn this screw there is only a small percentage dissipated in tending to burst the nut; consequently, its efficiency is greater than that of the **V**-thread. As will be seen from the accompanying figure, the thickness of the thread and the width of the space are made equal, in single-threaded screws, therefore the shearing thickness is greatly reduced, and consequently its strength is less than the **V**-thread. The durability is, however, greater than in any other form of screw, for there is a larger bearing surface presented in the best manner to resist pressure.

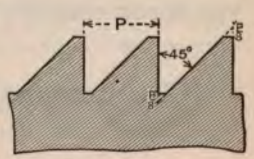
The Rounded Thread —This form is simply a modification of the square thread, in order to facilitate the quick engaging and



ROUNDED THREAD

disengaging of a leading motion screw by its nut in machine tools, or where a screw has to be subjected to rough usage. Its efficiency and durability are less than the square thread, but its strength is much greater, since the shearing thickness is greatly increased by the fillets at the bottoms of the thread.

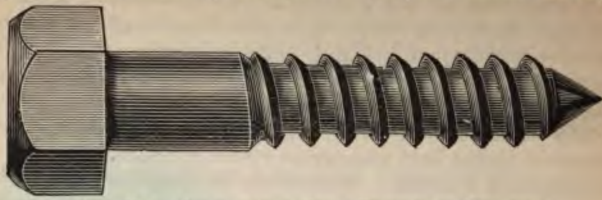
The Buttress Thread.—In such cases as the raising and lowering of heavy guns for the purposes of sighting and loading them, where the pressures are always in one direction, then this form of thread is adopted, because its strength is a maximum, the loss due to friction is a minimum, and there is very little tendency to burst the nut. The efficiency is quite equal to that of the square thread, although the durability is lessened by the fact that a certain amount of wear would diminish the depth of the thread. The strength is, however, nearly double, since the shearing thickness is double. It therefore possesses the advantages of the **V** and the square thread where pressures have to be applied in one direction.



BUTTRESS THREAD.

A slight modification of the buttress thread is used for rag-bolts. These bolts take a very firm hold of any material into which they *can be* screwed. Consequently, they are used for *screwing thick planks* of wood together, and binding down plates or

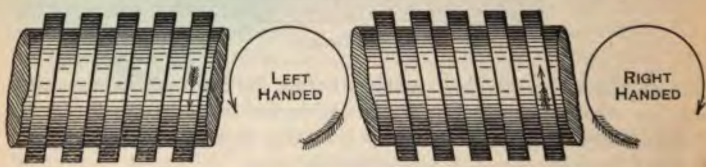
other planks where vibration and stresses would start and lessen the grip of the ordinary V-thread. They are much used by ship



THE RAG BOLT WITH SEMI-BUTTRESS THREAD.

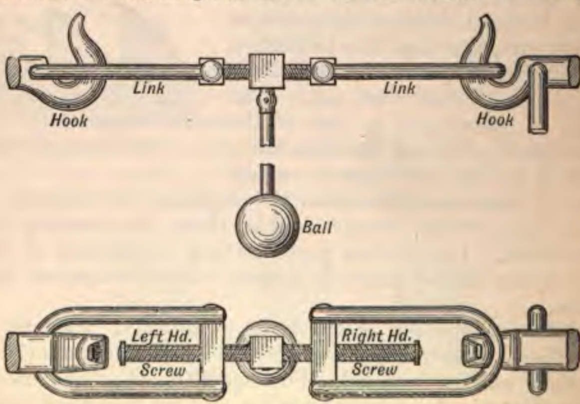
carpenters and erectors of light scaffolding, and are sometimes called holding-down bolts.

Right- and Left-hand Screws.—A right-hand screw, when being turned forward or into a nut, rotates in a right-handed way or in the direction of motion of the hands of a watch, whereas



a left-hand screw moves in the opposite or left-handed direction, as shown by the direction of the circular arrows in the above figure.

The Screw-coupling for Railway Carriages is a very good



SCREW COUPLING FOR RAILWAY CARRIAGES.

example of the use of right- and left-hand screws. When two carriages are brought together, the free link hanging from the hook of one of them is placed on the hook of the other one. The porter then turns the central lever by rotating the ball in a circle, thereby screwing both the right- and the left-hand screws into their respective nuts, which consequently draws the hooks toward each other, and couples the carriages tightly together.

EXAMPLE.—If the pitch of each screw is $\frac{1}{2}''$, the length of the lever arm or distance from the axis of the screw to the centre of the ball is $14''$; and if the railway porter pulls the ball with a force of 40 lbs. when the carriages are brought tightly together, what will be the tension on the screw threads?

ANSWER.—Here $p = \frac{1}{2}''$; $b = 2\pi R = 2 \times \frac{22}{7} \times 14'' = 88''$; $P = 40$ lbs.

The formula for the ratio of P to W in the case of a single screw given in this Lecture is

$$\frac{P}{W} = \frac{p}{b}, \text{ or } W = \frac{P \times b}{p}$$

$$\therefore W = \frac{40 \times 88''}{\frac{1}{2}''} = \mathbf{7040 \text{ lbs.}}$$

But there are two screws, and for every complete turn made by P, the stress W would be moved through twice the pitch of one screw or through $2 \times \frac{1}{2}'' = 1''$.

$$\therefore W = \frac{P \times b}{2p} = \frac{40 \times 88}{1} = \mathbf{3520 \text{ lbs.}}$$

NOTE.—We may answer this question directly from the "*Principle of Work.*" Students should be trained to work out each question from *first principles* rather than from formulæ; for, by a too free use of formulæ they are apt to lose sight of principles.

Let the lever make *one complete turn*, then *each* nut will advance along its own screw a distance *equal to the pitch*. Therefore the two nuts, and consequently the two carriages, will be brought nearer by a distance equal to *twice* the pitch, or, $= 2 \times p$.

By the principle of work, and neglecting friction—

Work got out = Work put in

$$\text{Or,} \quad W \times 2p = P \times 2\pi R$$

$$\therefore \quad W = \frac{P \times 2\pi R}{2p}$$

$$\text{Or,} \quad W = \frac{40 \times 2 \times \frac{22}{7} \times 14''}{2 \times \frac{1}{2}''} = \mathbf{3520 \text{ lbs.}}$$

Single, Double, and Treble-threaded Screws.—As has been previously stated, both the efficiency and the forward distance traversed in a single turn of a screw are directly as the pitch of

the thread, but the strength is proportional to the area of its cross section. Now, if for any purpose requiring a rapid movement of the nut or of a screw, the pitch must be increased; and if the screw consisted of a single-threaded square one, where the depth, thickness of the thread, and the width of the groove are each equal to half the pitch, the strength of the shaft upon which the screw is cut would be unnecessarily reduced. If the groove be made shallower and narrower then two threads, with two spaces having the same pitch as the single one, can be cut upon it so as to present about the same area of bearing surface to the pressure and at the same time afford quite as great a shearing thickness without interfering with the velocity ratio.* If a very great velocity ratio should be required, then three or more threads with corresponding grooves may be cut in the shaft and nut.

Backlash in Wheel and Screw-Gearings.—Backlash is the slackness between the teeth of wheels in gear or between a screw and its nut. Suppose that two wheels are in gear, and that you move one of them in a certain direction until it turns the other, and then reverse the motion; if you can now move the pitch circle through, say, $\frac{1}{8}$ inch, before the second wheel responds, this distance is the amount of backlash. In the same way, suppose you turn a screw in one direction until its nut moves, and then reverse the motion, the angle or proportion of a turn which you can now make before the nut responds, is the backlash of the screw and its nut. If a great amount of backlash be present in wheel-gearing, it causes vibration and a disagreeable rattling noise; and where severe stresses and sudden stoppages are common, the teeth are liable to be stripped. It can only be thoroughly prevented by cutting the teeth most accurately of the best rolling contact form by a tooth-cutting machine. All screws and nuts that are much worked are liable to backlash as they become worn, although when new they may have been very free from it, so that the best way of taking up the slack is to form the nut in two parts with flanges connected by screw-bolts, which may be tightened from time to time so as to take up the wear, and thus keep one side of the threads in one half of the nut, bearing hard against one side of the threads of the screw, and those in the other half against the other side.

* Refer to the Index for the page where the figure of the Fly Press appears. The screw of that machine is a double-threaded one.

LECTURE XIV.—QUESTIONS.

1. Explain how a screw is a combination of the lever and inclined plane, and illustrate your remarks. Find the theoretical advantage or ratio of W to P in the case of a screw of 1 inch pitch and 3·2 inches diameter; if the lever or spanner key be 7 feet long. *Ans.* 528 : 1.

2. Given a cylinder and a sheet of paper of sufficient size to cover the cylindrical surface, show how you would trace an evenly pitched spiral or screw line on the cylinder. Mark on your sketch the pitch, circumference, and angle of the screw-thread.

3. Trace a screw-thread line on a cylinder. Draw a triangle to represent the pitch, circumference and angle of the thread, and show the direction of all the forces on the supposition that there is a total pressure, W lbs., on the end of the cylinder acting parallel to its axis and balanced by a force, P lbs., acting at its circumference in a plane at right angles to the axis, with a total friction of F lbs. on the screw-thread.

4. What are the essential characteristics of a screw-thread? Upon which of these do (1) the efficiency, (2) the strength, (3) the durability of a screw depend?

5. Sketch and describe all the forms of screw-threads which you have seen in practice. State their representative advantages and disadvantages, and for which kind of work each kind is most suitable.

6. Define the pitch of a screw. In the Whitworth angular screw-thread, what is the angle made by opposite sides of the thread? To what extent is the thread rounded off at the top and bottom? Distinguish between a *single* and a *double-threaded* screw; in what cases should the latter be used? Why are holding down bolts made with angular threads?

7. Distinguish between a right-handed and a left-handed screw. Sketch the screw-coupling which is commonly used to connect two railway carriages, and explain the action of the combined screws. If the pitch of each screw is $\frac{3}{8}$ inch and the lever-arm from the axis of the screw to the centre of the ball is 12 inches, with what force will the carriages be pulled together by a force of 50 lbs. applied to the ball on the end of the arm? *Ans.* 5028·5 lbs.

8. Draw a single, double, and treble square-threaded screw to a $\frac{1}{10}$th scale, where the outside diameter of the screw-thread is 10 inches and the pitch 6 inches. Explain the advantages of using a double or treble thread instead of a single one for transmitting rapid motion against a considerable resistance.

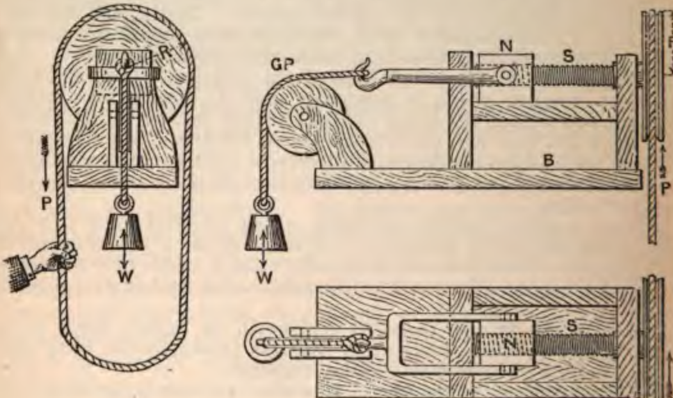
9. Why is the angular-threaded Whitworth or Seller's screw better adapted than the square, rounded, or buttress thread for the bolts which are used to bind pieces of machines, &c., together?

10. What is meant by *backlash*? How may backlash be prevented in a screw, and in wheel gearing?

LECTURE XV.

CONTENTS.—Efficiency, &c., of a Combined Lever, Screw, and Pulley Gear—Example I.—Bottle Screw-Jack—Example II.—Traversing Screw-Jack—Screw Press for Bales—Screw Bench Vice—Example III—Endless Screw and Worm-Wheel—Combined Pulley, Worm, Worm-Wheel and Winch Drum—Worm-Wheel Lifting Gear—Example IV.—Questions.

Efficiency, &c., of a Combined Lever, Screw, and Pulley Gear.—Construct an apparatus of the following description, having a horizontal Whitworth **V**-screw of, say, p'' pitch, with cylindrical ends and flanges supported by bearings, so that the screw cannot move longitudinally, but with a nut free to travel from one end of the screw to the other, along a slide or guide



APPARATUS FOR DEMONSTRATING THE ACTION AND EFFICIENCY OF SCREW GEAR.

INDEX TO PARTS.

W	represents	Weight to be lifted.	P	represents	Pull on pulley rope.
GP	,,	Guide Pulley.	R	,,	Radius of pulley.
N	,,	Nut.	B	,,	Base or support.
S	,,	Screw.			

which prevents it from turning round. Apply a force, P, to a rope passed over the V-grooved pulley of radius, R, keyed to the end of the screw shaft, until it moves the nut with the hook, rope, and weight, W, attached thereto, as shown by the accompanying side elevation, plan and end view of the apparatus.*

EXAMPLE I.—If the radius, R, of the turning-pulley be 12″, the pitch, p, of the screw 1″, and the gross pull, P, required to lift a weight of 100 lbs. be 4 lbs.: find (1) the velocity ratio; (2) the theoretical advantage; (3) the working advantage; (4) the work put in to lift W 1 foot; (5) the work got out; (6) the percentage efficiency; (7) the mean coefficient of friction.

ANSWER.—We have got in this question all the necessary data required to find the various answers except n, the number of turns which the screw will have to make in order to lift W 1 foot. Since the pitch of the screw is 1″, each turn thereof will elevate or lower the weight 1″, according as it is turned the one way or the other; consequently, if the screw makes 12 turns, the nut and the weight will move through 12″, therefore $n = 12$ turns.

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* It is evident that, in addition to the friction between the screw and the nut, there is friction at the several bearings, at the nut slide, and in the bending of the ropes. Consequently, if the student were to place in *succession* weights at W of, say, 10, 20, 30, 40 lbs., &c., and ascertain by aid of a Salter's spring balance (hooked into the rope which passes round the turning-pulley), the corresponding pulls required to lift these several weights, and to plot down the results on squared paper with the weights as abscissæ and the pulls as ordinates, and then to draw a line through the intersections of the vertical and horizontal lines drawn through the corresponding values, he would obtain a characteristic curve for the friction of the machine as a whole. If he took the precaution to balance the initial friction of the machine (when there was no weight attached at W) by hanging such a small weight at P as would just move the nut towards the turning-pulley, he would find upon repeating the above experiments (keeping the small additional weight on all the time) and replotting the results as now recorded by the spring balance, that the second frictional curve would approach much nearer to a straight line than the former one. In fact, its deviation therefrom would simply prove that the friction of the movable bearing surfaces *was not directly proportional to the load.* To arrive at the characteristic friction curve for the screw alone, he would have to find out by trial the proportion of the several pulls applied, which were spent in overcoming friction at all other points except between the screw and the nut. To those students who have the time and opportunity for carrying out experiments in applied mechanics, the apparatus illustrated above will prove interesting and instructive. The figures are drawn from the machine constructed in the author's engineering workshop for the purpose of enabling his students to make similar tests to those suggested above. A square, or a rounded, or a buttress-thread may be substituted for the V-Whitworth one, and sound information may thus be obtained about different forms of screws, which will make a stronger and more lasting impression on some students than by merely studying books.

By the principle of work :—

(1) *The Velocity Ratio* $= \dfrac{\text{P's distance in 1 turn of driving pulley}}{\text{W's distance in the same time}}$

Or, . . . $= \dfrac{\odot^{ce} \text{ of pulley}}{\text{pitch of screw}} = \dfrac{2\pi R}{p} = \dfrac{\mathbf{75{\cdot}4}}{\mathbf{1}}$

(2) *The Theoretical Advantage* . . $= \dfrac{\text{Weight lifted if there were no friction}}{\text{Pull applied}}$

" " " $= \dfrac{W_T}{P} = \dfrac{2\pi R}{p} = \dfrac{\mathbf{75{\cdot}4}}{\mathbf{1}}$

(3) *The Working Advantage* . . . $= \dfrac{W}{P} = \dfrac{100 \text{ lbs.}}{4 \text{ lbs.}} = \dfrac{\mathbf{25}}{\mathbf{1}}$

(4) *The Work Put in to lift W 1 foot* . $= 2\pi R n P$

" " " $= \dfrac{2 \times 22 \times 12'' \times 12 \times 4}{7 \times 12''} = \mathbf{301{\cdot}56 \text{ ft.-lbs.}}$

(5) *The Work Got out in raising W 1 foot* $= W \times 1' = 100 \text{ lbs.} \times 1' = \mathbf{100 \text{ ft.-lbs.}}$

(6) *The Percentage Efficiency* . . $= \text{Efficiency} \times 100$

" " " $= \dfrac{\text{Work got out}}{\text{Work put in}} \times 100$

" " " $= \dfrac{100 \text{ ft.-lbs.}}{301{\cdot}56 \text{ ft.-lbs.}} = \mathbf{33{\cdot}7\ \%}$*

(7) *The Co-efficient of Friction* . . $= \dfrac{P}{W} = \dfrac{4}{100} = \mathbf{{\cdot}04}$

Bottle Screw-Jack.—The importance of the screw as a simple machine for exerting great pressures, is very well exemplified by the screw-jack. This tool is used for replacing locomotives and railway carriages upon their rails, for elevating heavy girders into position, or for overcoming any great resistance through a small space which cannot be effected by a labourer and a lever. As will be seen from the accompanying figure it consists of a strong hollow bottle-shaped casting, with a projecting handle for facilitating the carrying of the tool from one place to another. In the upper end of the casting a square-threaded screw is cut

* It is evident that with such a low percentage efficiency the weight when hanging from the rope will not be able to overhaul the machine. …e student can calculate what pitch of screw would be required with the …e *co-efficient* of friction before overhauling could take place.

parallel with the axis, and into this nut there is fitted a steel screw terminating in a spherical head, having two holes bored through it at right angles to each other. Into one or other of these holes an iron lever bar is fixed, so that by pulling or pushing on the outer end of the bar the screw is turned, and thus the head is gradually raised from the base. To avoid the tearing, grinding action that would ensue between the head and the object acted upon, the former is provided with a loose crown fitted on a central pin projecting from the round head.

BOTTLE SCREW-JACK.

Let L = Length of the lever arm in inches from centre of jack to where the force is applied.
" p = Pitch of screw in inches.
" P = Pull or push applied at radius L.
" W = Weight lifted or resistance overcome.

Then, by *the Principle of Work*, and neglecting friction, we have in one turn of lever—

$$P \times \text{its distance} = W \times \text{its distance}$$

Or,
$$P \times 2\pi L = W \times p$$
$$\therefore P = \frac{W \times p}{2\pi L}$$

EXAMPLE II.—A weight of 10 tons has to be lifted by a screw-jack, in which the pitch of the screw is $\frac{1}{2}''$. What length of lever will be required if a force of 70 lbs. be applied at the end of it? (1) Neglecting friction; (2) if the modulus or efficiency of the tool is only ·4.

ANSWER.—(1) By the previous formula (neglecting friction)

$$L_1 = \frac{W \times p}{P \times 2\pi} = \frac{\overset{1120}{\cancel{22400}} \times \cdot 5'' \times \cancel{7}}{\underset{10}{\cancel{70}} \times \cancel{2} \times 22} = \frac{560}{22} = \mathbf{25\cdot\dot{4}\dot{5}''}$$

(2) Taking friction into account we see from the question that the efficiency is = ·4, therefore the percentage efficiency is 40, or 60 per cent. of the work put in is lost work required to overcome friction between the screw and its nut. But as the length of the lever is directly proportional to the work put in, the theoretical length of the lever found above is only 40 per cent. of the actual or working length required.

$$\therefore 40 : 100 :: 25\cdot45 : L_2$$

$$L_2 = \frac{100 \times 25\cdot45}{40} = 63\cdot6''$$

Traversing Screw-Jack.—It is very often convenient, when using a strong heavy screw-jack, to be able to move the head a short distance to one side or the other, when near the object to which it is to be applied; or, after having raised a load with one or more jacks, to be able to traverse the jacks forward or backward through a short distance until the load is brought into

TRAVERSING SCREW-JACK WITH RATCHET-LEVERS.
(By P. & W. MacLellan, Glasgow.)

the desired position. These movements may be effected with a jack of the form shown by the accompanying figure. Further, this jack is provided with a side foot-step attached to and projecting from the lower end of the vertical screw. This foot-step *can be placed* under the flange of a low beam or rail, where it would *be inconvenient* or perhaps impossible to get the top head underneath *the* same. The nut of the horizontal traversing screw is

formed in, or fitted to the bottom of the vertical casting, and this screw is turned by a ratchet-lever which may be slipped on to one or other of the squared ends of its shaft. The upward and downward movement of the vertical screw is also affected by a ratchet-lever, and in this case without turning the screw, for the ratchet-wheel is fixed to the nut of its screw. The pawl of the ratchet may be locked on one side or the other, so as to enable the ratchet-wheel and the vertical screw-nut to be turned round in either direction for elevating or lowering the load.

Screw Press for Bales.—When soft goods or hay have to be transported they may be squeezed into small bulk by means of a

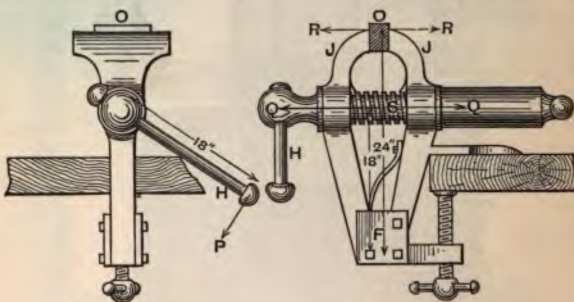
SCREW PRESS FOR BALES.
(By Loudon Bros., Glasgow.)

screw press, and bound firmly when under the press, by strips of hoop-iron passed round them and then riveted before the pressure is relieved. The bound bundle is then termed a bale. The operation will be understood by an inspection of the accompanying figure. The loose material is placed in the space between the rigid base and the movable plate of the press, the doors are closed and locked, the pressman applies himself to the end of the lever with a force, P, thereby turning the nut of the screw and forcing the movable plate downwards with a pressure, W, until the

desired compression of the goods has been attained. The doors are then opened and the strips of hoop iron (which were previously or are now placed in the grooves of the base and upper plate) are brought together and riveted. The lever is then turned in the opposite direction to relieve the pressure, and the bale is removed to the store or ship to make room for another quantity of goods being subjected to a similar action.

The same formula as we used for the screw-jack and for Example I. in connection with the combined lever, screw and pulley gear, naturally applies to this press, and to any similar appliance, such as a letter-copying press.*

Screw Bench Vice.—A bench vice is essentially an instrument for seizing and holding firmly any small object whilst it is being acted upon by a chisel, file, drill, saw, or emery cloth, &c. Looking at the figure which illustrates the following example, it will be seen that the vice is a combination of two levers, a square-threaded screw, and a nut. The object O to be gripped is placed



END VIEW. SIDE VIEW.

SCREW BENCH VICE.

between the serrated jaws JJ. The lever handle H, on being turned, forces the screw S into its long nut, and thereby presses forward the outer jaw upon the object, by aid of the flange on the screw-head. This jaw is a lever, having a fulcrum at F, and therefore the pressure on the object is less than that on the screw-collar in the proportion of SF to OF. The bent flat spring between the limbs of the fixed and movable jaws serves to force the movable jaw away from the fixed one when the screw is turned backwards, and thus relieves the object without having to pull

* *Refer to* the Index for the page where the illustration of the Fly-press *occurs. The statical* pressures produced by this machine when used for *punching holes,* &c., may be treated in the same way.

this jaw back by the hand. It will be observed that the fixed jaw should have been continued to the floor level by a vertical supporting leg, in the case of such a big vice intended for rough heavy engineering work.

EXAMPLE III.—Sketch an ordinary bench vice. Apply the principle of work to find the gripping force obtained when a man exerts a pressure of 20 lbs. at the end of a lever 18 inches long, the screw having four threads per inch, the length from the hinge to the screw being 18 inches, and the length from the hinge to the jaws being 24 inches. (S. & A. Exam. 1892.)

ANSWER.—Let P represent Pull on end of handle H = 20 lbs.

,,	Q	,,	Resistance offered by screw at S.
,,	R	,,	Reaction, or gripping force, exerted on object at O.
,,	L	,,	Length of handle H = 18 inches.
,,	p	,,	Pitch of screw S = $\frac{1}{4}$ inch.

Suppose the handle, H, to make one complete turn under the action of *a constant force*, P, at the extremity thereof, against *a constant resistance*, Q, acting along the axis of the screw.

[The student will observe that we *suppose* the forces P and Q to be *constant*, which is not correct for such a large movement as a complete turn of the handle, but which may be assumed here for the sake of simplicity. The reason for this is, that the resistance, R, will *vary* with the compression produced on the object at O. However, the *ratio* between P and R will remain a constant quantity.]

The work done *by* P during one turn of handle $= P \times 2\pi L$.
And ,, *on* Q during the same time $= Q \times p$.

But, by the *Principle of Work*—

$$\textit{Work done by } P = \textit{Work done on } Q$$

$$\therefore\ P \times 2\pi L = Q \times p$$

Interpolating the numerical values—

$$20 \times 2 \times \frac{22}{7} \times 18'' = Q \times \tfrac{1}{4}''$$

$$\therefore\ Q = \frac{20 \times 2 \times 22 \times 18 \times 4}{7} = 9051{\cdot}43 \text{ lbs.}$$

But by the *Principle of Moments*—

$$R \times FO = Q \times FS$$

$$\therefore\ R = \frac{18}{24} \times Q = \frac{3}{4} Q$$

$$\textit{i.e.,}\quad R = \frac{3}{4} \times 9051{\cdot}43 = \mathbf{6788{\cdot}57 \text{ lbs.}}$$

Endless Screw and Worm-Wheel.* — When a screw is rotated between fixed bearings so that it cannot move longitudinally, it is called an *endless screw*, because the threads of the screw seem to travel onwards without ending.† When such a screw gears with a toothed wheel, having its teeth set obliquely at the same angle as the threads of the screw so as to bear evenly thereon, the wheel is termed a *worm-wheel*. The endless screw is sometimes called the *worm*, no doubt from its resemblance to that well-known humble animal which, when coiled up for rest, would not turn upon any one unless trod upon.

By this arrangement, motion may be transmitted from one shaft to another at right angles to each other, without any possibility of the machine overhauling; for although the velocity ratio is very great, the efficiency is comparatively small—considerably under 50 per cent. with single-threaded screws—owing to the friction between the worm and the wheel.‡

It is most important for the student to comprehend that if *the screw be a single-threaded one, it must make as many turns as there are teeth on the wheel, for every revolution of the latter.* If the screw is a *double-threaded one*, then for each revolution thereof it drives the wheel through a distance equal to the distance *between two teeth* on the pitch circle, and if *treble-threaded* through the pitches of *three teeth*. Thus, if N equal the number of teeth in the worm-wheel, then, with a single-threaded screw, for every turn of the same, the wheel will move a distance of $\frac{1}{N}$; with a double-threaded worm $\frac{2}{N}$, and with a treble-threaded one $\frac{3}{N}$ and so on.

The endless screw and worm-wheel is used in a very great variety of circumstances, from the turning of a big marine engine when in port, to the delicate movements in a telescope or a microscope.

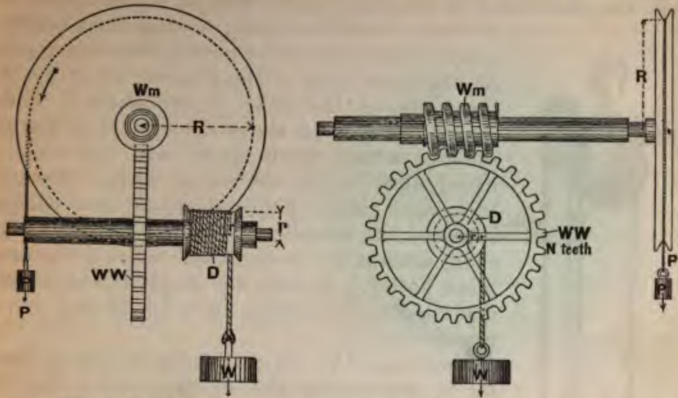
Combined Pulley, Worm, Worm-wheel and Winch Drum.—This combination is shown by the accompanying end and side views drawn from an experimental piece of apparatus in

* Refer to the next figure.

† The term *perpetual* screw would express more exactly its action, for when in motion, it continually screws the worm-wheel round.

‡ The greater the diameter of the screw and the smaller its pitch is, the better will be its bearing on the teeth of the wheel, but then the efficiency will be so small that there will be no chance of overhauling. This is the condition to be observed when the screw is intended to drive the wheel. If, however, it should be required to drive the screw by the wheel, or necessary that overhauling should take place, then the screw must be small in diameter, its pitch very great, and either double or treble threaded.

the Author's Laboratory, which is used by the students for ascertaining the efficiency of the machine, and for finding the co-efficient of friction between the endless screw and worm-wheel.



END VIEW. SIDE VIEW.

PULLEY, WORM, WORM-WHEEL AND WINCH DRUM.

INDEX TO PARTS.

P	represents	Pull applied to pulley.	N	represents	Number of teeth in WW.
R	,,	Radius of pulley.	D	,,	Drum, or diameter of winch barrel.
Wm	,,	Worm or endless screw.	r	,,	Radius of drum, D.
WW	,,	Worm wheel.	W	,,	Weight to be lifted.

By the *Principle of Work* (neglecting friction), if the drum, D, makes one turn, and if the worm be a single-threaded screw,

$$P \times \text{its distance} = W \times \text{its distance}$$

Or,
$$P \times 2\pi RN = W \times 2\pi r$$

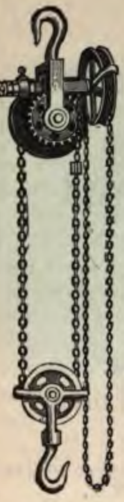
($\div$ both sides by 2π)

$$P \times RN = W \times r$$

$$\therefore \frac{P}{W} = \frac{r}{RN}*$$

* It will be evident to the student that, given any four of these five values, he can change this formula so as to find the fifth one; and, that he can experiment with this machine in precisely the same way as has been already explained in the case of the wheel and axle, block and tackle, Weston's *pulley block* and screw, &c., to ascertain its working advantage, *co-efficient of friction* and efficiency.

Worm-wheel Lifting Gear.—The accompanying figure shows a practical application of the endless screw and worm-wheel for the same purpose as the Weston's differential block is used—viz., the lifting of weights without fear of the tackle over-hauling. A light-driving endless chain passes over a V-grooved pulley having ridges or teeth on the inner sides of the grooves, so as to fit the pitch of the links of the chain. This pulley is keyed to the outer end of a worm spindle, whose screw gears with a worm-wheel fixed to or cast along with a second V-grooved ridged pulley or drum, over which is passed the movable end of a heavier lifting chain after it has been reeved under a snatch-block pulley. In fact, it is simply the previous experimental apparatus in a handy and compact form.



WORM-WHEEL LIFTING GEAR. (By Holt & Willets.)

EXAMPLE IV.—If in lifting tackle of the above description the driving pulley has a radius $R = 5''$, the number of teeth in the worm-wheel $N = 20$, and the driven pulley a radius $r = 5''$; what weight suspended from the snatch-block hook could be lifted by a force of 10 lbs. applied to the forward side of the light chain—(1) Neglecting friction, (2) if the modulus or efficiency of the whole apparatus were only $\cdot 25$?

ANSWER.—(1) Applying the previous formula, and taking account of the fact that the lifting chain is combined with a snatch-block, we have—

$$W = 2\frac{P \times R \times N}{r} = \frac{2 \times 10 \times \cancel{5} \times 20}{\cancel{5}''} = 400 \text{ lbs.}$$

(2) Owing to friction, weight of chain and snatch-block, the actual result obtainable is only $\cdot 25$, or 25 per cent. of this theoretical value; consequently

$$100 : 25 : 400 : x$$

$$x = \frac{25 \times 400}{100} = 100 \text{ lbs.}$$

LECTURE XV.—QUESTIONS.

1. A horizontal screw, of 1 inch pitch, is fitted to a sliding nut which is pulled horizontally by a cord passing over a fixed pulley, and having a weight, W, attached to it. To the free end of the screw there is fixed a pulley of 20 inches diameter, from the circumference of which a weight, P, hangs by a cord. Find the ratio of P to W. *Ans.* 1 : 62·8.

2. In a set of combined lever, screw, and pulley gear, like that illustrated before Example I. in this Lecture, R = 6", P = 2 lbs., W = 50 lbs., and the pitch of the screw is such that there are 2 threads to the inch; find (1) velocity ratio, (2) theoretical advantage, (3) working advantage, (4) work put in to lift W 1 ft., (5) work got out, (6) percentage efficiency. *Ans.* (1) 75·4 : 1; (2) 75·4 : 1; (3) 25 : 1; (4) 150·8 ft.-lbs.; (5) 50 ft.-lbs.; (6) 33·1 per cent.

3. Describe, with sketches, the construction of an ordinary lifting jack in which the weight is lifted by means of a screw and nut. If the screw be 1 inch pitch, the lever 20 inches long, and the pressure applied at the end of the lever be 30 lbs.; what weight can be lifted (neglecting friction)? (Take π = 3·1416.) (S. and A. Exam. 1899.) *Ans.* 1884·9 lbs.

4. In a *screw-jack*, where a worm-wheel is used, the pitch of the screw is $\frac{3}{8}$ inch, the number of teeth on the worm-wheel is 16, and the length of the lever is 10 inches; find the gain in pressure. *Ans.* P : W : : 1 : 1069.

5. What practical objection is there to the use of screw gear of any description for obtaining great pressure? Take for example the case of the screw-lifting jack. Sketch in vertical section and plan, and describe, a traversing one to lift say 20 tons. Explain how the screw of the jack is raised and lowered without being turned round.

6. Sketch and describe the construction and action of a screw press for pressing goods so as to make them into bales for transport. What force must be applied at the end of a screw press lever 8' 4" in length, in order to exert on the goods a total pressure of 22,000 lbs. when the pitch of the screw is 1"? If 60 per cent. be lost in friction, what pressure would result from the application of this force on the lever? *Ans.* 35 lbs.; 8800 lbs.

7. Sketch an ordinary bench vice. Apply the principle of work to find the gripping force obtained when a man exerts a pressure of 15 lbs. at the end of a lever 15 inches long, the screw having 5 threads per inch, the length from the hinge to the screw being 12 inches, and the length from the hinge to the jaws being 16 inches. *Ans.* 5303·6 lbs.

8. Explain, with a sketch, the manner in which the principle of work is applied in determining the relation of P to W in the case of the endless screw and worm-wheel. The lever handle which works the screw being 14" long, the number of teeth in the worm-wheel 20, and the load being a weight of 1000 lbs. hanging upon a drum 12" diameter on the worm-wheel shaft, find the force to be applied at the end of the lever handle in order to support the weight. (S. and A. Exam. 1887.) *Ans.* 21·43 lbs.

9. Explain the mechanical advantage resulting from the employment of an endless screw and worm wheel. The lever handle which turns an endless screw is 14" long, the worm, which has 32 teeth, and a weight, W, hangs by a rope from a drum 6" diameter, whose axis coincides with that of the worm-wheel. If a pressure P be applied to the lever handle, find the ratio of P to W. (S. and A. Exam. 1883.) *Ans.* P : W : : 3 : 448. If in this question the worm be changed to (1) a double, and (2) a treble-threaded screw, what will be the respective ratios of P to W? *Ans.* (1) 1 : 74·$\dot{6}$; (2) 1 : 49·$\dot{7}$.

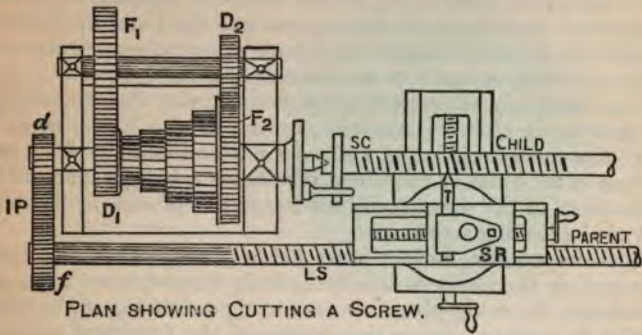
LECTURE XVI.

CONTENTS.—General Idea of the Mechanism in a Screw-cutting Lathe—Motions of the Saddle and Slide Rest—Velocity Ratio of the Change Wheels—Rules for Calculating the Required Number of Teeth in Change Wheels—Examples I. II.—Movable Headstock for a Common Lathe—Description of the Screw-cutting Lathe in the Author's Electrical Engineering Workshop, with a complete set of Detail Drawings—Questions.

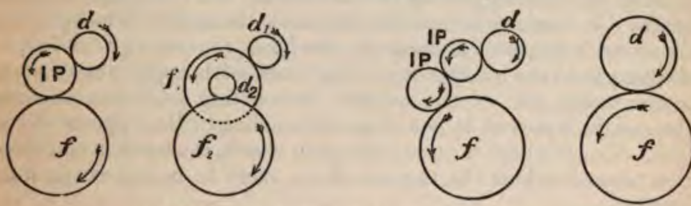
General Idea of the Mechanism in a Screw-cutting Lathe.—We will devote this Lecture to giving a general idea of the mechanism by which screws are cut in lathes, and the velocity ratio of the screw to be cut to the leading screw, together with a description of a complete set of illustrations prepared from working drawings of a new self-acting screw-cutting lathe.

Referring to the following figure, and to the general view of the 6-inch centre screw-cutting lathe (further on), it will be seen that the round metal bar on which the screw is to be cut is placed between the steel centres of the fixed and movable headstocks of the lathe. This bar has an eye-catch on its end next to the fixed headstock, which engages with a driving-stud connected to the face-plate. In order to obtain the necessary force to cut the screw, and to reduce the speed of the workshop motion shafts (in the case of a power lathe, or of the treadle shaft in the case of a foot lathe) to the required velocity, the fixed headstock is supplied with back motion gearing. The back wheels may be put into or out of gear with the lathe spindle wheels at pleasure, by a simple eccentric motion (in the case of the lathes herein illustrated), or, as is sometimes effected, by sliding the back shaft forward, so that its wheels clear those on the lathe spindle, and then fixing it there, by a tapered pin fitting through a hole in the framing and a groove cut in the shaft. But, when the back motion is required for the purpose of making a slow heavy cut, the follower F_2 is thrown out of gear with the stepped cone pulley, so that the driver D_1 (which is keyed to the cone) may *turn the follower* F_1; and the driver D_2 (which is keyed to the *same spindle* as F_1) rotate the follower F_2 (which is keyed to the *lathe spindle*), and hence revolve the face-plate and the bar, out of which the screw is to be formed.

On the back extension of the lathe spindle there is fixed a change wheel or small driver, *d*, which gears with a follower, *f* (keyed to the left-hand end of the leading or parent screw), either direct in the case of the cutting of a very finely pitched left-handed screw; or, through the intervention of a transmitting, or



PLAN SHOWING CUTTING A SCREW.



END VIEWS SHOWING CHANGE WHEELS.

FOR RIGHT-HANDED SCREWS. FOR LEFT-HANDED SCREWS.

GENERAL IDEA OF MECHANISM IN A SCREW-CUTTING LATHE.

INDEX TO PARTS.

SC	represents	Screw to be cut, or child.	F_1, F_2	represents	Followers of fixed headstock.
LS	„	Leading screw, or parent.	*d*, *f*	„	Driver and follower of change wheels.
SR	„	Slide rest.	IP	„	Idle pinion of change wheels.
D_1, D_2	„	Drivers of fixed headstock.			

what is technically termed an idle, pinion, IP, in the case of a medium-pitched right-hand screw. (See also the end views of the change wheels above.)

It will therefore be seen that there are two independent motions to be considered—(1) the reducing gear from the speed of the

driving cone to that of the lathe-spindle or bar to be operated upon; and (2) the multiplying or reducing gear between the lathe-spindle and the leading screw. The former of these will be at once understood from the figures, and from what was said in regard to wheel-gearing in Lecture XII.

We shall now consider the second motion. Remembering that the pitch of the *parent* or leading screw is fixed and unalterable, and that on its truth depends to a large extent the accuracy with which the *child*, or screw to be cut, can be formed, it will be clear that we have only to connect these two parallel shafts with suitable gearing in order to transmit, by aid of the "*copying principle*" the characteristics of the parent to the child.* This may be done in an equal or magnified or diminished degree, according as the pitch of the screw to be cut is equal or greater or less than that of the leading screw.

Motions of the Saddle and Slide Rest.—The base of the slide rest, or the saddle as it is technically termed, bears upon and is guided by the truly-planed shears (or upper framing of the lathe) parallel to the line joining the centres of the fixed and movable heads. In turning a *right-handed screw* the saddle is moved *from the movable headstock towards the fixed one, or from right to left*, by clasping it to the leading or guiding screw with a split nut attached to the under side of the saddle. In cutting a *left-handed screw* the saddle is moved by the same means, but in the opposite direction,—*i.e., from left to right*. In other words, it travels in the direction towards which the threads of the screw to be cut are inclined forward.

To the upper side of the saddle is bolted the slide-rest surmounted by the tool-holder. The rest is provided with two independent sliding motions, each actuated by a hand-turned screw, and guided by a true plane surface with dovetailed sides. These motions (for the purposes of turning parallel work and screws) are fixed at right angles to each other, the lower one being parallel to the centre line of the lathe, and the upper one at right angles thereto. Both motions are therefore independent of each other and of the sliding motion of the saddle. The turner is thereby enabled to adjust the cutting tool with great delicacy and accuracy with reference to the job to be operated upon, irrespective of the automatic travel of the supporting saddle.

Velocity Ratio of the Change Wheels.—As has been mentioned already, the change wheels are interposed between the

* It is reported that Sir Joseph Whitworth, feeling the importance of a *thoroughly true leading screw*, spent an immense deal of money upon the scraping and finishing of a parent screw for a first-class lathe, from which many of the best screws in this country have been copied.

back end of the lathe spindle and the leading screw, for the purpose of transferring motion to the saddle, and determining, that the cutting tool shall be moved through a definite pitch for each rotation of the cylinder to be turned or screwed. Every turn of the leading screw moves the saddle and cutting tool through a distance equal to its pitch, and consequently if the bar to be screwed, turns at the same rate as the leading screw, the pitch of the screw cut upon it, will be the same as that of the leading screw. If it moves faster than the leading screw, the pitch will be less; and if slower, the pitch will be correspondingly greater. It therefore follows as a matter of course, that if we fit wheels on the lathe spindle and on the leading screw of the same diameter, or having the same number of teeth, the screw being cut will have the same pitch as the leading screw. If we fix a small pinion, or one with few teeth, on the lathe spindle and a wheel of large diameter, or many teeth on the leading screw, the pitch of the screw to be cut will be small, compared with that of the leading screw. Or, if the number of turns per minute of the leading screw be greater than that of the screw being cut, the pitch of the latter will be greater than that of the former, and *vice versâ*.*

Rules for Calculating the Required Number of Teeth in Change Wheels.—The following rules simply express the previous reasoning in the form of proportion. In applying them, the student should again refer to the end views of the change wheels in the first figure of this Lecture.

$$\frac{\textit{Pitch of screw to be cut}}{\textit{Pitch of guiding screw}} = \frac{\textit{No. of teeth in 1st driver} \times \textit{No. in 2nd driver.}}{\textit{No. of teeth in 1st follower} \times \textit{No. in 2nd follower.}}$$

Let p_c = Pitch of screw to be cut *in inches, or fraction of inch, between two threads.*

" p_g = Pitch of guiding screw " " "

" d_1, d_2 = Diameters or number of teeth in drivers.

" f_1, f_2 = Diameters or number of teeth in followers.

Then,
$$\frac{p_c}{p_g} = \frac{d_1 \times d_2}{f_1 \times f_2}.$$

Or,
$$p_c \times f_1 \times f_2 = p_g \times d_1 \times d_2$$

* What was said in Lectures XII. XIII. and XIV. enables the student to see clearly the velocity ratio between the cut screw and the leading screw. We need scarcely remind the student that the above statements refer to the pitch of a screw as *the distance between two consecutive threads, and not to the number of threads per inch.* If the number of threads per inch of its length are taken as the pitch, instead of the distance between two threads, the reverse ratio will hold good. Since a pitch of $\frac{1}{4}''$ means 4 threads to the inch, *a pitch of* $\frac{1}{3}''$ means 3 threads to the inch, and a pitch of $\frac{1}{2}''$ means *2 threads to the inch.* Or, the number of threads per inch is *inversely proportional to the distance* between two consecutive threads of the screw.

When the train of wheels is a compound one, as in this case, the two intermediate multiplying or reducing wheels, f_1 and d_2, are fixed to any outstanding movable arm or quadrant at the left-hand end of the lathe, so as to bring them into gear with d_1 and f_2. (See second view of the previous figure.)

If the train of wheels is a simple one, as in the first, third, and fourth views referred to above, where there is only one driver, d, and one follower, f, with, when necessary, one or more idle pulleys, IP, simply for the purpose of connecting d and f and of giving f the desired direction of rotation, then—

$$\frac{p_c}{p_g}=\frac{d}{f}, \text{ or } p_c \times f = p_g \times d.$$

Should the pitch of a screw be expressed by the *number of threads per inch of its length*—as is usually the case in tables of screws and change wheels—then you can either convert this number into the pitch proper, by taking its reciprocal—(*i.e.*, by making the number of threads per inch the denominator of a fraction, with 1 for the numerator) or you may say—

Let t_c = Threads per inch of screw to be cut.
" t_g = Threads per inch of guiding screw.

Then, since the number of threads per inch are inversely proportional to the distance between any two consecutive threads,

$$\frac{t_c}{t_g}=\frac{p_g}{p_c}=\frac{f_1 \times f_2}{d_1 \times d_2}$$

Or, . . $t_c \times d_1 \times d_2 = t_g \times f_1 \times f_2$

If the train is a simple one, then

$$\frac{t_c}{t_g}=\frac{f}{d}; \text{ or, } t_c \times d = t_g \times f$$

Example I.—The lathe illustration further on, has a guiding screw of $\frac{1}{4}''$ pitch, or 4 threads to the inch. Calculate the number of teeth in the change wheel to be fixed to the end of the guiding or leading screw in order to cut a screw of 8 threads to the inch when the driver on the lathe-spindle has 40 teeth.

Compare the answer with the change-wheel table printed above the general view of the screw-cutting lathe, further on in this Lecture.

Answer.—Here $t_c = 8$; $t_g = 4$; $d = 40$; and you are required *to find* f.

By above formula,

$$\frac{t_c}{t_g} = \frac{f}{d}; \quad \text{or, } \frac{8}{4} = \frac{f}{40} \therefore f = \frac{8 \times 40}{4} = \textbf{80 teeth.}$$

By using the previous formula, we have $p_c = \frac{1}{8}''$ and $p_g = \frac{1}{4}''$

$$\therefore \frac{p_c}{p_g} = \frac{d}{f}; \quad \text{or, } \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{40}{f} \therefore f = \frac{\frac{1}{4} \times 40}{\frac{1}{8}} = \frac{8 \times 40}{4} = \textbf{80 teeth.}$$

It is at once evident from this example that you avoid having to multiply and divide by sometimes awkward fractions if you consider the number of threads per inch as the measure of the pitch of the screw, instead of the distance between two threads.

EXAMPLE II.—The guiding screw of a lathe is $\frac{1}{2}''$ pitch, and you are required to cut screws of $\frac{1}{10}''$ and $\frac{1}{20}''$ pitch respectively. Determine the number of teeth in the follower, given the use of a driver having 20 teeth.

ANSWER.—For a screw of $\frac{1}{10}''$ pitch, or 10 threads per inch, and using a driver of 20 teeth, we get by the above formula for a simple train,

$$\frac{t_c}{t_g} = \frac{f}{d}; \quad \text{or, } \frac{10}{2} = \frac{10}{2} \times \frac{10}{10} = \frac{100}{20} = \frac{f}{d}$$

For a screw of $\frac{1}{20}''$ pitch the number of threads per inch will be 20, and using a driver of 20 teeth, we find from the formula for a compound train—

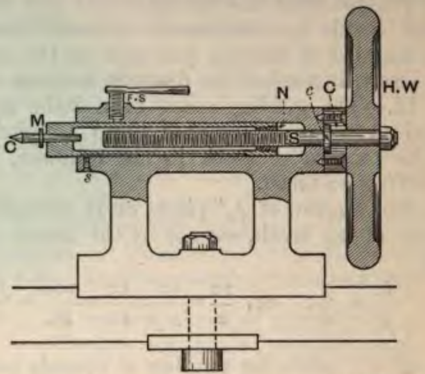
$$\frac{t_c}{t_g} = \frac{f_1 \times f_2}{d_1 \times d_2}$$

$$\therefore \frac{20}{2} = \frac{4 \times 5}{1 \times 2} = \frac{80 \times 100}{20 \times 40} = \frac{f_1 \times f_2}{d_1 \times d_2}$$

Here we multiplied numerator and denominator by 20, in order to obtain suitable wheels, of which d_1 will have 20 teeth. (See in the previous figure the second of the end views showing change wheels.)

Movable Headstock for a Common Lathe. — Before describing a complete screw-cutting lathe we will explain the use and construction of this part of a common small lathe for ordinary work. As will be seen from the accompanying rough sketch, it consists of a cast-iron poppet-head planed on its under side, so as to engage the breadth of the top of the shears. It may be bolted thereto in any desired position (along the length of the bed) by an underneath *iron plate* placed across the shears, and a single *vertical bolt.* The upper portion of the head is cylindrical, and *is bored for about* seven-eighths of its length to receive a round

hollow steel mandril, M, and for the remaining one-eighth to receive the spindle S. The mandril is fitted in front with a tapered centre, C, and behind with a screw nut, N. The centre is for carrying one end of the job to be operated upon by the turning tool, and the nut is for engaging the screwed part of the spindle S. On the back end of the spindle there is a collar, *c* (kept in position by a larger collar or guard, G, with small screws), and a hand-wheel, HW.* Consequently, by turning this wheel in



MOVABLE HEADSTOCK FOR A COMMON LATHE.

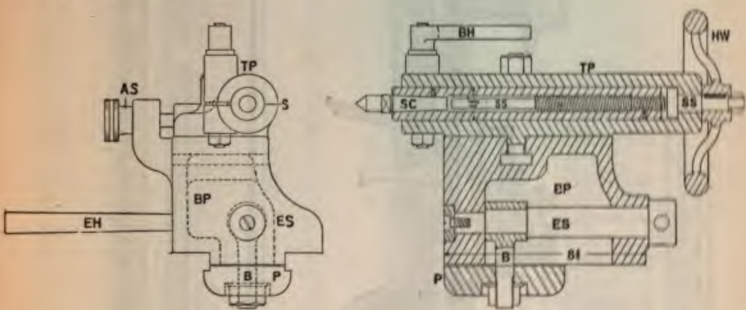
one direction the mandril and its centre are forced forward, and when moved in the opposite direction they are screwed backwards. To prevent the mandril turning round, it is fitted with a longitudinal slot on its underside, into which fits the flattened or rounded end of a small screw, *s*. A fixing stud, FS, with a handle, enables the mandril to be clamped to the head when it has been adjusted by the hand wheel and screwed spindle.

Description of a Screw-cutting Lathe.—By the favour of Messrs. John Lang & Sons we are enabled to give a general view, with a complete set of reduced working drawings, carefully indexed to every detail, of the very strong and superior 6-inch centre screw-cutting lathe, lately presented to the Author's Electrical Engineering Laboratory and Engineering Workshop by Mr. Andrew Stewart, of Messrs. A. & J. Stewart, and Clydesdale. This lathe weighs, with all its chucks and supernumerary parts, over 15 cwt. It has a bed 6 feet long, and admits a bar 3 feet

* *This arrangement of collar and guard is neither good nor strong, although frequently adopted in the case of small foot-lathes. The collar should be inside the bored head, behind the nut N.*

2 inches between its centres. The bed is 9½ inches broad and 6½ inches deep. The *gap* is 9 inches wide and 6 inches deep; consequently the lathe can swing a job of 20 inches diameter clear of the leading screw, and one of 24 inches diameter when this screw is withdrawn from its bearings. The *speed-cone* has three pulleys, each 2½ inches broad, the diameter of the largest being 8 inches and that of the smallest 4 inches.

The makers have planed and scraped the *bed* to a true bearing surface, and have so fixed the gap piece that it cannot wear loose or spring the bed.



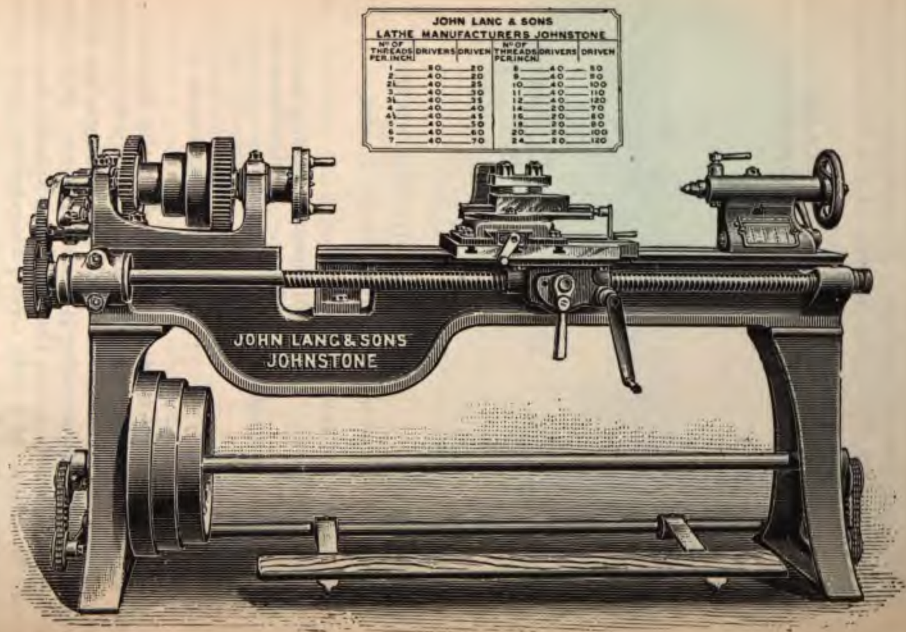
END VIEW. LONGITUDINAL SECTION.

MOVABLE HEADSTOCK OF SCREW-CUTTING LATHE.

INDEX TO PARTS.

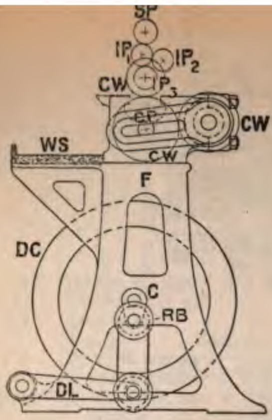
BP	represents	Bottom part.	BH	represents	Binding handle.
TP	,,	Top part.	AS	,,	Adjusting screw.
S	,,	Spindle, or mandril.	ES	,,	Eccentric spindle.
SC	,,	Steel centre.	EH	,,	Eccentric handle.
SS	,,	Steel screw.	B	,,	Bolt for clamping.
HW	,,	Hand wheel.	P	,,	Plate under B.

The *movable or loose headstock* is gripped to the bed by an eccentric motion worked by a handle, so that it may be instantly clamped in position without the trouble of finding a key to fit the usual nut, and then screwing it gradually home. The upper part of this head, which carries the mandril or spindle, has a side adjustment by means of a side screw, whereby the steel centre may be truly aligned with the corresponding centre of the fast headstock, or it may be moved to the one side or to the other in the case of taper turning. A small oil-holder is cast on the back side of the head to facilitate the oiling of the steel centre without having to look for an oil-can.

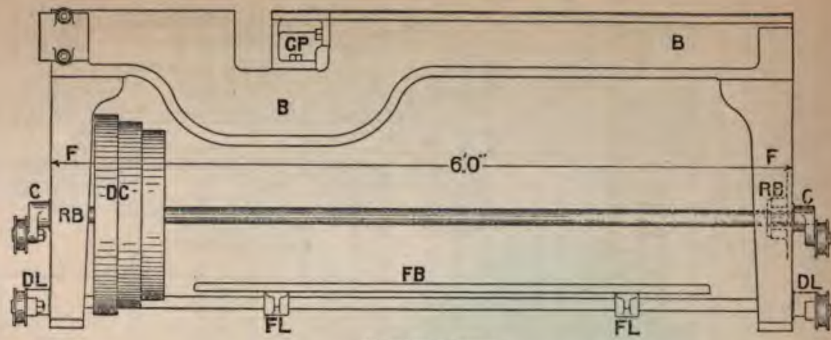


SELF-ACTING SCREW-CUTTING TREADLE LATHE WITH HAND SURFACING MOTION.

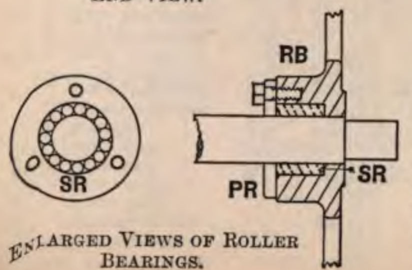
BED, FRAMINGS AND FOOT-DRIVING GEAR FOR SCREW-CUTTING LATHE.



END VIEW.



FRONT VIEW.



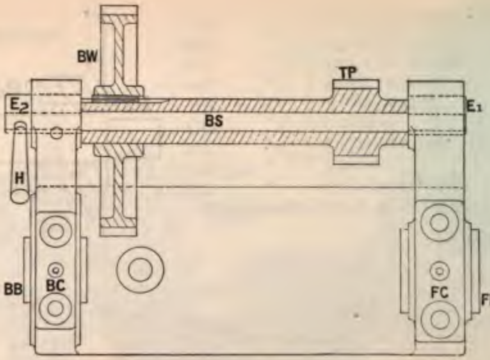
ENLARGED VIEWS OF ROLLER BEARINGS.

INDEX TO PARTS.

B	represents	Bed, or shears.	SR	represents	Steel rollers.
GP	"	Gap piece.	PR	"	Plate for keeping rollers in position.
FF	"	Frames.	CW	"	Change-wheels.
DC	"	Driving cone.	CP	"	Change-plate or quadrant for attaching CW.
CC	"	Cranks.	$IP_{1, 2, 3}$	"	Idle pinions connecting CW and SP.
FB	"	Foot board.	SP	"	Spindle pinion.
FL	"	Foot levers.			
DL	"	Driving levers.			
WS	"	Wooden shelf.			
RB	"	Roller bearings.			

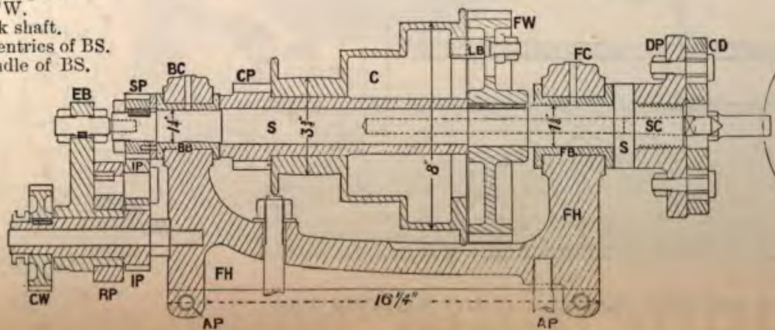
INDEX TO PARTS.

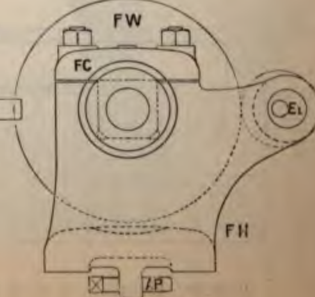
FH for Fast headstock.
AP „ Adjusting pins.
FB „ Front bush.
FC „ Front cover.
BB „ Back bush.
BC „ Back cover.
S „ Spindle.
C „ Cone for speeds.
CP „ Cone pinion (or D_1 in formula).
BW „ Back wheel (or F_1 in formula).
TP „ Tube and pinion (or D_2 in formula).
FW „ Front wheel (or F_2 in formula).
LB „ Locking bolt of FW.
BS „ Back shaft.
E_1, E_2 „ Eccentrics of BS.
H „ Handle of BS.



INDEX TO PARTS.

DP for Driving plate.
CD „ Clements driver.
SC „ Steel centre.
SP „ Spindle pinion (or d in formula).
IP „ Idle pinions.
EB „ End brackets.
RP „ Reversing plate.
CW „ Change-wheels (or f in formula).

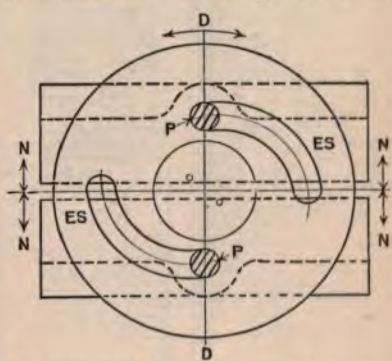




The *spindle* of the *fast headstock* is made of hard crucible steel ground accurately cylindrical, where it fits into parallel gun-metal bearings. These bearings are of extra diameter and length. This spindle is bored hollow for 12 inches of its length, in order to admit small rods for making terminals and screws in electrical engineering work. The *speed-cone* is turned inside and outside, and properly balanced. A specially strong and simple *reversing gear* has been fitted to the back end of this headstock, whereby the machine-cut steel pinions for turning right and left hand screws may be put into or out of gear by simply depressing or elevating a reversing handle. The *back-motion gear* is actuated by means of a handle and eccentrics on each end of the back-motion shaft; whilst the *front wheel* (or last follower, F_2, as we have symbolled it in the formula) is locked to the cone or thrown out of gear therewith in the usual way—viz., by a bolt fitting into a sliding slot in the cone and a projecting nut on the side of the toothed wheel.

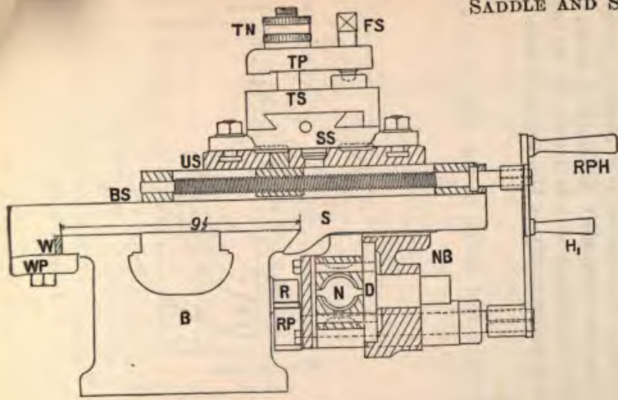
The *saddle* has T slots on its upper side for the purpose of bolting work to it that requires boring out, and which necessitates the removal of the slide rest. A quick hand traverse motion is provided for the saddle by means of a rack and pinion motion, quite independent of the sliding motion of the leading screw. The leading screw is turned to the standard pitch of $\frac{1}{4}$ inch, or four threads to the inch. The engaging nut is made in halves, so that it may grip the leading screw fairly at the top and bottom of the threads.*

* In order to make the construction and action of the split nut which engages the leading screw clearer, we show here an enlarged view with the halves of the nut, N← →N, slightly apart, and the disc handle removed, so as to bring into full view the two eccentric slots, ES, which guide the two steel pins, P and P, fixed on N and N. By comparing this view with the others under heading "Saddle and Slide," the student will see how, by merely turning the disc handle DH the disc D is moved round through nearly a quarter of a circle, and the eccentric slots ES cause the pins, P, P, to move closer to or further away from the centre of the disc D, and consequently move the two parts of the nut, N, N, in or out of gear with the leading screw.

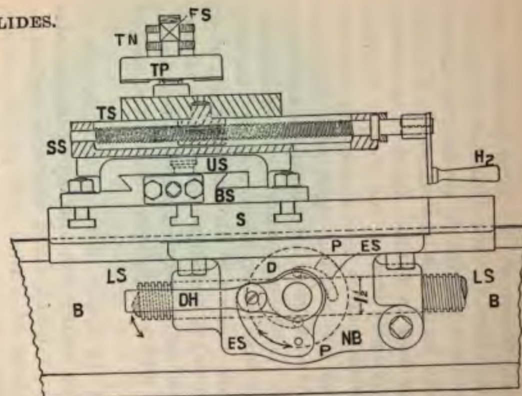


ENLARGED VIEW OF SPLIT NUT FOR LEADING SCREW, &c.

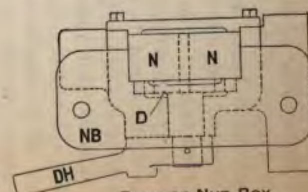
SADDLE AND SLIDES.



CROSS SECTION.



LONGITUDINAL SECTION.



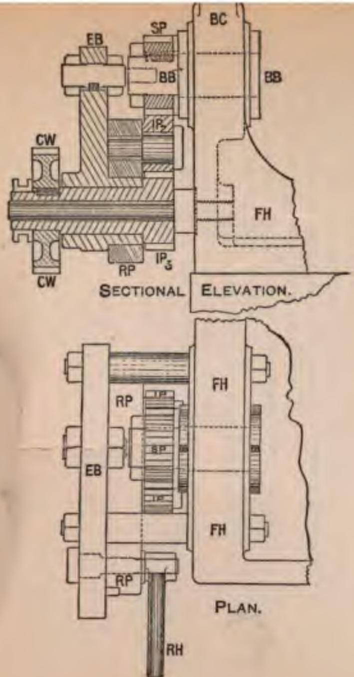
PLAN OF NUT BOX.

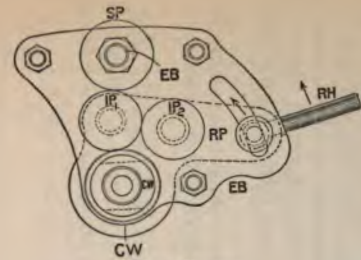
INDEX TO PARTS.

B for Bed, or shears.
S „ Saddle.
WP „ Wedge plate for S.
W „ Wedge for S.
BS „ Bottom slide.
H_1 „ Handle for BS.
US „ Under slide.
SS „ Swivel slide, for taper turning.

TS for Top slide.
H_2 „ Handle for TS.
TP „ Top plate.
TN „ Thumb nut.
FS „ Fixing stud.
LS „ Leading screw.
NB „ Nut box for N.
N „ Nut in halves for engaging LS.

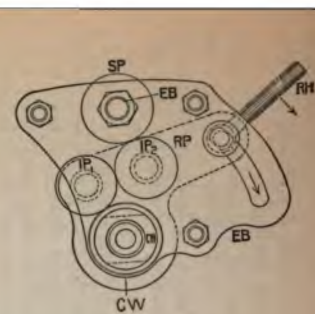
D for Disc with ES.
ES „ Eccentric slots.
PP „ Pins working in ES.
DH „ Disc handle.
R „ Rack on B.
RP „ Rack pinion.
RPH „ Rack pinion handle for turning RP and moving S by hand.





END VIEW.

Showing IP$_1$ *in gear with* SP *for cutting a Right-handed Screw.*



END VIEW.

Showing IP$_2$ *in gear for cutting a Left-handed Screw.*

DIFFERENT VIEWS OF THE CHANGE-WHEEL GEAR.

INDEX TO PARTS.

FH	represents	Fast headstock.
BB	,,	Back bush.
BC	,,	Back cover.
SP	,,	Spindle pinion.
IP$_1$	,,	Idle pinion for right-hand screws.
IP$_2$	,,	Idle pinion for left-hand screws.
EB	represents	End bracket for supporting RP, &c.
RP	,,	Reversing plate to carry IP$_1$, IP$_2$.
RH	,,	Reversing handle.
CW	,,	Change-wheels, with change-wheel nut, CN.

A *compound slide rest* is fitted to the top of the saddle, having large bearing surfaces with adjustments for taking up the wear, and a swivel arrangement for conical boring.

All the toothed wheels, including the change-wheels, have had their teeth cut directly from the solid casting, by the makers' special tool for that purpose, so that back-lash, and consequently noise and vibration arising from fast-speed driving may be minimised as far as possible.

The *driving shaft* has anti-friction steel roller-bearings. It is connected to the foot-treadle at each end by a pulley, chain, and crank. The driving-cone is so stepped that the belt has equal tension on any corresponding pair of driving and driven pulleys. It is sufficiently heavy to act as a fly-wheel. It is balanced along with the treadle to secure an easy, steady drive. A power-drive may be applied if desired, but the author believes that, as students should work in pairs or in sets of three in a laboratory, they will take a deeper interest in their experiments if they have turned out everything by their own skill and labour, than if motive power were freely supplied to them.

Of *heavy chucks* there are a very complete set, including a four-jaw expanding chuck, clements driver, drill chucks for both the fast and loose head spindles, &c.

The student should now go over each drawing *most carefully* by aid of the corresponding index to parts, and, if they feel inclined, by comparing the drawings with an actual screw-cutting lathe.

LECTURE XVI.—QUESTIONS.

1. Sketch the fast headstock of a double-geared lathe, and explain the contrivance for increasing or diminishing the speed of the mandril. In the headstock of a lathe a pinion of 20 teeth drives a wheel of 60, and a second pinion of 20 drives another wheel of 60; compare the rates of rotation of the first driving pinion and of the mandril of the lathe. *Ans.* 1 : 9.

2. Why is a lathe often back-geared? Sketch a section through the headstock showing the arrangement. If the two wheels have 63 and 63 teeth respectively, and each pinion has 25 teeth, find the reduction in the velocity ratio of the lathe spindle due to the back-gear. (S. and A. Exam. 1891.) *Ans.* 6.35 : 1.

3. Make a vertical longitudinal section through the *movable* or loose headstock of a lathe, showing precisely the manner in which a screw and nut are applied to produce the necessary movement of the centre which supports the work. Name the materials of which the several parts are made. (S. and A. Exam. 1888.)

4. What is the use of the guide-screw in a lathe? Where is it usually placed? Show by sketches the precise manner in which the slide rest is connected with or disengaged from the guide-screw. (S. and A. Exam. 1890.)

5. Describe and show by sketches the means by which the slide rest of a lathe may be connected with the leading screw. If the slide rest traverses the bed at the rate of $1\frac{1}{6}$ feet when the leading screw makes 56 revolutions, what is the pitch of the screw thread? (S. and A. Exam. 1892.) *Ans.* $\frac{1}{4}$ inch.

6. Sketch and describe the mechanism by which the saddle of a screw-cutting lathe can be made to travel automatically in either direction along the lathe bed while the speed pulleys run always in the same direction. (S. and A. Exam. 1890.)

7. How is the *copying principle* applied in a screw-cutting lathe? Describe a method of throwing a self-acting screw-cutting lathe in and out of gear, and of reversing it by means of a belt and overhead pulleys. (See Fig. 5 in Lecture XI.)

8. Explain the use of the quadrant for change wheels in a screw-cutting lathe by making a sketch showing it in its position on a lathe with the wheels in gear. (See the general and the end views of the 6″ screw cutting-lathe bed, and index for the part marked CP.)

9. Explain the mode in which *change wheels* are employed in a screw-cutting lathe. The leading screw being of $\frac{1}{3}$-inch pitch, arrange, on a sketch, the change wheels as required for cutting a screw of 15 threads to the inch, marking the numbers on each wheel.

10. Sketch and describe the mechanism for cutting a screw with five threads to the inch in a lathe where the guide screw has three threads to the inch. Assign suitable numbers to the wheels which you would employ. (S. and A. Exam. 1889.)

11. The leading screw of a lathe is $\frac{1}{2}$-inch pitch, and *right-handed*. Sketch and describe the arrangement whereby you would employ the lathe for cutting a screw of $\frac{1}{4}$-*inch pitch*, and *left-handed*.

12. *Describe the operation of cutting a screw in a lathe, showing the wheels required, and how they are placed to cut a right-handed screw with eight threads to the inch in a lathe whose leading screw is of* $\frac{1}{2}$-*inch pitch.*

13. Explain the use of change wheels in a screw-cutting lathe. desired to cut a screw of $\frac{3}{8}$-inch pitch in a lathe with a leading scr four threads to the inch, using four wheels. If both screws be handed, what wheels would you employ? (S. and A. Exam. 1887.)

14. The leading screw in a self-acting lathe has a pitch of $\frac{1}{2}$ inch; an arrangement of change wheels for cutting a screw of $\frac{3}{8}$-inch pitch.

15. You are required to cut a left-handed screw of five threads t inch in a lathe fitted with a right-handed guide-screw of $\frac{1}{2}$-inch p Show clearly by the aid of sketches the change wheels which you w employ for the purpose, indicating how they would be respectively ca and the number of teeth in each wheel. (S. and A. Exam. 1891.)

LECTURE XVII.

CONTENTS.—Hydraulics—Definition of a Liquid—Axioms relating to a Liquid at Rest—Transmission of Pressure by Liquids—Pascal's Law —"Head" or Pressure of a Liquid at Different Depths—Total Pressure on a Horizontal Plane immersed in a Liquid—Sir William Thomson's Wire-testing Machine—Total Pressure on any Surface immersed in a Liquid—Examples I. II.—Questions.

Hydraulics.—Hitherto the student's attention has been confined to solid bodies, which were supposed to remain perfectly rigid and unchanged when acted upon by forces. We shall now direct his consideration to the properties and applications of another great division of matter—viz., liquids—which possess the marked opposite character of mobility under the action of forces. In nature we do not meet with either perfectly solid or perfectly liquid bodies; and consequently the practical engineer, when applying the formulæ of the physicist to his machines and hydraulic works, has to make certain allowances according to circumstances, with the aid of constants predetermined by experience and experiment.

The most common and the most useful liquid with which the engineer has to deal is that of water. Hence the term "hydraulic engineer," as applied to persons who direct and guide the action of waters, as in the case of the water supply for a town, or for navigation purposes, or for the transmission of force and power. The term hydraulics, therefore, comprehends *hydro-statics, which is the science of liquids in equilibrium*, and *hydro-dynamics, the science of liquids in motion*. We shall only have space in this manual for an elementary inquiry into the former of these two divisions of hydraulics.

Definition of a Liquid.—*A liquid is a collection of particles which are perfectly movable about each other.* In consequence of this property, a liquid requires some external force or resistance to keep its particles together, such as the sides of a vessel; for its molecules can be displaced by the smallest force, and are readily divided from each other in any direction.*

* *The late Prof. Clerk* Maxwell distinguished solids from liquids in the *following manner*:—"Bodies which can sustain longitudinal pressure,

Axioms relating to a Liquid at Rest.—It follows directly from the above definition, that when equilibrium exists—

(1) *The surface of a liquid at rest is horizontal;*

(2) *The surface of a liquid at rest is everywhere perpendicular to the force which acts upon it;*

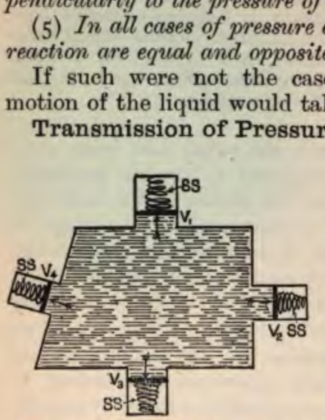
(3) *A liquid at rest acted on by a force presents a surface which is everywhere perpendicular to the direction of the force;*

(4) *A surface supporting a liquid at rest reacts everywhere perpendicularly to the pressure of the liquid;*

(5) *In all cases of pressure on or from liquids at rest, action and reaction are equal and opposite.*

If such were not the case, equilibrium could not exist, and motion of the liquid would take place.

Transmission of Pressure by Liquids.—Take a tight vessel filled with a liquid and fitted with four frictionless piston-valves, V_1, V_2, V_3, V_4, of the *same area.* Let the outward pressure on these valves be balanced by spiral springs, arranged so that they indicate the forces applied to them. Now apply an *inward* force of, say, 1 or 5 or 10 lbs. to the spiral spring of valve V_1, then instantly the other three springs of the other valves will register an outward pressure of the same amount as that applied. If the other valves had been of different areas from valve V_1, their springs would have registered pressures corresponding with the ratio of their areas to the area of valve V_1. Or the pressure per square inch on valve V_1 is communicated throughout the liquid to the other valves, and to every square inch of the internal surface of the vessel, with undiminished effect.



TRANSMISSION OF PRESSURE BY LIQUIDS.

Pascal's Law.—*Fluids transmit pressure equally and in all directions.** In the case of solids pressure is only transmitted

however small that pressure may be, without being supported by lateral pressure, are called solids, and those which cannot are termed liquids." *A perfect liquid is therefore one in which there is absolutely no resistance to a change of shape,* although there may be practically an infinite resistance to change of volume. We say practically because, although liquids are more or less compressible to a very small extent, yet the amount is so small as to be negligible in the case of most engineering problems.

* Here the word fluid has been used instead of liquid, as being more general, since the term fluid includes both liquids and gases. Refer to 3, Lecture I., for the distinction between a liquid and a gas.

along the line of its action, and therefore we have in this law an exemplification of the fundamental distinction between solids and fluids. In Lecture XIX. we will explain several machines that depend upon the principle enunciated by Pascal's law for their action.

Head or Pressure of a Liquid at Different Depths.—Imagine a very small horizontal area, a (for instance, a *square inch*), situated at a depth or height, h, inches from the free surface of a liquid, and that the *vertical* column from, a, to the surface becomes solidified without in any way disturbing equilibrium. It is evident that the horizontal and the vertical forces on the solid column must be separately in equilibrium, otherwise motion would ensue. But the only vertical forces are the weight of the column downward and the pressure of the surrounding liquid upwards on the base, a. Therefore,

The pressure upwards = weight of the prism.

Or, . . . $p = haw.$

Where, w, is the weight of every inch of its height or the weight of a cubic inch of the column. But the area, a, and the weight, w, are constant quantities for any particular unit of area and kind of liquid. Hence—

Pressure varies directly as the depth from the free surface.

Or, $p \propto h$.

The technical term "*head*" expresses the above fact in a single word. For, when speaking of the working pressure per square inch due to a supply of water for a mill wheel or turbine, we say it has 10 or 20 or 30 feet of head, meaning thereby the pressure due to a difference of level of so many feet, from the free surface of the water as it enters the supply pipe to the free surface of the tail race or discharge pipe. Since every foot of "head" of water gives in round numbers a pressure of 1 lb. per square inch, we might have said that the pressure was 5 or 10 or 15 lbs. respectively per square inch. Consequently,

Pressure varies directly as the head.

Total Pressure on a Horizontal Plane immersed in a Liquid.—Take a vessel of *any shape* having a horizontal base, and fill it with a liquid to any known height. Then from the above proof it follows that,

The Total Pressure on the base = { *height in inches from base to surface × area of base in square inches × weight of a cubic inch of the liquid.*

For, pressure per sq. in., $p = haw$, when, $a = 1$ square inch.

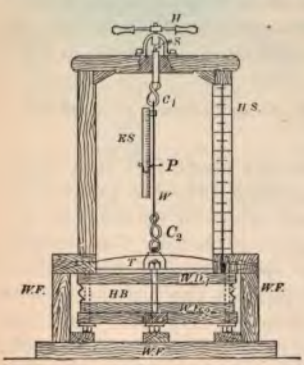
Consequently, if the total area of the horizontal plane be equal to, a, square inches, instead of 1 square inch.

The Total Pressure $= haw$.

This proves that the shape of the vessel containing the liquid, and the total weight of water in the vessel, *do not* in any way *affect the total pressure on the base.* For, it depends solely on the difference of level between the base (or immersed plane) and the free surface, on the area immersed, and on the weight per unit volume or specific gravity of the liquid.

This property results in what used to be termed the *hydrostatic paradox*, which is very well illustrated by Sir Wm. Thomson's apparatus for testing the tensile strength and percentage elongation of the sheathing wires used for covering and protecting the insulated conductors of submarine cables.

Sir Wm. Thomson's Wire-testing Machine, or Hydrostatic Paradox.—W represents the wire to be tested, which is fixed to the clips C_1 C_2. HB is a circular hydrostatic bellows, 3′ diameter, with india-rubber sides. WD_2 is the bottom wooden disc attached by bolts to an iron tripod T, which is connected at its centre to the clip C_2; while WD_1 is an upper wooden disc rigidly fixed to the wooden framing WF. H is a handle keyed to the screwed spindle S. HS is a hydrostatic scale, fixed behind the vertical glass tube which is fitted into a short brass cylinder passing through WD_1 and into HB. ES is the scale for measuring the percentage elongation. The upper end of this scale is fixed to the wire W, and the lower end is free. There is a clip pointer P which is affixed to each wire before testing it, and moved up or down until it is opposite to the zero of the scale ES.



THOMSON'S HYDROSTATIC WIRE-TESTING MACHINE.

Method of Testing Wire by this Machine.—(1) Turn the handle H backwards until C_1 is as far down as it can get. (2) Fix wire in clips, and attach the pointer P so as to be opposite the zero of scale ES. (3) Turn the handle H forward, thus lifting WD_2, and *stretching* the wire, by forcing water up the glass tube in front of *HS. This gives the necessary* "head," h, or pressure due to the *difference* in level between the free surface in the glass tube and *the bottom* of the wooden base WB_2. The area in square inches

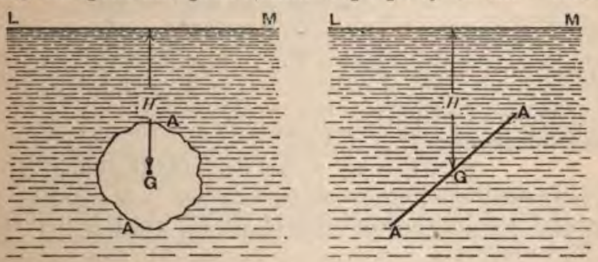
of this base gives, a, and hence the total pull on the wire is $= haw$. (4) Note the elongation by the scale ES, and the total tensile stress by the scale HS, at the moment the wire breaks. WD_2 falls upon stops, so as not to injure the india-rubber hydrostatic bellows HB.

This machine was used in 1872–73 by the Author and others in testing all the sheathing wire for the Western and Brazilian Company's cables. The homogeneous wire gave an average of 55 tons per square inch.

In this machine we see that, owing to the *quaqua versus* principle enunciated above a few pounds weight of water can produce a stress of many hundreds or even thousands of pounds by simply giving it "*head*," through a small tube in connection with an enlarged area.

When the sides of a vessel taper towards the top, as in the case of a wine bottle, the liquid pressing vertically upwards upon them produces a reaction on the base, which makes up for the want of weight of liquid which would be naturally due to direct vertical pressure in the case of a cylindrical vessel.

Total Pressure on any Surface immersed in a Liquid.—Let a surface of *any shape* be immersed in a liquid of *any kind* to *any depth*, as illustrated by the following figures. Then, by applying the previous proofs, and a property of the "centre of



END VIEW. SIDE VIEW.

PRESSURE ON ANY SURFACE IMMERSED IN A LIQUID.

gravity" (which affirms that the *mean perpendicular distance from any plane, is equal to the distance from the c.g. of the surface to that plane*), we find, that the *total pressure* on the immersed surface is represented by the following equation :

$$P = HAW.*$$

Where P = Pressure (total) in lbs.

,, H = Height from *c.g.* to free surface in feet.*

,, A = Area in square feet.*

,, W = Weight of a cubic foot of the liquid.*

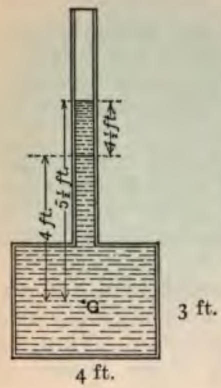
* *This formula will be* easily remembered by Englishmen, for they are

EXAMPLE I.—Find the total pressure on the bottom of a cubical tank having a bottom $4' \times 4'$ and filled with water to a depth of $4'$.

ANSWER.—By the above formula—

$$P = HAW.$$
$$P = 4' \times (4' \times 4') \times 62{\cdot}5 \text{ lbs.} = \mathbf{4010 \text{ lbs.}}$$

We may here remark that 62·4 lbs. is nearer the weight of a cubic foot of fresh water, and that a tank $4' \times 4' \times 4'$ holds 100 gallons = 1000 lbs. = 1·8 tons of fresh water.



EXAMPLE II.—A rectangular tank for holding water has a vertical side whose dimensions are 3 feet vertical by 4 feet horizontal. An open pipe is inserted into the cover of the tank, and water is poured in until the level in the pipe is 7 feet above the base of the tank. Find the pressure on the vertical side and the reduction of pressure when the water in the pipe is allowed to sink $1\frac{1}{2}$ feet. (The weight of a cubic foot of water = 62.5 lbs.) (S. and A. Exam. 1890.)

ANSWER.—In the first case,

Height from *c.g.* of side to free surface $= H_1 = 5{\cdot}5'$.
Area of this vertical side in sq. ft. $= A = 3' \times 4'' = 12$ sq. ft.
Weight of a cubic foot of water $= W = 62.5$ lbs.

By the above formula,
The total pressure $P_1 = H_1AW$.

$$\therefore P_1 = 5{\cdot}5' \times 12 \times 62.5 = \mathbf{4125 \text{ lbs.}}$$

In the second case, when the free surface is lowered by $1\frac{1}{2}$ ft., everything remains the same except the H, which is now reduced from H_1 to $H_2 = 4'$.

By the formula,

$$P_2 = H_2AW.$$
$$\therefore P_2 = 4 \times 12 \times 62.5 = 3000 \text{ lbs.}$$

Consequently, the reduction in pressure is the difference between these pressures.

$$\text{Or } (P_1 - P_2) = 4125 \text{ lbs.} - 3000 \text{ lbs.} = \mathbf{1125 \text{ lbs.}}$$

fond of saying HAW ! HAW ! The student will observe that we have suddenly jumped from heights in inches to those in feet, areas in square inches to those in square feet, and weights of cubic inches to those in cubic feet. This is because the usual units of measurement in hydraulics are feet, square feet, and cubic feet, and because the weight of water is generally reckoned by 62.5 lbs. per cubic foot,

Lecture XVII.—Questions.

1. Define the terms liquid, hydro-statics, hydro-dynamics, and hydraulics.

2. Give the chief properties of a liquid, stating wherein it differs from a solid and a gas.

3. Describe and illustrate any experiment, other than the one referred to in this Lecture, to prove the law of transmission of pressure by liquids. State Pascal's law.

4. Describe the nature of fluid pressure. A mass of stone when placed in water feels lighter than when it is situated in the open air. Will you explain the cause of this fact, and state the difference of weight per cubic foot of water displaced?

5. What is meant by "head" in relation to water supplies for developing power? Give an example.

6. Explain how the pressure on the base of a vessel is quite independent of the shape of the vessel, and the total weight of water it holds. Illustrate your remarks by showing a series of connected vessels of very different shapes, but with each of their bottoms of the same size and on the same level, and filled with water to the same height.

7. Sketch and describe Sir Wm. Thomson's wire-testing machine, and explain how such a great force is obtained thereby from such a small quantity of water.

8. How is the pressure of water on a given area ascertained? A tank, in the form of a cubical box, whose sides are vertical, holds 4 tons of water when quite full; what is the pressure on its base, and what is the pressure on one of its sides? *Ans.* 4 tons; 2 tons.

9. A water tank is 13 feet square and 4 feet 6 inches deep; find the pressure upon one of the sides when the tank is full. *Ans.* 8226·56 lbs.

10. State approximately the increase of pressure to which a diver would be exposed when working at a depth of 50 feet below the surface of fresh water. *Ans.* About 22 lbs. per square inch.

11. In the vertical plane side of a tank holding water, there is a rectangular plate whose depth is 1 foot and breadth 2 feet, the upper edge being horizontal, and 8 feet below the surface of the water; find the pressure on the plate. *Ans.* 1062·5 lbs.

12. The base of a rectangular tank for holding water is a square, 16 square feet in area. The sides of the tank are vertical, and it holds 250 gallons of water when quite full. Find the depth of the tank and the pressures on each side and on the base when quite filled with water. (S. and A. Exam. 1888.) *Ans.* 2·5 feet; 781·25 lbs.; 2·500 lbs.

13. A rectangular tank for holding water has a vertical side whose dimensions are 4 feet vertical by 5 feet horizontal. An open pipe is inserted into the cover of the tank, and water is poured in until the level in the pipe is 10 feet above the base of the tank. Find the pressure on the vertical side and the reduction of pressure when the water in the pipe is allowed to sink 2 feet. *Ans.* 10,000 lbs.; 2500 lbs.

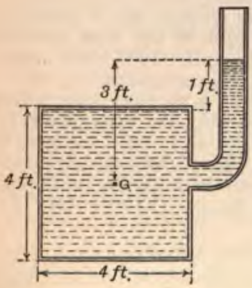
LECTURE XVIII.

CONTENTS.—Useful Data regarding Fresh and Salt Water—Examples I. II. III. IV.—Centre of Pressure—Immersion of Solids—Law of Archimedes—Floating Bodies—Example V.—Atmospheric Pressure—The Mercurial Barometer—Example VI.—Low Pressure and Vacuum Water Gauges—Example VII.—The Siphon—Questions.

Useful Data regarding Fresh and Salt Water.—We will commence this Lecture by giving some useful data regarding the weights, &c., of fresh and salt water, and then work out a few more examples for the pressures on immersed surfaces, finishing with the immersion of solids in fluids, &c.

FRESH WATER:
- Specific gravity * = 1.
- 1 cubic foot weighs 62·5 lbs., or 1000 oz.
- 1 gallon weighs 10 lbs., or 160 oz.
- 1 ton occupies 35·84 cubic feet.
- 1 atmosphere = 14·7 lbs. per sq. in. = 29·92 in. mercury = 33·9 (say 34) ft. head of water.
- 1 foot of head = ·43 lb. on sq. in.
- 1 lb. on the sq. in. = 2·308 ft. head.
- H.P. in a waterfall = cubic ft. per minute × head × 62·5 ÷ 33,000.

SALT WATER:
- Specific gravity * = 1·026.
- 1 cubic foot weighs 64 lbs.
- 1 gallon weighs $10\frac{1}{4}$ lbs.
- 1 ton occupies 35 cubic ft., or $218\frac{1}{4}$ gallons.



EXAMPLE I.—A cubical box or tank with a closed lid, the length of a side of which is 4 feet, rests with its base horizontal, and an open vertical pipe enters one of its sides by an elbow. The tank is full of water, and the pipe contains water to the height of 1 foot above the top of the tank. What are the pressures of water on the top, bottom, and sides of the tank? (Given the weight of a cubic foot of water = $62\frac{1}{2}$ lbs.) (S. and A. Exam. 1887.)

* Specific gravity is the ratio of the weight of a given bulk of a substance, to the weight of the same bulk of pure water.

ANSWER.—(1) For the pressure on the *top*—

The depth of *c.g.* of the top from free surface $= H = 1'$.

$\therefore$ Total pressure on top $= HAW = 1' \times (4' \times 4') \times 62{\cdot}5$ lbs. $=$ **1000 lbs.**

(2) For the pressure on *the bottom*—

The depth of *c.g.* of the bottom from the free surface $= H = 5'$.

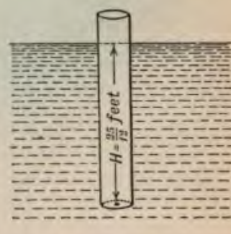
$\therefore$ Total pressure on bottom $= HAW = 5' \times (4' \times 4') \times 62{\cdot}5$ lbs. $=$ **5000 lbs.**

(3) For the pressure on each of the *sides*—

The depth of *c.g.* of *each side* from the free surface $= H = 3'$.

$\therefore$ Total pressure on each side $= HAW = 3' \times (4' \times 4') \times 62{\cdot}5$ lbs. $=$ **3000 lbs.**

EXAMPLE II.—A cylindrical vessel, 30 inches long and 6 inches in diameter, is sunk vertically in water, so that the base, which is horizontal, is at a depth of 25 inches below the surface of the water. Find the upward pressure in pounds on the base of the vessel. The weight of a cubic foot of water is $62\frac{1}{2}$ lbs., and $\pi = 3{\cdot}1416$. (S. and A. Exam. 1889.)



ANSWER.—The depth of *c.g.* of the base from the free surface is $= H = \frac{25}{12} = 2\frac{1}{12}' = 2{\cdot}08\dot{3}$ feet.

The area of the base $= A = \frac{\pi}{4}d^2 = \frac{22}{7} \times {\cdot}5' \times {\cdot}5' = {\cdot}196$ sq. ft.

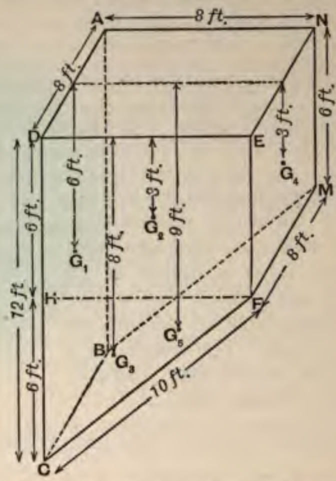
The weight of a cubic foot $= W = 62{\cdot}5$ lbs.

$\therefore$ The total pressure on base $= HAW = 2{\cdot}08\dot{3} \times {\cdot}196 \times 62{\cdot}5 =$ **25·5 lbs.**

EXAMPLE III.—A water tank, 8 feet long and 8 feet wide, with an inclined base, is 12 feet deep at the front and 6 feet deep at the back, and is filled with water. Find the pressure in lbs. on each of the four sides and on the base; water weighing $62\frac{1}{2}$ lbs. per cubic foot.

ANSWER.—In answering a question of this kind the student will find it best to draw a figure representing the water tank and the positions of the centres of gravity of *each side* and of the base in the manner shown by the accompanying illustration. The only point that presents any difficulty is the *c.g.* of the side DEFC, and of the correspondingly opposite one. This might be done by first finding the *c.g.* of the □ DEFH, viz., G_2; second, of the △ HFC, viz., G_3; third, by joining these two points with a line G_2G_3,

and taking a distance along it from G_2 towards G_3 inversely proportional to the areas of the □ DEFH and the △ HFC; this would give a point G_6 the *c.g.* of the whole side = 4·6′ from surface. But it will evidently be easier to treat the pressures on the □ and △ separately, and then to add them together in order to obtain the total pressure on the whole side DEFC.



SHOWING POSITIONS OF THE CENTRES OF GRAVITY.

G_1	represents	centre of gravity	of area	ABCD
G_2	,,	,,	,,	DEFH
G_3	,,	,,	,,	HFC
G_4	,,	,,	,,	ENMF
G_5	,,	,,	,,	BCFM.

Let H_1, H_2, &c., represent depths of G_1, G_2, &c.

Then $H_1 = \frac{1}{2}DC = 6'$; $H_2 = \frac{1}{2}EF = 3'$.

G_3 is $\frac{1}{3}$ of HC below the line HF (see Lecture III., *re* position of *c.g.* of certain areas).

$$\therefore H_3 = 6 + \tfrac{6}{3} = 8'; \ H_4 = \tfrac{1}{2}EF = 3'.$$

G_5 is at a depth below the surface = the mean between the edges BC and FM of the base BCFM.

$$\therefore H_5 = \tfrac{1}{2}(DC + EF) = \tfrac{1}{2}(12 + 6) = 9'.$$

Total pressure on area—

$$ABCD = H_1A_1W = 6' \times (12' \times 8') \times 62{\cdot}5 = 36{,}000 \text{ lbs.}$$
$$DEFH = H_2A_2W = 3' \times (6' \times 8') \times 62{\cdot}5 = 9{,}000 \text{ lbs.}$$

$$HFC = H_3A_3W = 8' \times (\frac{6'}{2'} \times 8') \times 62{\cdot}5 = \mathbf{12000\ lbs.}$$

$$DEFC = DEFH + HFC = 9000 + 12000 = \mathbf{21000}\ ,,$$

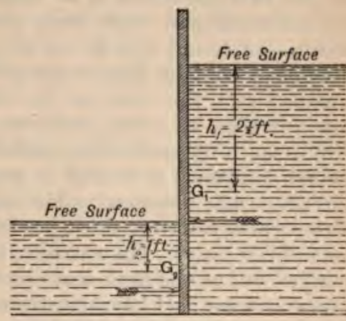
$$ENMF = H_4A_4W = 3' \times (6' \times 8') \times 62{\cdot}5 = \mathbf{9000}\ ,,$$

$$BCFM = H_5A_5W = 9' \times (10' \times 8') \times 62{\cdot}5 = \mathbf{45000}\ ,,$$

EXAMPLE IV.—A sluice gate is 4 feet broad and 6 feet deep, and the water rises to a height of 5 feet on one side and 2 feet on the other side. Find the pressure in pounds on the gate.

ANSWER.—The net pressure on the sluice gate is evidently equal to the difference of the pressures on the two sides.

Total pressure on—



NETT PRESSURE ON SLUICE GATE.

$$\text{Back side} = H_1A_1W = 2{\cdot}5' \times (4' \times 5') \times 62.5 = 3125 \text{ lbs.}$$
$$\text{Front side} = H_2A_2W = 1' \times (4' \times 2') \times 62.5 = 500\ ,,$$

Subtracting the front from the back pressure we get the net pressure $\Big\}$ = **2625 lbs.**

Centre of Pressure.—In the case of a plane area immersed in a liquid, the "*centre of pressure*" is the point at which the resultant of all the pressures of the fluid acts. If the plane be horizontal, the resultant naturally acts at the centre of the figure, and therefore the *centre of pressure* agrees with the *centre of gravity* of the figure. In the case of a vertical rectangle, having one of its edges in the surface of liquid, like a dock-gate or a sluice, the *centre of pressure* will be at a point $\frac{2}{3}$ of the depth from the free surface and at the middle of the breadth of the immersed portion. We will have to prove this in our Advanced Course, and perhaps refer to the position of the centre of pressure in other cases.

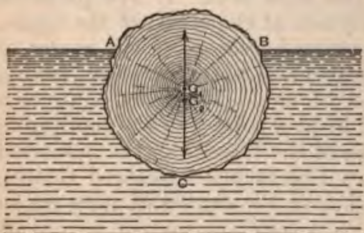
Immersion of Solids.—Law of Archimedes.—If a solid be immersed in any *fluid* (whether liquid or gas), it displaces a quantity of that fluid equal to its own volume. This is evident from the principle of impenetrability—viz., "*two bodies cannot occupy the same space at the same time.*"

Hence we have a simple method of determining the volume of

M

any irregular body by plunging it into a liquid, and noting the cubic contents of the liquid displaced, by letting it run into a measure of known capacity, such as a graduated jar. This principle was first discovered by Archimedes, a philosopher of Syracuse, in the year 250 B.C. The story of this discovery is related by Vitruvius, who states that Hero, a king, sent a certain weight of gold to a goldsmith to be made into a crown. Suspecting that the workman had kept back part of the gold, he weighed the crown, but found that it was the same as the weight of the gold previously sent by him to the goldsmith. He was, however, not satisfied with this test, so he consulted Archimedes, and asked him whether he could find out if the crown was adulterated. Not long afterwards the philosopher, on going into his bath (which happened to be full of water), observed that a quantity of the water was displaced. He immediately conjectured that the water which ran over must be equal to the volume of the immersed part of his body. He was so overjoyed at the discovery that he jumped out of the bath and ran naked to the king, exclaiming, Εὕρηκα! εὕρηκα! (I have discovered! I have found out!) He then began to experiment with the crown by taking a quantity of pure gold of the same weight, and observed its displacement in water. Next he ascertained by the same process the volume of the same weight of silver, and finally the volume of the crown, which actually displaced more water than its equivalent weight of pure gold. In this interesting manner the fraud of the artificer was detected, to his great astonishment and chagrin, and a Law of Nature was discovered.

Floating Bodies.—A body is said to float in a fluid when it is entirely supported by the fluid. In order that a body may float, the forces acting upon it must be in equilibrium. Now, as may be seen from the case illustrated by the accompanying figure, there are only two forces to be considered—viz., the weight of the body acting *vertically downwards* through its centre of gravity G_1, and the pressure of the liquid acting *vertically upwards* through the centre of gravity G_2 of the displaced fluid. The horizontal pressures of the liquid on the body are in equilibrium by themselves, and simply tend to compress it so that they do not affect the question. The upward pressure of the liquid must be equal to the weight of the body (for



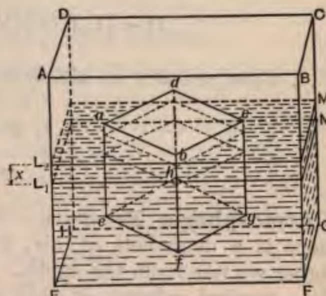
CONDITIONS OF EQUILIBRIUM IN THE CASE OF A FLOATING BODY.

it is equal to the weight of the volume of the fluid displaced), otherwise the body would rise or sink. Further, the centres of gravity of the body and of the displaced fluid must be in the *same vertical line*. For if not, there would be an arm of a couple between them, which would enable the two forces to turn the body. But it is in equilibrium. Hence the conditions of equilibrium for a floating body are—

1. *The weight of the body must be equal to the weight of the fluid displaced.*

2. *The centres of gravity of the body and of the fluid displaced must be in the same vertical line.*

As a natural deduction from the above proof we conclude that a body cannot float in a liquid of less specific gravity than itself. A solid glass or metal ball will float in mercury, but not in water. If the specific gravity of a body be the same as that of a liquid, it will float totally submerged. If the body and the liquid are each *incompressible*, the body will float indifferently at any depth. If the body be incompressible, but be placed in a compressible fluid, such as air, the body will rise or fall until it finds a place where its mean specific gravity is the same as that of the displaced gas. This is exemplified by the case of a balloon filled with a gas lighter than air. It rises until it arrives at a height from the earth where the combined weight of the machine and the gas contained therein are equal to the weight of the same volume of air.*



EXAMPLE V.—A rectangular tank, 4 feet square, is filled with water to a height of 3 feet. A rectangular block of wood, weighing 125 lbs., and having a sectional area of 4 square feet, is placed in the tank, and floats with its sides vertical and with this section horizontal. How much does the water rise in the tank, and what is now the pressure on one vertical side of the tank? (S. and A. Exam. 1892.)

ANSWER.—The answer to the first part of this question depends on the application of the principle enunciated by Archimedes—viz., that the volume of the water displaced by the block of wood is equal to the volume of the wood immersed.

Let x represent the height that the water rises in the tank when the block of wood is placed therein.

* *We must leave the subject of metacentres, &c., to our Advanced Course.*

Then, *the weight of water displaced in lbs.* $= \left\{\begin{array}{l}\text{the volume of water displaced by wood} \\ \text{in cubic feet} \times 62\frac{1}{2}\text{ lbs. (the weight of} \\ \text{a cubic foot of water.}\end{array}\right.$

$$\therefore 125 = V \times 62\tfrac{1}{2} = V \times \frac{125}{2}$$

$$\therefore V = 2 \text{ cubic feet.}$$

Owing to this volume of water being displaced, the level of the water in the tank will rise from L_1M_1, to L_2M_2. But the volume of water between these two surfaces is equal to the volume $L_1M_1L_2M_2$, *minus* the volume taken up by the block between those two surfaces. The volume included between the two surfaces $= 4^2 \times x$ cubic feet. The volume intercepted by the block $= 4 \times x$, since the cross-sectional areas of the tank and block are 4^2 and 4 square feet respectively.

$$\therefore 4^2 \times x - 4 \times x = 2$$

$$\therefore x = \frac{2}{12} = \mathbf{\frac{1}{6}} \textbf{ foot, or 2 inches.}$$

Next, we have to find *the pressure on one of the vertical sides of the tank*. Here the depth of the centre of gravity of the area of the side subjected to pressure below the free surface of the water is

$$H = \tfrac{1}{2}L_2E = \tfrac{1}{2}(3' + \tfrac{1}{6}') = \frac{19}{12} \text{ feet.}$$

$\therefore$ *Total pressure on side* $= P = HAW$

$$\therefore P = \frac{19'}{12} \times (4 \times 3\tfrac{1}{6}) \text{ sq. ft.} \times 62\tfrac{1}{2} \text{ lbs.}$$

Or, ... $\mathbf{P = 1253{\cdot}47}$ **lbs.**

Atmospheric Pressure.—Surrounding the earth's surface there is a deep belt of air, which gets rarer and lighter the higher we rise from the earth. If we consider the case of a complete vertical column of this air, we find that it produces an average pressure on the earth's surface of about 15 lbs.; or, in other words, we say that the atmosphere produces an average pressure of 15 lbs. on the square inch, for we find that it will balance a vertical column of mercury of about 30 inches, or a vertical column of water of 34 feet. We do not experience any inconvenience from this normal pressure of the atmosphere, because we are so constituted as to be able to resist it. Should we, however, enter the closed compressed air-chamber of the under-*ound* workings of a railway tunnel (such as those in operation

at the present time for the construction of the Glasgow Central Railway), or the caissons of a great bridge while they are being sunk (as in the case of the Forth Bridge), or go down into the sea in a diving-dress or diving-bell, then we do feel a *most uncomfortable* sensation in our ears, eyes, &c. Or, if we climb a very high mountain, or rise far into the air in a balloon, we have a somewhat similar sensation, but due to an opposite effect—viz., a diminution from the normal pressure to which we are accustomed.

The Mercurial Barometer.—The pressure of the atmosphere is usually measured by a mercurial barometer, which consists of a vertical tube of glass about 33 inches long, of uniform calibre, hermetically sealed at the top end, into which has been carefully introduced mercury freed from air. The lower end dips into an open dish containing a quantity of that liquid metal. Consequently the pressure of the atmosphere acting on the mercury in the open dish forces it up inside the tube to a height directly proportional to its pressure, since there is supposed to be a perfect vacuum between the upper surface of the mercury and the closed end of the glass tube.

EXAMPLE VI.—Suppose the height of mercury as registered by a mercurial barometer is 30 inches, and that the specific gravity of mercury be taken as 13·6, what would be the height in feet of a water column which would support the same atmospheric pressure?

ANSWER.— $1 : 13{\cdot}6 :: 30 \text{ inches} : x$

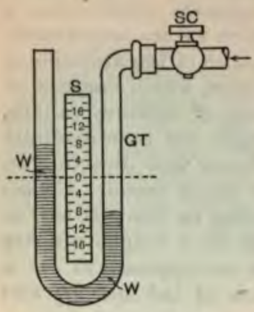
$$\therefore x = 30 \times 13{\cdot}6 = 408'' = \mathbf{34\ feet.}$$

Low Pressure and Vacuum Water Gauges.*—It is often necessary for the engineer to measure *low pressures* or *vacuums* of gases. For example, in the supply of illuminating gas to a town, or in the pressure of air feeding a boiler furnace by natural or forced draught, or the vacuum produced by a chimney-stalk; or, in the case of the vacuum in a coal mine produced by a furnace below the earth, or by a guibal fan situated near the upcast shaft, &c. In such cases, as well as in many others where low pressures have to be observed, the force is not reckoned by pounds per square inch, or by inches of mercury sustained in a vertical column, but by the number of inches of water which the pressure will support or which the vacuum will detract from the atmospheric pressure.

The accompanying figure illustrates the apparatus usually employed in determining such low pressures. It consists of a

* For a description of mercurial pressure and vacuum gauges, as well as Bourdon's high-pressure and vacuum gauges, refer to the Author's Elementary and Advanced Books on "Steam and Steam Engines."

simple bent **U** glass tube with a scale between the vertical legs of the **U**, divided into inches and tenths of an inch, so that either the pressure or the vacuum may be read off in inches of water pressure, according as the forward pressure from the point of supply is positive or negative in respect to the pressure of the atmosphere. For example, let the leg of the **U** tube next the cock be connected to the gas pipe of a house, then the pressure of the gas supply acts on the water in the right-hand leg of the tube, and forces it downwards, whilst the water in the other leg rises correspondingly. The reading observed on the scale S, below or above the zero or equilibrium line, has of course to be doubled in order to ascertain the exact total pressure in inches of water. If the **U** tube be connected to a vacuum or negative pressure, then the water rises in the inner leg of the **U** tube, owing to the greater pressure of the atmosphere on the outer limb, and the inches of water representing the amount of the vacuum are accordingly read off in the same way. For example, if the apparatus be connected to the base of a steam boiler chimney, or to the inlet of a guibal fan creating a draught in a coal mine, then the suction produced forms a vacuum which requires the supply of atmospheric air, and consequently the air presses on the open water of the outer limb of the **U** tube, and forces it downwards. The vacuum is therefore observed and recorded by adding the inches of water below and above the zero line.



GAS PRESSURE GAUGE.

INDEX TO PARTS.

SC represents Steam or gas cock.
GT " Glass tube.
S " Scale.
W " Water.

EXAMPLE VII.—A difference of level is observed of 4 inches between the outer and inner limbs of a **U** tube water-gauge. What is the pressure of the gas supply in lbs. per square inch?

ANSWER.—A vertical column of 34 feet of water corresponds to 15 lbs. pressure on the square inch. Consequently,

$$(34' \times 12'') : 4'' :: 15 \text{ lbs.} : x$$

$$x = \frac{15 \times 4}{34 \times 12} = \frac{34}{5}, \text{ or nearly } \tfrac{1}{7} \text{ of a lb. per sq. in.}$$

The Siphon is simply a bent tube for withdrawing liquids from a higher to a lower level by aid of the atmospheric pressure. 's used in chemical laboratories and works for emptying acids

from carboys, in breweries and distilleries for extracting beer from vats and spirits from casks, in the crinal glass tube of Sir Wm. Thomson's recorder for conveying ink from the ink-pot to the telegraph message-paper; and on a large scale for draining low-lying districts, such as the fens of Lincolnshire.

THE SIPHON.

The conditions for the successful working of a siphon are, that—

1. The liquid shall be carried by the outer limb of the tube to a *lower level* than the surface of the supply.

2. The vertical height from the free surface of the liquid being drained to the top of the bend of the siphon *shall not be greater than* the height of the water barometer at the time—say only 30 feet —on account of the necessary deduction of 3 or 4 feet to be made from the full height of 34 feet, due to having to overcome the friction of the pipe.

3. The end of the siphon dipping into the liquid to be drained, shall *not become uncovered.*

To start the siphon, either the tube must be filled with liquid, the ends closed, and the siphon inverted, with the shorter limb under the fluid to be drained, before uncovering the ends; or, whilst the shorter limb is in position a vacuum must be formed in the siphon tube by extracting the air from the end of the longer leg.

The principle upon which the siphon acts is as follows :—

A vacuum having been formed in the tube, the pressure of the atmosphere acting on the free surface of the liquid to be drained, forces it up the shorter limb, and having turned the highest point of the ∩ it naturally descends the longer limb by the action of gravity with a velocity proportionate to the difference of level between the outlet and the free surface of the source of supply. The outflowing liquid is always acting as a water-tight piston at the bend of the ∩, and in this way keeping up the vacuum there, until either the inlet and the outlet free surfaces come to a level (when the siphon stops for want of "head"), or, when the difference of level between the free surface of the supply and the top of the bend exceeds the height supportable by the atmosphere, when it stops for want of breath or atmospheric pressure.

Lecture XVIII.—Questions.

1. What are the respective specific gravities and the weights per cubic foot and per gallon of fresh and of salt water?

2. A cylindrical vessel, 120 inches long and 10 inches in diameter, is sunk vertically in water, so that the base, which is horizontal, is at a depth of 100 inches below the surface of the water. Find the upward pressure in pounds on the base of the vessel. *Ans.* 284·2 lbs.

3. A cubical box or tank with a closed lid, the length of a side of which is 5 feet, rests with its base horizontal, and an open vertical pipe enters one of its sides by an elbow. The tank is full of fresh water, and the pipe contains water to the height of 10 feet above the top of the tank. What are the pressures of water on the top, bottom, and sides of the tank? *Ans.* 15,625 lbs.; 23,437·5 lbs.; 19,531·25 lbs.

4. A water tank 10′ long, 10′ wide, with an inclined base 10′ deep at one end and 5′ at the other end, is filled with fresh water. Find the pressure in pounds on each of the four sides and on the base. *Ans.* 31,250 lbs.; 7,812·5 lbs.; 18,229·16 lbs.; 52,500 lbs.

5. A lock gate is 12 feet wide, and the water rises to a height of 8 feet from the bottom of the gate. What pressure in pounds does it sustain? The weight of a cubic foot of water is 62½ lbs. *Ans.* 24,000 lbs.

6. A vertical rectangular sluice gate, measuring 2 feet horizontal by 3 feet vertical, is immersed so that its upper side is 4 feet below the surface of the water pressing on it. Find the pressure on the gate: you are required to explain the reasoning on which your calculation is founded. (S. and A. Exam. 1891.) *Ans.* 2062·5 lbs.

7. What is meant by the "centre of pressure" in the case of a plane surface immersed in a liquid? If the plane be a horizontal circle, where does the centre of pressure act? If it be a vertical rectangle 10 feet wide and 6 feet deep, immersed in water so that the upper edge of the rectangle is flush with the surface of the water, where does the "centre of pressure" act? *Ans.* at the centre of the circle; 4 feet below surface of water.

8. State the law discovered by Archimedes, and the conditions for a body in equilibrium floating in a liquid. A cylinder 10 feet long and 2 feet in diameter floats in fresh water, with 2 feet projecting from the surface; find the weight of the cylinder. *Ans.* 1,571·42 lbs.

9. A rectangular tank, 5 feet square, is filled with water to a height of 7¾ feet. A rectangular block of wood, weighing 312·5 lbs., and having a sectional area of 5 square feet, is placed in the tank, and floats with its sides vertical and with its section horizontal. How much does the water rise in the tank, and what is now the pressure on one vertical side of the tank? *Ans.* 3 inches; 10,000 lbs.

10. The mercurial barometer registers 31″; calculate the height of columns of fresh and of salt water that will balance the corresponding pressure.

11. Sketch and describe a mercurial barometer. State how it is made, and how it acts as a register of the pressure of the atmosphere.

12. Describe some simple form of gauge which would enable you to measure the pressure at which gas is supplied, and explain the principle on which it is constructed.

13. Sketch and explain the action of the siphon, and give a few practical examples of its use. Also state under what circumstances it fails to work.

LECTURE XIX.

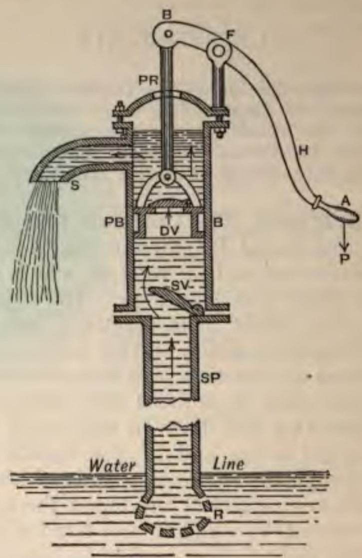
CONTENTS.—Hydraulic Machines—The Common Suction Pump—Example I.—The Plunger, or Single-acting Force Pump—Example II.—Force Pump with Air Vessel—Continuous-delivery Single-acting Force Pump without Air Vessel—Combined Plunger and Bucket Pump—Double-acting Force Pump—Example III.—Questions.

Hydraulic Machines.—The Common Suction Pump consists of a bored cast-iron barrel PB, terminating in a suction pipe, SP, fitted with a perforated end or rose R, which dips into the well from which the water is to be drawn. The object of the rose is to prevent leaves or other matter getting into the pump, that might clog and spoil the action of the valves. At the junction between the barrel and suction pipe there is fitted a suction valve SV, of the hinged clack type faced with leather. The piston or bucket B is worked up and down in the barrel of the pump by the force P, applied to the end of the handle H, being communicated to it through the connecting link of the hinged piston-rod PR. In the centre and at the top of the bucket is fixed the clack delivery valve DV, which is also faced with leather in order to make it water-tight. The bucket is sometimes packed with leather; but, as shown by the figure, a coil of tightly woven flax rope wrapped round the packing groove would be more suitable in the present instance.

Action of the Suction Pump.—(1) Let the barrel and the suction pipe be filled with air down to the water-line, and let the bucket be at the end of the down stroke. Now raise the bucket to the end of the up-stroke by depressing the pump handle. This creates a vacuum below DV; therefore the air which filled the suction pipe *only*, expands, opens SV, and fills the additional volume of the barrel. Consequently, according to Boyle's law, its pressure must be diminished in the *inverse ratio* to the enlargement of its volume.* This enables the pressure of the atmosphere

* The student may refer to Lecture XII. of the Author's Elementary Manual on "Steam and the Steam Engine," for an explanation and demonstration of Boyle's law; where it is shown that if p=the pressure of a gas and v=its volume, then at a uniform temperature pv=a constant, or p varies as $\frac{1}{v}$.

(which acts constantly on the surface of the water in the well) to force a certain quantity of water up the suction pipe, until the weight of this column of water and the pressure of the air (between it and the delivery valves) balance the pressure of the outside atmosphere.



COMMON SUCTION PUMP.

INDEX TO PARTS.

H	represents	Handle.	SP	represents	Suction pipe.
P	,,	Push or pull at A.	R	,,	Rose.
F	,,	Fulcrum of H.	SV	,,	Suction valve.
PR	,,	Plunger rod.	B	,,	Bucket or piston.
PB	,,	Pump barrel.	DV	,,	Delivery valve.
S	,,	Spout.			

(2) In pressing the bucket to the bottom of the barrel by elevating the handle, the suction valve closes and the delivery valve opens, thereby permitting the compressed air in the barrel to escape through the delivery valve into the atmosphere.

(3) Raise and depress the piston several times so as to produce the above actions over again, and thus gradually diminish the volume of the air in the pump to a minimum. Then water will ve been forced by the pressure of the atmosphere up the suction and into the pump, if the bucket and the valves are tight,

and if the delivery valve when at the top of its stroke be not more than the height of the hydro-barometric column above the water line of the well.*

(4) The bucket now works in water instead of in air. In fact, the machine passes from being an air-pump to be a water one. During the down-stroke water is forced through the delivery valve. During the up-stroke this water is ejected through the spout; at the same time more water is forced up through suction pipe and valve to supply the place of the vacuum created by the receding piston. The water is therefore discharged *only* during the up-stroke in the case of the pump illustrated by the figure. Should it, however, be fitted with an air-tight piston-rod and pump cover, and should the pump handle be moved rapidly, more water will be taken into the barrel than can escape from the spout during the up-stroke. Consequently, the compression of the pent-up air between the surface of the water in the barrel and the cover, will cause the water to flow out in a more or less continuous stream during the down-stroke. In other words, the top cover and the portion of the pump above the spout may be converted into an air vessel, the precise action of which will be explained later on.

EXAMPLE I.—If the cross area of the bucket of a suction pump be 20 sq. in. and if water be raised 24 ft. from its surface in the well, what is the pull on the pump rod?

ANSWER.—The pull P on the pump rod is evidently equal to the weight of a column of water of height $H = 24$ ft., and the area of the bucket in sq. ft. $= A = 20 \div 144$. Therefore, by the formula employed for the pressure of a liquid on a surface in Lectures XVII. and XVIII.—

$$P = HAW,$$

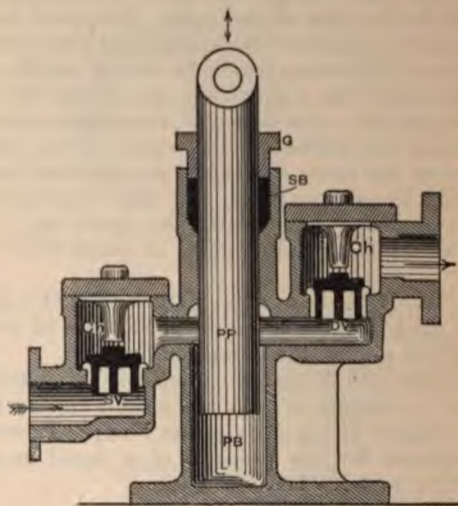
$$P = 24' \times \frac{20'}{144} \times 62{\cdot}5 = \mathbf{208} \text{ lbs.}$$

The Plunger, or Single-acting Force Pump.—The upper or outer end of the barrel of this pump is provided with a stuffing-box and gland, through the air-tight packing of which the solid pump plunger works.

During the up or outward stroke of the plunger a vacuum is

* Theoretically, such a pump should be able to lift water from a depth of 34 feet below the highest part of the stroke of the delivery valve, but practically, owing to the imperfectly air-tight fitting of the piston and the valves, it is not used for withdrawing water from wells more than 20 to 25 feet below this position of the delivery valve. In fact, such a pump *frequently requires a bucket or two* of water to be poured into it above the *delivery valve in order to* make it work at all, if it should have been left *standing for some time* without being worked.

created in the pump barrel, and consequently air is expanded into it from the suction pipe. This pipe is attached to the flange of the suction valve-box. During the down or inward stroke the suction valve closes, and the pent-up air in the barrel is forced through the delivery valve. This action goes on precisely in the manner just explained in the case of the suction pump, until the water rises into the barrel. Then the inward stroke of the plunger drives water through the delivery valve to any desired height (or against any reasonable back pressure, as in the case of a feed



THE PLUNGER FORCE PUMP.

INDEX TO PARTS.

SV	represents	Suction valve.	PB	represents	Pump barrel.
DV	"	Delivery valve.	PP	"	Pump plunger.
Ch	"	Checks for valves.	SB and G	"	Stuffing box and gland.

pump for a steam boiler) consistent with the strength of the pump and the power applied.

The eye of the plunger may be attached to a connecting-rod actuated by a hand lever, as in the case of the suction pump, or it may be worked from one eccentric or crank revolved by a steam engine or other motor.

By whichever way it is worked, the force applied to the plunger must be sufficient to overcome the friction between the plunger

and the packing, the resistance due to sucking the water from the source of supply, and of driving the same up to the place where it is delivered.

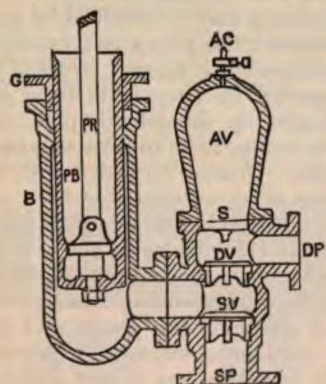
With this pump (as in the case of the suction pump), the water is only delivered during one out of every two strokes of the plunger, and consequently, in an intermittent or pulsating fashion. In order to make the supply continuous we have to use one or other of the devices about to be described.

EXAMPLE II.—In a single-acting plunger force pump the cross area of the plunger is 10 sq. in., and its distance from the surface of the water in the well, when at the end of its outward or suction stroke, is 20 ft. During the inward stroke the water is pumped up to a height of 100 ft. above the end of the plunger. What forces are required to move the pump plunger during (1) an "out," and (2) an in-stroke (neglecting the forces to overcome friction).

ANSWER.—(1) $P_1 = H_1AW = 20' \times \frac{10}{144} \times 62 \cdot 5 = \mathbf{86 \cdot 8}$ lbs. pull.

(2) $P_2 = H_2AW = 100' \times \frac{10}{144} \times 62 \cdot 5 = \mathbf{434}$ lbs. pressure.

Force Pump with Air Vessel.—In the following figure of a force pump the only points of difference worth noticing *between*



FORCE PUMP WITH AIR VESSEL.

INDEX TO PARTS.

SP	represents	Suction pipe.
SV	"	Suction valve.
B	"	*Barrel of pump.*
PB	"	*Plunger barrel.*
PR	"	*Plunger rod.*
DV	represents	Delivery valve.
S	"	Stop for DV.
AV	"	Air vessel.
AC	"	Air cock.
DP	"	Delivery pipe.

it and the previous one are :—(1) The plunger, instead of being solid, is a hollow trunk or barrel, with the connecting rod fixed to an eye-bolt at its lower end.

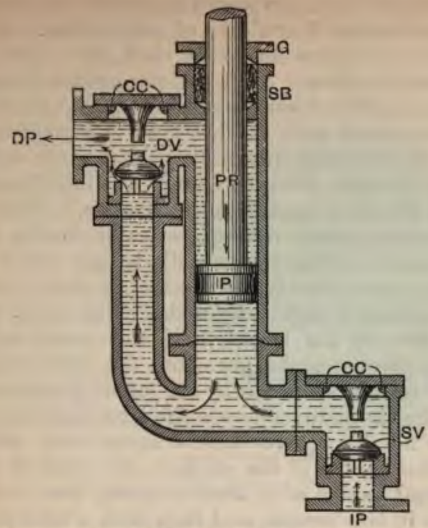
(2) The suction and the delivery valves are both at one side, instead of being fixed on opposite sides of the pump.

(3) There is an air vessel.

Action of the Air Vessel.—During the inward or delivery stroke of the plunger, part of the water forced from the barrel goes up the delivery pipe, and the remainder enters the air vessel, and consequently compresses the air in AV. During the outward or non-delivery stroke of the plunger the compressed air in the air vessel presses the rest of the water into the delivery pipe. In this simple way a continuous flow of water is maintained in the delivery pipe, and with far less shock, jar, and noise than in the previous case. Where very smooth working is required, an air vessel is also put on to the suction side of the pump. Should the air in the air vessel become entirely absorbed by the water, the fact will be noticed at once, by the noise and the intermittent delivery. Then the pump should be stopped, the air cock AC opened, and the water run out. When the air vessel is full of air, the air cock should be shut and the pump started again.

Continuous-delivery Pump without Air Vessel. — A fairly continuous delivery may be obtained by making the plunger of the piston form, and the pump rod exactly half its area, as shown by the accompanying figure. During the down stroke, half the water expelled by the piston from the under side of the pump barrel goes up the delivery pipe, and the other half is lodged above the piston, to be in turn sent up the delivery pipe during the up-stroke. Where very high pressures are required, such as in the filling of an accumulator ram, pumps working on this principle, but of the following form, are frequently used. The action is precisely the same as in the one just described, and the same index letters have been used, so that the student will have no difficulty in understanding the figure; more especially as the directions of motion of the piston and of the ingoing and outflowing water have been marked by straight and feathered arrows respectively. Where sea or acid water is used it may be necessary to fit the pump barrel, PB, with a brass liner, L, to prevent corrosion.

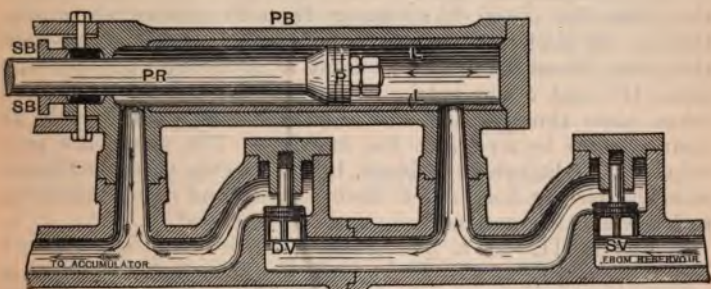
In accumulator and other kinds of high-pressure work it is not advisable to use air vessels, because you cannot prevent the water which enters the vessel absorbing air and carrying the same with it to the hydraulic machines, where its presence would be most objectionable, and because with, say, 750 to 1000 or more lbs.



CONTINUOUS-DELIVERY FORCE PUMP WITHOUT AIR VESSEL.

INDEX TO PARTS.

IP	represents	Inlet pipe.	DV	represents	Delivery valve.
SV	„	Suction valve.	CC	„	Cover and check to DV.
CC	„	Cover and check to SV.	DP	„	Discharge pipe.
P	„	Piston.	SB	„	Stuffing-box.
PR	„	Pump-rod.	G	„	Gland.



CONTINUOUS-DELIVERY FORCE PUMP.

As used in Connection with the Armstrong Accumulator.

(See Indexes to previous Figures.)

pressure per square inch, you would require a very large and very strong air vessel before it could be of any service. If a pressure of only 750 lbs. per square inch were used, then, since the normal pressure of the atmosphere is 15 lbs. per square inch, the air in the air vessel would be compressed to $\frac{15}{750}$, or $\frac{1}{50}$th of its original volume, in accordance with Boyle's law. Consequently, with an air vessel of 50 cubic feet internal capacity, there would be only 1 cubic foot of air in it, when the pump was in full action.

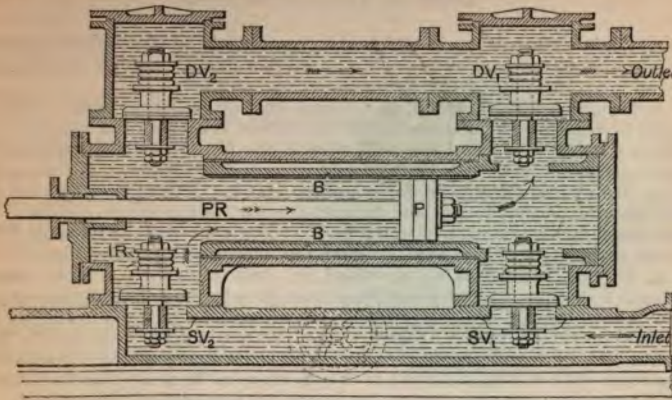
Combined Plunger and Bucket Pump.—We have already seen that a suction pump discharges water during the outward stroke, and that a plunger pump discharges water during the inward stroke; consequently, by combining these two kinds, we get a double-acting pump. By making the cross area of the plunger half that of the barrel, half the water raised by the bucket during the up-stroke goes into the delivery pipe, whilst the other half fills the space left by the receding plunger. During the down-stroke the plunger forces the latter half up the delivery pipe. We do not happen to have a figure with which to illustrate these remarks, but if the student will first of all sketch a complete vertical section of a suction pump like that shown by the first figure in this lecture, and then draw a solid plunger, with stuffing-box and gland, like that in the second figure, in place of the pump rod and open cover in the suction pump, it will form a useful exercise in the designing of such a pump.

Double-acting Force Pump.—The pumps which we have hitherto considered are all single-acting in this sense, that they do not both suck and discharge water during every stroke. This can, however, be accomplished by having two sets of suction and delivery valves placed at each end of the pump barrel, as shown by the accompanying figure. Then, during the outward stroke of the piston the pump draws water from the source of supply through the inlet pipe and suction valve SV_1. At the same time the piston forces the water in front of it through the delivery valve DV_2 and outlet pipe. During the inward stroke, suction takes place through SV_2 and discharge through DV_1, all as clearly shown by arrows in the drawing. The valves are provided with india-rubber cushions, IR, to ease the shock and minimise the jarring noise due to their reaction and natural reverberation when they are suddenly opened and closed.

EXAMPLE III.—In a double-acting force pump the vertical height from the surface of the well to the point of delivery is 100 feet. If the area of the piston equal 1 square foot, what is the stress on the piston-rod during each stroke?

ANSWER.—Here we need not distinguish between the force *quired during* suction and delivery, for both actions take place

during each stroke. We have only to deal with the net force required to elevate a column of water to a height of 100 feet.



DOUBLE-ACTING FORCE PUMP.*

INDEX TO PARTS.

SV_1 SV_2	represent	Suction valves.	B	represents	Barrel (liner).
DV_1 DV_2	"	Delivery valves.	P	"	Piston (solid).
IR	"	India-rubber cushions.	PR	"	Piston-rod.

Neglecting friction, the stress on the piston rod will therefore be the weight of a column of water of height 100′ and cross area = 1 sq. ft.

$\therefore P = HAW = 100' \times 1' \times 62 \cdot 5 =$ **6250** lbs. pull and push.

If 30 per cent. of the force applied be spent in overcoming friction, what will then be the stress on the pump-rod. Here 6250 is only 70 per cent. of the whole stress, for 30 per cent. of the whole is lost force.

$$\therefore 70 : 100 :: 6250 : x$$

$$x = \frac{6250 \times 100}{70} = \mathbf{8928 \cdot 5} \text{ lbs. pull and push.}$$

* We are indebted for the above figure to Professor H. Robinson's book on "Hydraulic Machinery," published by Messrs. Charles Griffin & Co. Students should refer to Lecture XXIV. of the Author's Elementary Manual on "Steam and the Steam Engine" for detailed illustrations and description of the air and circulating pumps of the SS. "St. Rognvald."

LECTURE XIX.—QUESTIONS.

1. Explain the manner in which the pressure of the atmosphere is made serviceable in the case of the common suction pump. Sketch and explain by an index the details of this pump.

2. Describe, with a sketch, an ordinary suction or lifting pump, and explain its action. If the diameter of the bucket is 4″, and the spout is 20′ above the free surface of the well, what is the tension on the pump-rod in the up-stroke? *Ans.* 109 lbs.

3. Sketch and describe a force pump, drawing a section so as to show the packing of the plunger and the construction of the valves. How is an air-vessel applied to such a pump? Why is the air-vessel dispensed with when pumping water into an accumulator? (S. and A. Exam. 1890.)

4. Explain the use of an air-vessel in connection with a force pump. Sketch a section through a double-acting force pump, showing the valves and the connection of the pump with the air-vessel, and explain the action of the pump. (S. and A. Exam. 1887.) Water is forced up to 100 feet above the air-vessel; what proportion of the volume of the air-vessel is occupied with water, and what is the pressure of the air therein? *Ans.* 74·6 per cent.; 43·4 lbs. per sq. in. above the atmospheric pressure.

5. The leverage to the end of the handle of a common force pump is five times that to the plunger, and the area of the plunger is 5 square inches; what pressure at the end of the lever handle will produce a pressure of 45 lbs. per square inch on the water within the barrel? *Ans.* 45 lbs.

6. A force pump is used to raise water from a well to a tank. The piston has a diameter of 1·6″, and is 20′ above the free surface of the water in the well, and 40′ below the mouth of the delivery pipe leading into the tank. Find the force required to work the pump—(1) Neglecting friction; (2) when 30% is spent in overcoming friction; (*a*) when sucking, (*b*) when forcing, (*c*) what is the work put in and got out per double stroke of 6″? *Ans.* (*a*) (1) 17·45 lbs.; (2) 24·93 lbs.; (*b*) (1) 34·9 lbs.; (2) 49·86 lbs.; (*c*) 37·39 ft.-lbs.; 26·17 ft.-lbs.

7. What is the difference between a double-acting and a single-acting pump? The area of the plunger of a force pump being 3 square inches, find the pressure upon it when water is forced up to a height of 20′. *Ans.* 26·04 lbs.

8. Describe, with a sketch, some form of pump which will deliver half the contents of the barrel at each respective up-stroke and down-stroke of the pump-rod. Name the valves. (S. and A. Exam. 1892.)

9. Sketch and describe a "double-acting force pump." If the diameter of the piston be 12″, the stroke 3′, the distance from pump to well 20′, from pump to position for delivering the water 40′, and if the number of strokes per minute be 40, what is (1) the theoretical horse-power required to work the pump, (2) the actual, if 30 per cent. of the power be spent against friction. *Ans.* (1) 10·71; (2) 15·3.

LECTURE XX.

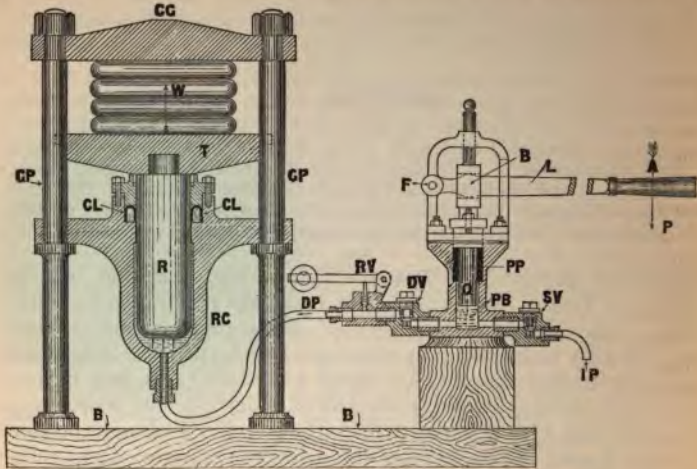
CONTENTS.—Bramah's Hydraulic Press—Bramah's Leather Collar Packing—Examples I. II.—Large Hydraulic Press for Flanging Boiler Plates—The Hydraulic Jack—Weem's Compound Screw and Hydraulic Jack—Example III.—The Hydraulic Bear or Portable Punching Machine—The Hydraulic Accumulator—Example IV.—Questions.

Bramah's Hydraulic Press.—This useful machine was invented by Pascal, but he could not make the moving parts water-tight. Bramah, about the year 1796, discovered a means by which this difficulty was effectually overcome; and thus the instrument has been handed down to us under his name. As may be seen from the following figure, it consists of a single-acting force pump in connection with a strong cylinder containing a plunger or ram, which is forced outwards from the cylinder through a tight collar by the pressure of the water delivered into the cylinder from the force pump.

From what was said in Lecture XIX. about force pumps, we need not particularise about this part of the machine, except to say that the suction and delivery valve boxes can be disconnected from the pump, and the valve cover-checks removed at any time for the purpose of examining the parts, or of regrinding the valves into their seats. The plunger extends through a stuffing-box and gland filled with hemp packing, and is guided by a centrally bored bracket bolted to the top flange of the pump. The lever fits through a slot in this guide-bar, whereby it has an easy free motion, when communicating the force applied through it to the pump plunger. The relief-valve RV has a loaded lever, adjusted like the lever safety valve in Lecture IV., so as to rise and let the water escape when the pressure exceeds a certain amount. It may also be used for taking the pressure of the object under compression, or for lowering the ram R by simply lifting the little lever and pressing down the table T, when the water flows easily from the cylinder, and out of DP by the relief valve. The delivery pipe DP is made of solid drawn brass, and the ram cylinder is carefully rounded at the bottom end, instead of being flat, in order that it may be naturally of the strongest shape.*

* *In the case of large cylinders for very great pressures, the lower or*

The guide pillars are securely bolted to the base B by nuts and iron washers, not shown. The cup leather packing CL deserves special attention, because it formed the chief improvement by



VERTICAL SECTION OF A BRAMAH HYDRAULIC PRESS,
Made in the Engineering Workshop of The Glasgow Technical College.

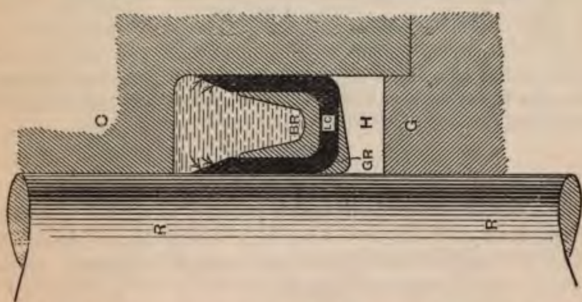
INDEX TO PARTS.

L	represents	Lever.	DV	represents	Delivery valve.
P	"	Pressure on L at A.	RV	"	Relief valve.
F	"	Fulcrum of L.	DP	"	Delivery pipe.
B	"	L's connection with plunger's guide-rod.	RC	"	Ram cylinder.
			R	"	Ram or plunger.
			CL	"	Cup leather packing.
PP	"	Pump plunger.	T	"	Top, table, or T piece.
Q	"	Reaction or stress on plunger PP.	W	"	Weight lifted, or total pressure on R.
PB	"	Pump barrel.	CG	"	Cross girder.
IP	"	Inlet pipe.	GP	"	Guide pillars.
SV	"	Suction valve.	BB	"	Base block.

inner end of the cylinder should be carefully rounded off, both inside and outside. For, if left square, or nearly square, the crystals formed in the casting of the metal naturally arrange themselves whilst cooling in such a manner as to leave an initial stress, and consequent weakness, inviting fracture along the lines joining the inside to the outside corners of the cylinder end. The severe shocks and stresses to which this weak line of division is subjected during the working of the press would sooner or later force out the end of the cylinder, in the shape of the frustrum of a cone, unless the cylinder had been made unnecessarily thick and strong at the bottom end.

Bramah on Pascal's press. It consists of a leather collar of ∩ section, placed into a cavity turned out of the neck of the cylinder, and kept there by the gland of the cylinder cover. The following figure shows an enlarged section of Bramah's packing suitable for a huge press, where the desired shape of the leather collar LC is maintained by an internal brass ring, BR, and an outside metal guard ring GR, resting on a bedding of hemp H. It will be observed at once, from an inspection of this figure, that the water which leaks past the easy fit between the plunger or ram R, and the cylinder C, presses one of the sharp edges of the leather collar against the ram, and the other edge against the side of the bored cavity in the neck of the cylinder, with a force directly proportional to the pressure of the water in the cylinder. By this simple automatic action, the greater the pressure in the cylinder the tighter does the leather collar grip the ram and bear on the cylinder's neck.

Bramah's Leather Collar Packing.—This collar is made from a flat piece of new strong well-tanned leather, thoroughly soaked in water, and forced into a metal mould of the requisite



ENLARGED VIEW OF BRAMAH'S LEATHER COLLAR FOR A BIG HYDRAULIC PRESS.

INDEX TO PARTS.

R	represents	Ram.	BR	represents	Brass ring.
C	„	Cylinder.	GR	„	Guard ring.
G	„	Gland of C.	H	„	Hemp bedding.
LC	„	Leather collar.			

size and shape until it has assumed the form of a U collar. The central or disc portion of the leather is then cut out, and the circular edges are trimmed up sharp in the bevelled manner shown by the above figure.

Formula for the Pressure on the Ram of a Bramah Press.—Referring again to the first figure in this Lecture, it will be found that by taking moments about the fulcrum at F, we obtain the pressure or reaction Q on the plunger of the force pump. Therefore, neglecting weight of lever and friction, we get—

$$P \times AF = Q \times BF. \quad \therefore\ Q = \frac{P \times AF}{BF}$$

Further, by Pascal's law for the transmission of pressure by liquids, enunciated in Lecture XVII., we know that the statical pressure Q is transmitted with undiminished force to every corresponding area of the cross section of the ram.

Or, . . Q : W : : area of plunger : area of ram.

$\therefore$ W $\times$ area of plunger $=$ Q $\times$ area of ram.

$$W \times \pi r^2 = Q \times \pi R^2$$

Where r = radius of plunger, and R = radius of ram, both in the same unit. Substituting the previous value for Q, and dividing each side of the equation by π, we get—

$$W \times r^2 = \frac{P \times AF}{BF} \times R^2$$

$$\therefore\ W = \frac{P \times AF}{BF} \times \frac{R^2}{r^2}$$

Since the radius of a circle is directly proportional to its diameter, we may write the formula thus, where D is the diameter of the ram and d the diameter of the plunger, both in the same unit—

$$W = \frac{P \times AF}{BF} \times \frac{D^2}{d^2}$$

EXAMPLE I.—In a small Bramah press, P = 50 lbs., AF = 20 in., BF = 2 in., area of plunger = 1 sq. in., whilst area of ram = 14 sq. in. Find W, neglecting friction and weight of lever.

ANSWER.—By the above formula—

$$W = \frac{P \times AF}{BF} \times \frac{\pi R^2}{\pi r^2}$$

Interpolating values, we get— $W = \frac{50 \times 20}{2} \times \frac{14}{1} = \mathbf{7000\ lbs.}$

EXAMPLE II.—In Bramah's original press at South Kensington the ram is 3 inches in diameter, and it acts at a distance of 6 inches from the fulcrum, which is at one end of a lever 10 feet 3 inches long, carrying a loaded scale-pan at the other end. What should be the pressure of the water in the press in order to *lift a weight* of 3 cwt. in the scale-pan, neglecting the weight of *the lever?* Make a diagram of the arrangement. (*S. and A. Exam. 1892.*)

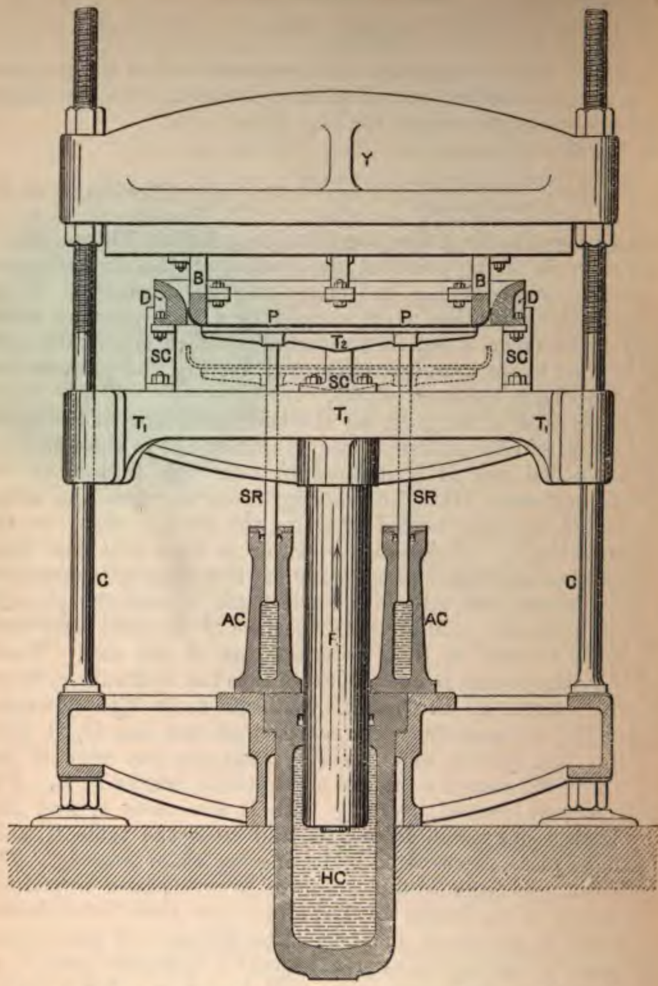
ANSWER.—Here $d = 3$ in., consequently the area of the plunger $= \frac{\pi}{4}d^2 = \cdot 7854 \times 3'' \times 3'' = 7$ sq. in.; BF $= 6''$; AF $= 10'\ 3'' = 123''$; P $= 3$ cwt. $= 3 \times 112 = 336$ lbs.; and we have to find the pressure per sq. in. on the ram that will balance P, acting with the stated advantage, since the area of the ram is not given.

By the above formula—

$$W = \frac{P \times AF}{BF} \times \frac{\text{area of 1 sq. in.}}{\text{area of plunger}} = \frac{336 \times 123''}{6''} \times \frac{1 \text{ sq. in.}}{7 \text{ sq. in.}} = \mathbf{984} \text{ lbs. per sq. in.}$$

Large Hydraulic Press for Flanging Boiler Plates, &c.—As an example of the practical application of the Bramah press to modern boiler-making, the accompanying illustration shows the form which it takes when worked by a high-pressure water supply derived from a central accumulator, which may at the same time be used to work cranes, punching, riveting, and other similar machine tools, in the same works.

The operation of flanging, say the end tube-plates of the cylindrical barrel of a locomotive boiler, is carried out in the following manner:—The ram R is lowered to near the bottom of the hydraulic cylinder HC, thus leaving room to place the boiler plate (which has been heated all round the outside edge) on the movable table T_2. High-pressure water is then admitted from the central accumulator to the auxiliary cylinders AC, thus forcing the side rams SR, SR, with their table T_2, and the plate P, vertically upwards, until the upper surface of the plate bears hard against the bearers B, B, or internal part of the dies. Water from the same source is now admitted into the hydraulic cylinder HC, which forces up the ram R, with its table T_1, supporting columns SC, SC, and the external part of the dies D, D, until the latter has quietly and smoothly bent the hot edge of the plate round the curved corner of the internal bearer B, B. The ram R is now lowered, carrying with it the table T_1 and dies D, by letting out water from HC, and then the table T_2, with the flanged plate, are lowered by letting out water from AC. The plate is removed from its table, allowed to cool, placed in position in the barrel of the boiler, marked off for the rivet holes, drilled and riveted in the usual manner. The student will now understand what a useful and powerful servant a hydraulic press is to the engineer in the hands of a skilful workman, for it can be made to do work in the manner indicated above in far less time, and with far greater certainty of uniformity and exactitude, than the boiler-smith could turn out, with any number of hammermen *to help him. It is fast* replacing, the steam-hammer for cressin *work, and the steam* or belt-driven punching and riveti



LARGE HYDRAULIC PRESS FOR FLANGING BOILER PLATES.*

* The above figure is a reduced copy of one from Prof. Henry Robinson's book on "Hydraulic Machinery," published by Messrs. Charles Griffin & Co., but it has been indexed according to the Author's style of symbols, and described in an elementary manner.

INDEX TO PARTS.

HC	represents	Hydraulic cylinder.
R	"	Ram of HC.
C, C	"	Columns supporting Y.
Y	"	Yoke or cross-head.
BB	"	Bearers of the internal die ring.
P	"	Plate to be flanged.
DD	"	Dished die or external die ring.
SC	"	Supporting columns for DD.
T_1	"	T-piece or movable table for DD.
T_2	"	T-piece or movable table for P.
SR	"	Side rams for T_2.
AC	"	Auxiliary cylinders.

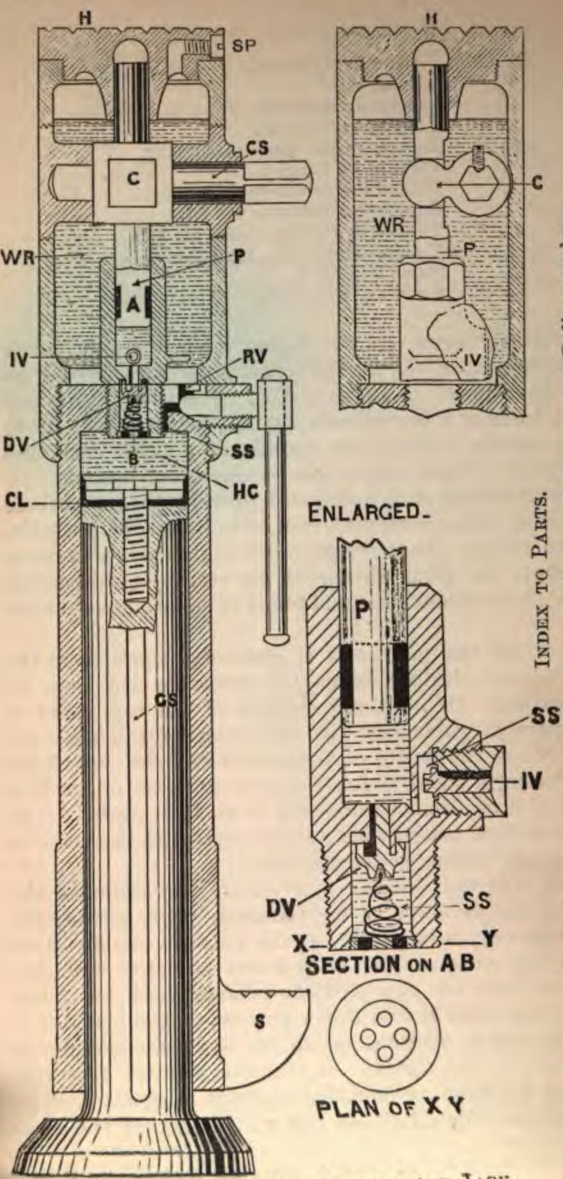
machines, the steam screw and wheel-gear worked cranes, screw and wheel-gear hoists, as well as the screw press for making up bales of goods mentioned in Lecture XV. For with it, you can bring to bear a force of a few pounds on the square inch or as many tons, by merely turning the handle of a small cock, and with a certainty of action unattainable by any other means.

The Hydraulic Jack is a combined hydraulic press and force pump, arranged in such a compact form as to be readily portable, and applied to lifting heavy weights through short distances. It therefore effects the same objects as the screw-jack described in Lecture XV., but with less manual effort or greater mechanical advantage.

The base on which the jack rests is continued upwards in the form of a cylindrical plunger, so as to constitute the ram of the hydraulic cylinder HC. Along one side of this ram there is cut a grooved parallel guide slot GS, into which fits a steel set pin, screwed through the centre of a nipple cast on the side of the cylinder (not shown in the drawings) for the purpose of guiding the latter up and down without allowing it to turn round. The top of the ram is then bolted with a water-tight cup leather CL, by means of a large washer and screw-bolt.

The action of this cup leather is precisely the same as the leather collar in the cylinder of the Bramah press already described; but it has only to be pressed by the water in one direction —viz., against the sides of the truly-bored cast-steel cylinder, instead of against both the ram and the cylinder neck, as in the previous case. The head H and upper portion of the machine is of square section, and is screwed on to the hydraulic cylinder in the manner shown by the figure. It contains a water reservoir WR, which may be filled or emptied through a small hole by taking out the screw-plug SP.* In the centre line of the head-

* *This screw plug SP is* slackened back a little to let the air in or o



THE HYDRAULIC JACK.

INDEX TO PARTS.

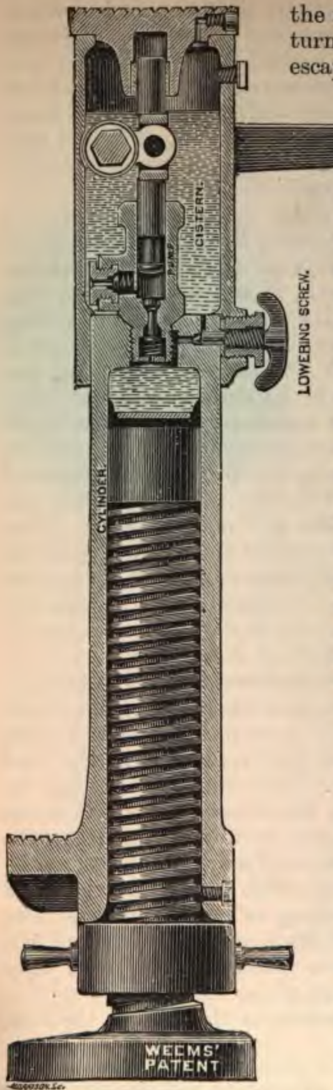
H	represents	Head for placing under a load.
SP	"	Screw plug for filling and emptying [WR.
WR	"	Water reservoir.
C	"	Crank.
CS	"	Crank shaft.
P	"	Pump plunger.
IV	"	Inlet valve.
DV	represents	Delivery valve.
SS	"	Spiral springs for IV and DV.
RV	"	Relieve valve.
HC	"	Hydraulic cylinder.
CL	"	Cup leather-packing.
CS	"	Guide slot along the length of ram.
S	"	Step for placing under a load.

piece there is placed a small force pump, the lower end of which is screwed into the centre of the upper end of the hydraulic cylinder. This pump is worked by the up-and-down movement of a handle placed on the squared outstanding end of the turned crank shaft CS. To the centre of the crank shaft there is fixed a crank C, which gears with a slot in the force-pump plunger P, and thus the motion of the handle is communicated to the pump plunger in a reduced amount, corresponding to the inverse ratio of the lengths of the handle and the crank from the fulcrum or centre of the crank shaft. By comparing the right-hand section of the water reservoir, and the section on the line AB, with the vertical left-hand section of the jack, it will be seen where the inlet and delivery valves IV and DV are situated. On raising the pump plunger P, water is drawn from WR into the lower end of the pump barrel through IV, and on depressing the plunger this water is forced through the delivery valve DV into the hydraulic cylinder, thus causing a pressure between the upper ends of the cylinder and the ram, and thereby forcing the cylinder, with its grooved head H, and footstep S, upwards, and elevating whatever load may have been placed thereon. Both the inlet and outlet valves are of the kind known as "mitre valves." They have a chamfer cut on one or more parts of their turned spindles, so as to let the water in and out along these channels. The valves are assisted in their closing action by small spiral springs SS, bearing in small cups or hollow centres, as shown more clearly in the case of DV by the enlarged section on AB.

Weems' Compound Screw and Hydraulic Jack.—This is a jack combining some of the advantages of the ordinary screw-jack with those of the hydraulic one. It is often desirable to be able to bring the head or footstep into trial contact with the load before applying the water pressure. This can easily be done by turning the nut at the foot of the screw, cut on the ram of the jack. The arrangement will at once be understood from the figure. It will be observed that the load may also be lowered by turning this nut, or by the screw-tap which permits water to flow from the cylinder back into the cistern, as in the previous case. The bottom nut may be screwed hard up to the foot of the hydraulic cylinder, so as to sustain the whole load, and thus prevent overhauling through leakage of the water.

When it is necessary to lower the load or the head of the jack,

of the top of the water reservoir when working the jack. There is generally another and separate screw plug opening (as will be seen by the following figure of Weems' patent jack) for filling or emptying the water reservoir, quite independent of the above-mentioned one, which is used in this case for both purposes.



WEEMS' COMPOUND SCREW AND HYDRAULIC JACK.

the relief valve or lowering screw, is turned so as to permit the water to escape from the hydraulic cylinder back into the water reservoir, as clearly shown by the drawing. This may be done very gently by simply giving this screw a very small part of a complete turn; in other words, by throttling the passage between the hydraulic cylinder and the water reservoir. Or it may be done quickly by turning it through one or more revolutions. This passage can then be closed by screwing the plug home on its seat.

Mr. Croydon Marks, in his book on "Hydraulic Machinery," illustrates and describes another method of lowering the jack-head (first introduced by Mr. Butters, of the Royal Arsenal, Woolwich), where, by a particular arrangement, the inlet and delivery valves are acted upon by an extra depression of the handle, and consequent movement of the pump plunger. He also gives the main dimensions, with a drawing, of the standard 4-ton pattern as used by the British Government, where the ram has a diameter $D = 2''$, the pump plunger a diameter $d = 1''$; and the ratio of the leverage of the handle to the crank is 16 to 1. Therefore from the previous formula we find that,

The Theoretical Advantage =

$$\frac{W}{P} = \frac{AF}{BF} \times \frac{D^2}{d^2} = \frac{16}{1} \times \frac{2^2}{1^2} = \frac{64}{1}$$

And he instances two trials by Mr. W. Anderson, the Inspector-general of Ordnance Factories, to determine the efficiency of these jacks, where, with a pressure on the end of the working handle of 76 lbs., the theoretical load should have been 76 lbs. × theoretical advantage $= 76 \times 64 = 4864$ lbs., instead of which it was only 3738 lbs.;

$$\therefore \quad 4864 \text{ lbs.} : 3738 \text{ lbs.} : 100 : x$$

$$\text{Or,} \quad x = \frac{3738 \times 100}{4864} = \mathbf{77}\ \% \text{ efficiency}$$

In a second trial, a load of 1064 lbs. required a pressure of 22 lbs. on the handle, and consequently the efficiency at this lighter load, as might be expected, was less, or only 74 %.

Example III.—With a hydraulic jack of the dimensions given above, and of 77 % efficiency, it is desired to lift a load of 4 tons; what force must be applied to the lever handle?

Answer.—By the previous theoretical formula,

$$W = \frac{P \times AF}{BF} \times \frac{D^2}{d^2}$$

$$\therefore P = \frac{W \times BF}{AF} \times \frac{d^2}{D^2}$$

$$= \frac{4 \times 2240 \times 1}{16} \times \frac{1^2}{2^2} = \mathbf{140\ lbs.}$$

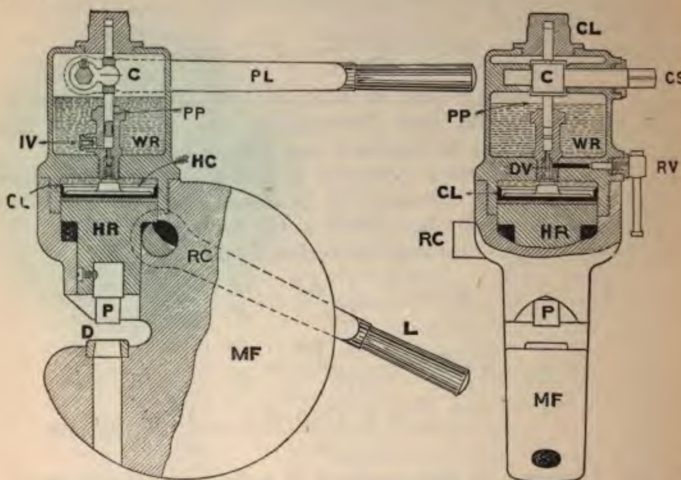
But the efficiency of the machine is only 77 %: consequently 140 lbs. is 77 per cent. of the force required—

$$\therefore 77 : 100 :: 140 \text{ lbs.} : x \text{ lbs.}$$

$$x = \frac{140 \times 100}{77} = \mathbf{181{\cdot}81\ lbs.}$$

The Hydraulic Bear, or Portable Punching Machine.—This is another very useful application of the hydraulic press and force pump. It is used in every iron or steel shipbuilding-yard and bridge-building works. By comparing the drawing with the index to parts, and taking into consideration the fact that its construction and action are so very similar to the hydraulic jack already described in full detail, we need say nothing more than direct the student's attention to the action of the raising cam, and to the means by which the apparatus is lifted and suspended. In order to raise the punch for the admittance of a plate between it and the die D, the relief valve RV must first be turned backwards, and the lever L depressed. This causes the corner of the *raising cam RC* to force the hydraulic ram HR upwards, and the *water from the hydraulic* cylinder HC back into the water

reservoir WR. The relief valve may now be closed and the plate adjusted in position. Then the pump lever can be worked up and down until the punch P is forced through the plate, and the punching drops through the die D and the hole in the metal frame MF, on to the ground, or into a pail placed beneath to receive it.



SIDE VIEW AND SECTION. END VIEW AND SECTION.

THE HYDRAULIC BEAR, OR PORTABLE PUNCHING MACHINE.

INDEX TO PARTS.

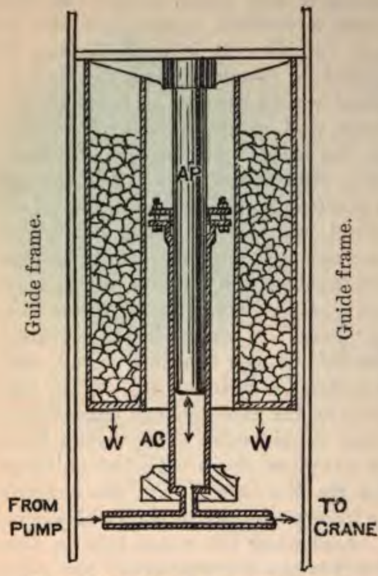
PL	represents	Pump lever.	HC	represents	Hydraulic cylinder.
CS	,,	Crank shaft.	CL	,,	Cup leather.
C	,,	Crank.	HR	,,	Hydraulic ram.
PP	,,	Pump plunger.	RC	,,	Raising cam.
WR	,,	Water reservoir.	L	,,	Lever for RC.
IV	,,	Inlet valve.	P	,,	Punch.
DV	,,	Delivery valve.	D	,,	Die ring.
RV	,,	Relief valve.	MF	,,	Metal frame.

The whole bear is suspended by a chain (worked by a crane or other form of lifting tackle) attached to a shackle, whose bolt passes through a cross hole in the back of the metal frame MF, just above, but a little to the front of the centre of gravity of the machine. This hole and shackle are not shown in the drawing, but the student can easily understand that the hole would be ...d a little above where the letters RC appear on the side view

and that the chain would pass clear of the pump lever, since it works well to the right-hand side of the bear.

The Hydraulic Accumulator.—The demand for hydraulic power to work elevators, cranes, swing bridges, dock gates, presses, punching and riveting machines, &c., being of an intermittent nature—at one moment requiring a full water supply at the maximum pressure, and at another a medium quantity, whilst in many cases all the machines may be idle—it is evident that if an engine with pumps were devoted to supplying this demand in a direct manner, the power thereof would have to be equal to the greatest requirements of the plant, and would have to instantly answer any and every call from the same. In the case of a low-pressure supply, as for lifts, this difficulty is best overcome by placing one tank in an elevated position at the top of the hotel or building where the lift is required, and another tank below the level of the lowest flat. Then a small gas engine working a two- or three-throw pump, or a Worthington duplex steam pump, may be used to elevate the water more or less continuously from the lower to the higher tank. The "head" of water in the elevated tank will, if sufficient, work the lift at the required speed, and the discharged water from the hydraulic cylinder will enter the lower tank, to be again sent round on the same cycle of operations. Should the lift be stopped for any considerable time, then a float in the upper tank, connected by a rope or chain with the shifting fork for the belt-driven pumps (in the case of the gas engine) will force the belt over on to the loose pulley, or shut off the steam from the Worthington pump. And when the water falls in the upper tank, the float will cause a reverse movement of the rope and shift the belt to the tight pulley, or open the steam valve, and so start the pumps. When the pressures required are great, such as for cranes, &c., where 700 lbs. on the square inch is considered a very medium pressure, an elevated tank would be out of the question, for it would have to be fully 1600 feet high in order to exert this force and to overcome friction. Under these circumstances recourse is had to a very simple and compact arrangement called an accumulator, of which a lecture diagram is herewith illustrated, without any details of cocks or valves, and automatic stopping and starting gear. A steam engine or other motor works a continuous delivery pump, of the combined piston and plunger type, without the aid of an air vessel, as illustrated by the fourth and fifth figures in Lecture XIX. The water from the pump enters the left-hand branch pipe leading into the foot of the accumulator cylinder, and forces up the accumulator ram with its cross head or top **T** piece, and the attached weight or dead load, until the ram has reached nearly to the end of its stroke. Then

the top of the T piece or a projecting bracket on the side of the wrought-iron cylinder containing the dead load, engages with and lifts a small weight attached to a chain passing over a pulley fixed to the guide frame or to the wall of accumulator house. This chain is connected directly to the throttle valve of the steam engine supply pipe, or to the belt shifting gear (if the pump is driven by belt gearing), and being provided with a counter-weight, the motor and pump are automatically stopped by the raising of the weight and the chain in the accumulator house. Should the water which has been forced into the accumulator cylinder be now used by a crane or other machine, the load on the ram causes it to follow up and keep a constant pressure per square inch on the water. The starting weight naturally falls as the receding T piece or bracket descends, thus pulling the starting chain, and opening the steam engine throttle valve, or shifting the belt from the loose to the fixed pulley, and again setting the pump to work. Should the hydraulic machines be working continuously, then the pump is kept going, for the water from it passes directly on to the machines, and only the surplus water finds its way into the accumulator cylinder if the pump's supply exceeds the demand of the machines for water.



THE HYDRAULIC ACCUMULATOR.

INDEX TO PARTS.

AC for Accumulator cylinder.
AP ,, Accumulator plunger or ram.
W ,, Weight or load contained in an annular cylinder of wronght iron and suspended from the top of T-piece or crosshead.

The annular cylinder of wrought iron is generally filled with scrap iron, iron slag, or sand, or other inexpensive weighty material. The accumulator cylinder AC has a stuffing-box and gland at its upper end. A coil of hemp woven into a firm rectangular section and smeared with white lead is placed in the *bottom of the* stuffing-box. The gland is screwed down on the top

of this packing until the normal pressure of the water in the cylinder cannot leak past it. Cup leather packing is seldom used for this simple form of accumulator; just the ordinary packing that would be used for pump rods is found to answer all requirements. This is the simplest form of accumulator which we have described, but it requires the greatest load for a certain hydraulic pressure per square inch. There are several other forms of accumulators, and several most interesting appliances such as capstans, cranes, bridges, punching and riveting machines, &c., are worked by them, which we would have liked to have described here, but the limits of our space and the complexity of their construction necessitate our deferring this pleasure to our Advanced Course.

Example IV.—Describe and sketch in section a hydraulic accumulator, showing how the ram is kept tight in the cylinder. A hydraulic press, having a ram 16 inches in diameter, is in connection with an accumulator which has a ram 8 inches in diameter and is loaded with 50 tons of ballast; what is the total pressure on the ram of the press? (S. and A. Exam. 1892.)

Answer.—The first part of the question is answered by the previous figure and by the text.

By Pascal's Law the pressure *per square inch* in the accumulator is equal to the *pressure per square inch in the* hydraulic press. Consequently—

$$\frac{\text{Total Pressure on Press}}{\text{Total Load on Accumulator}} = \frac{\text{Cross Area of Press}}{\text{Cross Area of Accumulator}}$$

$$\frac{P}{50 \text{ tons}} = \frac{\pi}{4} \times 16^2 \Big/ \frac{\pi}{4} \times 8^2$$

$$\therefore P = \frac{50 \times 16 \times 16}{8 \times 8} = \mathbf{200 \text{ tons.}}$$

LECTURE XX.—QUESTIONS.

1. Draw a section through a hydrostatic press, showing the cylinder, ram, and force pump, together with the valves. Why is the base of the cylinder of a large press rounded instead of being flat as in a steam cylinder? If the diameter of the ram is 9 times that of the force pump, and if Q be the pressure on the pump, what is the pressure exerted by the ram, neglecting friction? *Ans.* 81 P.

2. Explain by aid of a sketch the mode of packing the ram of a hydraulic press and explain how it acts. The force which actuates the force pump is applied at the end of a lever giving a mechanical advantage of 14 to 1, and the area of the plunger of the pump is 1 square inch. What pressure must be applied to the end of the lever to produce a pressure of 1 ton per square inch on the water enclosed in the press? *Ans.* 160 lbs.

3. In the force pump of a press the area of the plunger is $\frac{1}{3}$ of a square inch, the distance from the fulcrum of the lever handle to the plunger is 2 inches, and the distance from the fulcrum to the other end of the lever is 2 feet; what pressure per square inch is exerted on the water underneath the plunger, when a weight of 20 lbs. is hung at the end of the lever handle? *Ans.* 720 lbs. per square inch.

4. In what way do you estimate the theoretical advantage gained by the use of the hydraulic press? In a small press the ram is 2 inches and the plunger $\frac{1}{2}$ inch in diameter; the length of the lever handle is 2 feet, and the distance from the fulcrum to the plunger is $1\frac{1}{2}$ inches. Find the pressure exerted on the ram when 10 lbs. is hung at the end of the lever. *Ans.* 2560 lbs.

5. In an hydraulic press with two pumps the plungers are $2\frac{1}{2}$ and 1 inch in diameter, and each is worked by a similar lever, which is acted on by the same force. When the larger pump alone is at work the pressure on the ram is 40 tons; what will it be when the smaller plunger is only working? *Ans.* 250 tons.

6. An hydraulic press, which is used for making lead pipes, has a ram 20 inches in diameter, while the ram which presses the lead is 5 inches in diameter. Find the pressure per square inch on the lead when the hydraulic gauge indicates 1 ton per square inch. Sketch a sectional elevation of the press, and show the packing of the hydraulic ram. (S. and A. Exam. 1891.) *Ans.* 16 tons.

7. How is the pressure taken off the object under compression when required, in a hydraulic press? Sketch the arrangement. What is the proportion of the diameters of the plunger and ram when the theoretical advantage gained thereby is 100 to 1, neglecting friction? (S. and A. Exam. 1888.) *Ans.* 1 to 10.

8. Make a rough sketch, and write a short description of the hydraulic lifting jack. It may be arranged on any system that you are acquainted with. Show clearly how the valves act and how the jack is lowered.

9. Sketch and describe the hydraulic bear or portable punching machine. Explain how the punch is raised and how the tool is handled.

10. Sketch and describe the construction of a vessel suitable for storing up a supply of water under pressure, and intended for actuating hydraulic machinery. If the plunger of this vessel be 17 inches in diameter, what load will bring the pressure of the water to 700 lbs. per square inch? *Ans.* 158,950 lbs.

11. Sketch and describe the hydraulic accumulator for storing up water

er pressure. If the ram of the accumulator be 6 inches in diameter, .t load will be required to produce a water pressure of 500 lbs. on the ıre inch? To what head of water would this pressure correspond? and A. Exam. 1887.) *Ans.* 14,142·8 lbs. and 1152 feet.

:. An hydraulic accumulator, with a ram of 16 inches in diameter, is ıected with an hydraulic press whose ram is 26 inches in diameter. load on the accumulator is 80 tons; what force would the press ·t? Make a vertical section through the accumulator, showing its struction. (S. and A. Exam. 1889.) *Ans.* 211·25.

LECTURE XXI.

CONTENTS.—Motion and Velocity—Uniform, Variable Linear and Circular Velocity—Unit of Velocity—Acceleration—Unit of Acceleration—Acceleration due to Gravity—Graphic Representation of Velocities—Composition and Resolution of Velocities—Newton's Laws of Motion—Formulæ for Falling Bodies—Formulæ for Linear Velocity with Uniform Acceleration—Centrifugal Force due to Motion in a Circle—Experiments I. II. III.—Example I.—Balancing Fast-speed Machinery—Centrifugal Stress on the Arms of a Fly-wheel—Example II.—Energy—Potential Energy—Kinetic Energy—Accumulated Work—Accumulated Work in a Rotating Body—The Fly-wheel Radius of Gyration—Example III.—The Fly Press—Questions.

Motion and Velocity.—(1) Motion is the opposite of rest, for it signifies change of position.

(2) *Velocity* is the rate at which a body moves, or rate of motion. It is considered *absolute* when it is measured from some fixed point, and *relative* if it refers to another body in motion at the same time.

(3) *Uniform Velocity* takes place when the rate of motion does not change—*i.e.*, when the body moves over equal distances in equal times.

(4) *Variable Velocity* takes place when the rate of motion changes—*i.e.*, when a body moves with either a constantly increasing or decreasing velocity. For example, a stone pitched into the air rises with a gradually *decreasing* velocity, but falls with a gradually increasing rate of motion.

(5) The *Unit of Velocity* is the velocity of a body which moves through unit distance in unit time. The British unit of velocity is therefore 1 *foot in* 1 *second*. In physical problems velocity is generally expressed in *feet per second*, but for convenience the engineer reckons the piston speed of engines in *feet per minute*, and the public speak of the speed of a man walking, of a horse trotting, or of a train, in miles per hour.

(6) *Linear Velocity* is the rate of motion in a straight line, and is measured, as we have just stated, in feet per second or per minute, or in *miles* per hour.

If $v =$ the velocity ; $l =$ the distance ; and $t =$ the time—

$$\text{Then } v = \frac{l}{t}; \text{ or } l = vt; \text{ or } t = \frac{l}{v}.$$

(7) *Circular Velocity* is the rate at which a body describes an angle about a given point—for example, the number of revolutions per minute of a pulley; but circular velocity may also be measured by the feet per second or per minute which a point at a known distance from the centre of motion moves.

(8) *Acceleration.*—In the case of variable velocity, the *rate of change of the velocity* is termed *the acceleration*, and may be either positive or negative—*i.e.*, it may be an increasing or a decreasing rate.

(9) The *Unit of Acceleration* is that acceleration which imparts unit change of velocity to a body in unit time; or in this country it is *an acceleration of* 1 *foot per second in one second.*

(10) The *Acceleration due to Gravity* is considerably greater than the above unit, and varies at different places on the earth's surface. At Greenwich it is 32·2 feet per second in one second. In Elementary Applied Mechanics questions we will indicate it by the symbol g, and consider $g = 32$ feet per second *in one second.*

Graphic Representation of Velocities.—The *linear* velocity of a point (such as the *c.g.* of a body) may be represented in the same way as we have hitherto represented a force. A line drawn from a point with an arrow-head indicates the direction of motion, and the length of the line to scale the magnitude of the velocity. (See p. 3, Lecture I.)

Composition and Resolution of Velocities.—Velocities may be compounded and resolved in exactly the same way as we treated forces by the parallelogram and triangle of forces, &c., in Lecture VIII.

Newton's Laws of Motion.—I. *A body in motion, and not acted on by any external force, will continue to move in a straight line and with uniform velocity.*

II. *When a force acts upon a body in motion, the change produced in the quantity of motion is the same, both in magnitude and direction, as if the force acted on the body at rest.*

*The change in the quantity of motion is therefore proportional to the force applied, and takes place in the direction of that force.**

III. *If two bodies mutually act upon each other, the quantities of motion developed in each in the same time are equal and opposite.*

Or, *Action and reaction are equal and opposite.*

These three laws were first stated clearly by Sir Isaac Newton as the result of *inductive* reasoning. Having observed certain facts, he set about investigating what would be the consequence if his conjectures as to these facts were applied to particular

* Here "quantity of motion" means "momentum," or mass × velocity, but mass = weight in lbs. ÷ acceleration due to gravity.

∴ Quantity *of motion* or momentum $= Mv/g$. See footnote, Lecture I, *page 2.*

cases. Finding that his estimate of the probable result came true, he formulated a general law in accordance with his observations and reasonings.

The student has already conceived the truth of the first and third laws in the reasonings and applications of force to matter, treated of in the previous Lectures. We will now give in as brief a form as possible the formulæ for falling bodies, because they naturally lead on to the formulæ for "*centrifugal force*" on a rotating body, and to the "energy stored" up in a moving body, both of which are of great interest and importance to the young engineer. The experimental and algebraical proofs of these formulæ are given in Elementary Manuals on Theoretical Mechanics, and we must either assume that the student has studied these, or ask him to assume their truth in the meantime.

Formulæ for Falling Bodies.—If a body falls freely *in vacuo* under the action of gravity from *rest* through a height h feet; then (since gravity produces a constant acceleration in the velocity of the body) at the *end* of each successive second the velocity of the body will be increased by g, or 32 feet. Let v be the velocity of the body at the end of t seconds,

Then, . . $v=gt$; but $v^2=2gh$

$$\therefore \quad h=\frac{v^2}{2h}=\frac{g^2t^2}{2g}=\tfrac{1}{2}gt^2$$

$$\text{And,} \quad t=\frac{v}{g}=\frac{\sqrt{2gh}}{g}=\sqrt{\frac{2h}{g}}$$

Formulæ for Linear Velocity with Uniform Acceleration.—Suppose that instead of the uniform accelerating force of gravity we have any other constant force of F lbs. acting on a body, and if this force moves the body through a distance of l feet along a *perfectly smooth horizontal* plane, the above formulæ naturally becomes*—

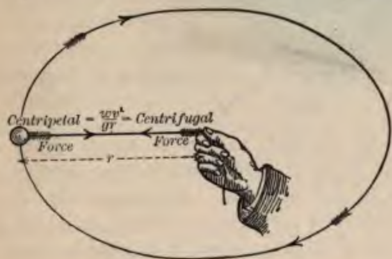
Then, . . $v=at$; but $v^2=2al$

$$\therefore \quad l=\frac{v^2}{2a}=\frac{a^2t^2}{2a}=\tfrac{1}{2}at^2$$

$$\text{And,} \quad t=\frac{v}{a}=\frac{\sqrt{2al}}{a}=\sqrt{\frac{2l}{a}}$$

* We intentionally use the letter l for length or distance, and a for acceleration. Most writers use the word "space" for distance and the symbol s; but space is of *three* dimensions, and involves the idea of *volume*. It cannot therefore be, strictly speaking, used to represent distance or length, which is only of one dimension. The letter f is also often used for acceleration; but f naturally represents a force, so we prefer to use, a, for acceleration, in order to be consistent with our notation.

Centrifugal Force due to Motion in a Circle.—EXPERIMENT I.—When a body—such as a stone—is attached to a cord and whirled round and round in a circle, the hand experiences a pull in the direction of the string as if it were in tension under the action of a force, and the faster the body is moved the greater becomes the stress on the string, just as David of old must have felt it before he let go that pebble from his sling which went so straight for Goliath's brow. The stone is constantly tending to fly off at a tangent, and is only kept moving in the circular path by the reaction pulling it towards the centre of motion. The pull *from* the centre of motion is called the *centrifugal* or *centre-flying force*, and the exactly equal and opposite reaction is termed the *centripetal* or *centre*-seeking force. It may be proved by geometry that each of these forces is equal to



EXPERIENCING THE EFFECT OF CENTRIFUGAL FORCE.

the weight of the body × the square of the velocity ÷ the acceleration due to gravity × the radius of the circle described by the body.

Or, . . . $$P = \frac{Wv^2}{g\,r} \text{ lbs.}$$

Where P = Pull on the cord, or the centrifugal force in lbs.
,, W = Weight of the body in lbs.
,, v = velocity of the body in feet per second,
,, g = gravity's acceleration = 32′ per second *in one second*,
,, r = radius from centre of motion to *c.g.* of body in feet.

* At present the student must accept the above formula as correct. We shall have occasion to deduce the formula by aid of geometry in the Advanced Course. In the formula $\frac{W}{g}$ = the mass of the body, so that *if M represents the mass*, the formula may be written $P = \frac{Mv^2}{r}$ poundals.

EXPERIMENT II.—Take a pail and half fill it with water. Attach a rope to the centre of the handle, and swing it round and round your head. The water does not fall out, even if you swing it in a vertical plane, if the velocity be sufficient to cause the centrifugal force to be greater than the force of gravity.

EXAMPLE I.—A small tin pail, containing 1 lb. of water, with a rope attached to its handle, is to be whirled in a vertical circle. If the distance from the hand or centre of motion, to the surface of the water be 2 feet, what is the least number of revolutions per minute that you can give it in order not to spill any of the water?

ANSWER.—Here P must be at least equal to 1 lb., for $W = 1$ lb. and $r = 2$ feet, whilst $g = 32$.

By the formula—

$$P = \frac{Wv^2}{gr}$$

$$\therefore v = \sqrt{\frac{P \times g \times r}{W}} = \sqrt{\frac{1 \times 32 \times 2}{1}} = \sqrt{64} = 8 \text{ ft. per second.}$$

Now a circle of 2 feet radius = 12·56 feet circumference. Therefore, 12·56/8 = 1·57 revolutions per second, and $1{\cdot}57 \times 60 = 94{\cdot}2$ revolutions per minute. Consequently, if you whirl the pail at 100 revolutions per minute, there will be no fear of any water coming out of it even when it is upside down at the highest part of the circle.

EXPERIMENT III.—Turn a disc of wood with a small barrel on one side of the centre. Fit the wheel and the barrel so truly with a turned axle that when the axle is supported by eye hooks at each end for bearings, a cord wound round the barrel and then pulled sharply, will cause the wheel to revolve freely at a high speed without vibration or oscillation. Now bore a hole through the disc near its circumference, and run in molten lead into this hole. Again spin the wheel rapidly, when it will be found to hobble to such an extent as to shake itself almost out of the bearings.

The centrifugal force due to the unbalanced piece of lead asserts itself so thoroughly that when it reaches the highest position of its revolution round the axis, it overcomes gravity, and lifts the whole wheel and barrel clean out of the bearings. It thereby creates such a disturbance as to leave a distinct impression on the mind of the student.

Next bore another hole through the disc of the same size as the former one, and at the same distance from the axle, but diametrically *opposite* to the front hole, and run in the same weight of

lead into it. Again spin the wheel, and it will be found to run smoothly.

This experiment conveys to the young engineer a most useful lesson, for it not only shows him the effect of centrifugal force due to want of balance, but it also gives him an idea how to rectify the evil.

Balancing Fast-speed Machinery.—All fast-speed machinery, whether revolving or reciprocating, should as far as possible be most carefully balanced, in order to prevent centrifugal force coming into play and creating that horrid vibration and noise with which it is always more or less accompanied. There is nothing tends so much to the heating of bearings, and to the quick wearing out of brasses and other bearing surfaces as unbalanced moving parts; besides which, at very high velocities they become actually dangerous, and have frequently been known to cause destruction to life and property.*

Centrifugal Stress on the Arms of a Fly-Wheel.—If the arms of a fly-wheel or pulley are not properly proportioned to resist the centrifugal force due to the mass of the revolving rim; or, if the casting has been carelessly cooled, so as to set up internal stresses between the arms and the boss or the rim, the wheel may give way. In fact, there is no fly-wheel or pulley made that would not burst, under the very great stress of centrifugal force, if you only ran it fast enough. The student will observe from the formula that the centrifugal force or stress on the arms of a fly-wheel is directly proportional to the square of the velocity, so that by merely doubling the number of revolutions per minute you quadruple the stress on the arms, and if the speed be increased three times, the stress becomes nine times as great.

EXAMPLE II.—Each segment of a fly-wheel, with its corresponding arm to which it is attached, weighs 1000 lbs., and the mass may be taken as collected at a distance of 4 ft. from the axis of the wheel. If each arm has a breaking stress of 100,000 lbs., what is the maximum number of revolutions per minute that the fly-wheel could be run at without breaking the arms, neglecting the binding strength of the rim of the wheel?

ANSWER.—By the previous formula for centrifugal force—

$$P = \frac{Wv^2}{gr}$$

$$100{,}000 = \frac{1000 \times v^2}{32 \times 4} \quad \therefore v^2 = 12{,}800$$

* See Mr. Laidlaw's paper and the discussion on the "Balancing of High-speed Machinery," in the Transactions of the Institution of Engineers and *Shipbuilders* of Scotland for Session 1890–91.

$$\therefore v = 113 \text{ ft. per second, fully.}$$

$$\text{Or, } v = 113 \times 60 = 6780 \text{ ft. per minute.}$$

Now, the circumference of a circle of $4'$ radius $= 25$ ft.

$$\therefore \frac{6780}{25} = 271 \text{ revolutions per minute.}$$

Energy.—In applied mechanics energy means the *capability* of doing work.*

Potential Energy *is energy due to position of a body* with respect to the earth or some lower place. For example, when a body of 10 lbs. is lifted 10 ft. high, it has a potential energy of 100 ft.-lbs.; for it takes that amount of work to lift the 10 lbs. through the 10 ft.; and if then allowed to fall, it would naturally give out the same quantity of work, either in overcoming friction, or, if it fell freely, it could be usefully employed to that amount and no more.

Potential energy may also be due to a condition of a body, such as the potential energy in the coiled spring of a watch or clock, which when wound up does work in moving the mechanism. We have also the case of potential energy in a lump of coal, which when burned gives out heat, that will raise steam to be used in a steam engine for doing work. Or, in the case of an electric battery, where plates of copper and zinc are respectively placed in solutions of sulphate of copper and zinc, and on being suitably connected by wires to an electric motor, will give out electrical energy, which may be converted into mechanical work by the motor, and thereby effect some useful purpose.

Kinetic Energy *is energy due to motion.* For example, in the first instance of potential energy the weight of 10 lbs., in falling freely down through 10 ft., had *stored* up in it, *due to its motion*, an amount of accumulated work equivalent to 100 ft.-lbs.

Accumulated Work.—If a body of weight W lbs. be raised to a height h feet above the earth

The potential energy stored up $= \mathrm{W}h$ (ft.-lbs.)

Now, if the body be allowed to fall freely, under the action of gravity, through h feet, it would have a velocity at the end of time t seconds of v feet per second.

Referring back to the formulæ for falling bodies previously given in this lecture we see that—

$$h = \frac{v^2}{2g} \quad \therefore \mathrm{W}h = \frac{\mathrm{W}v^2}{2g} \text{ ft.-lbs.}$$

* We have specially avoided using this term hitherto, as students are [apt] to confuse it with force, work, and power.

Therefore the *kinetic energy* or *accumulated work* stored up in a moving body is expressed by the formula—

$$\frac{Wv^2}{2g}$$

If a body of weight W lbs. were impressed forward along a perfectly smooth plane for a distance of l feet, by a force F lbs., causing an acceleration of, a, feet per second; then the previous set of formulæ for linear velocity would apply when the reaction from the plane cancelled the force of gravity.

Here, . . $F = \frac{W}{g}a$; and $l = \frac{v^2}{2a}$

But the *Work Done* through distance $l = F \times l$

And . . $F \times l = \frac{W}{g}a \times \frac{v^2}{2a} = \frac{Wv^2}{2g}$ ft.-lbs.

Therefore in this case the accumulated work stored up in the moving body would be expressed by the formula—

$$\frac{Wv^2}{2a}$$

Accumulated work in a Rotating Body.—If a body of W lbs. be concentrated at a distance of r feet from the centre of motion, and be rotated so that it has a velocity of v feet per second, then

$$\textit{The Accumulated Work} = \frac{Wv^2}{2g} \text{ ft.-lbs.}$$

The Fly-wheel of a steam engine is an excellent example of accumulated work. If the pressure of steam in the cylinder and the point of cut-off be kept constant, and if one or other of the machines which are being driven by the engine be thrown out of circuit—or, in other words, if the belt be moved to the loose pulley—the load on the engine will be lessened, and the engine will have a tendency to increase in speed. If, however, it be provided with a very heavy fly-wheel, the surplus power of the engine will be stored up in the fly-wheel, so that the increase of speed will not be so great as if it had a light one, or none at all. If a machine should be suddenly brought into circuit again after a short time, then the load on the engine will be as quickly increased; but the stored-up energy in the fly-wheel will enable it to overcome this sudden demand for power, so that the speed of the engine will not be greatly altered. The fly-wheel, therefore, acts as *a regulator of speed, not only for alterations of load, but also for the variable pressures which exist in the cylinder of an engine.* This is

particularly noticeable in the case of gas engines, where the almost instantaneous explosion of gas in the cylinder at the beginning of a stroke creates an immense force, which would urge the piston forward at lightning speed, if it were not for the very heavy fly-wheel with which the engine is provided. The fly-wheel stores up some of this sudden force and gives it out again during the intervening strokes when there is no explosion, thus tending to a uniformity of speed which would be conspicuous by its absence if the gas engine had only a light fly-wheel, or none at all. In fact, the motion of gas engines would be so erratic without fly-wheels as to prevent their application to many purposes for which they are admirably adapted when aided by *very* heavy ones.

Radius of Gyration.—It will be evident, almost without explanation, that in the case of a fly-wheel or a rotating disc, those parts which are furthest from the centre of motion must accumulate more energy than those of the same weight which are nearer to that centre, because they move at a greater velocity. There is, however, for every body a *mean radius of rotation*, termed the "*radius of gyration*," which is at such a distance from the centre of motion, that if the whole mass of the body were concentrated there, the same kinetic energy or accumulated work would be developed at the same speed or number of revolutions per minute. The length of this mean radius varies with the shape of the rotating body, and requires a knowledge of higher mathematics for its computation; so we will assume that in the case of a fly-wheel it is at the *c.g.* of the rim, or that the distance is given in any question requiring solution.

EXAMPLE III.—A fly-wheel weighing 10,000 lbs. has a mean radius of rotation, $r = 5$ feet, and turns normally at 100 revolutions per minute. Owing to the load being diminished, the speed increases to 110 revolutions per minute; what reserve power is stored up in the fly-wheel fit to overcome any sudden increase of load?

ANSWER.—Let v_1 = the velocity in feet per second, at the normal speed n_1 revolutions per minute,

And v_2 = the velocity at the increased speed n_2 revolutions per minnte;

$$v_1 = 2\pi r n_1 = \frac{2 \times 22 \times 5 \times 100}{7 \times 60} = 52{\cdot}4 \text{ ft. per sec.}$$

$$v_2 = 2\pi r n_2 = \frac{2 \times 22 \times 5 \times 110}{7 \times 60} = 57{\cdot}6 \text{ ft. per sec.}$$

Stored energy at speed $n_1 = \frac{Wv_1^2}{2g}$

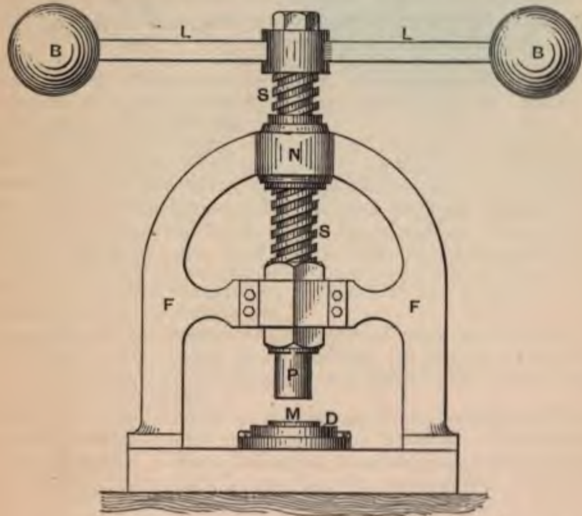
" " $n_2 = \frac{Wv_2^2}{2g}$

Reserved stored energy $= \frac{Wv_2^2}{2g} - \frac{Wv_1^2}{2g} = \frac{W}{2g}(v_2^2 - v_1^2)$

" " $= \frac{10000}{2 \times 32}(57 \cdot 6^2 - 52 \cdot 4^2)$

" " $=$ **89,375 ft.-lbs.**

The Fly-press.—This machine is used, in the form shown by the figure, either for embossing or stamping pieces of metal with



THE FLY-PRESS.

INDEX TO PARTS.

D represents	Disc supporting M.	S represents	Screw.
M ,,	Metal to be stamped.	N ,,	Nut for S.
P ,,	Punch or die.	L ,,	Lever arms.
F ,,	Frame of machine.	B ,,	Balls or weights.

some design, or for punching thin metal plates. The piece of metal M, to be embossed or punched, is laid on a disc D, and the *die or punch P is* caused to come down on M with a large *amount of stored-up* energy, due to the operator taking hol

one or other of the heavy balls B, and giving them a very rapid turn round. The result of this movement is to send the quickly pitched square-double-threaded screw rapidly through its nut N, thereby forcing the guided square carrying the punch straight downwards, and causing the latter to overcome the resistance of the hard metal. Neglecting friction at the screw and the guide, and considering the combined weight of the two balls as $= W$ lbs., and $v =$ their velocity in feet per second at the instant the punch meets the metal M, then—

The stored energy, or energy of the blow, $= \dfrac{Wv^2}{2g}$ ft.-lbs.

If . $l =$ Length the punch or die goes into the metal in feet,
And $R =$ Resistance overcome (mean) in lbs.,

Then . . . . $Rl = \dfrac{Wv^2}{2g}$ ft.-lbs.

LECTURE XXI.—QUESTIONS.

1. A body moves in a circle with a uniform velocity; show that it must be acted on by a constant force tending towards the centre, and find the magnitude of the force in terms of the radius of the circle, and of the mass and velocity of the body.

2. A body weighing $2\frac{1}{2}$ lbs., fastened to one end of a thread 4 feet long, is swung round in a circle, of which the thread is the radius; what will be its velocity when the tension of the thread is a force of 20 lbs. ($g = 32$)? *Ans.* 32 feet per second.

3. When an unbalanced wheel is set in rapid rotation, a considerable amount of shake and vibration is experienced. You are required to explain this result from first principles, and to state the mechanical laws which appear to be at work. How would you calculate the amount of pull that this unbalanced weight exerts?

4. What primary law in mechanics asserts itself when some revolving piece of machinery moves at a high velocity, and is unbalanced? A weight of 1 lb. is placed on the rim of a wheel 2 feet in diameter, which revolves upon its axis and is otherwise balanced. The linear velocity of the rim being 30 feet per second, what is the pull on the axis as caused by the weight of 1 lb.? *Ans.* 28·1 lbs.

5. A segment of a fly-wheel, with the arm to which it is attached, weighs 3500 lbs., and the mass of the portion may be taken as collected at a distance of 8 feet from the axis of the wheel, which makes 40 revolutions per minute. What is the force tending to pull away the segment and arm from the boss of the wheel? (S. and A. Exam. 1889.) *Ans.* 15,365 lbs.

6. Define kinetic energy. How does it differ from potential energy? If a velocity of 300 feet per second is impressed on a weight of 10 lbs., what is the measure of the energy now imparted to the weight? (S. and A. Exam. 1891.) *Ans.* 14,062·5 ft.-lbs.

7. State the rule for finding the amount of work stored up in a given weight when moving with a given velocity. A weight of 6 cwt. moves with a velocity of 20 feet per second; how many units of work are stored up in it? *Ans.* 4200 ft.-lbs.

8. Write down the formula for the amount of energy stored up in a given weight when moving with a given velocity. Describe, with a sketch, the action of a fly-press. If each ball of the press weighs 50 lbs., and the work stored up in the balls is 400 ft.-lbs., find the velocity with which they are moving. Take the number 32 to represent g. (S. and A. Exam. 1888.) *Ans.* 16 feet per second.

9. Account for the storing up of energy in a rotating fly-wheel. If the weight of the rim be doubled while the rate of rotation remains unchanged, how much is the energy increased? *Ans.* Twice.

10. State the formula for the energy stored up in a fly-wheel, on the supposition that the whole of the material is collected in a heavy rim of given mean radius. Apply the formula to show (1) the effect of doubling the number of revolutions per minute; (2) the effect of doubling the weight; (3) the effect of increasing the mean radius in the proportion of 3 to 2. (S. and A. Exam. 1890.)

11. A fly-wheel weighs $2\frac{1}{2}$ tons, and its mean rim has a velocity of 40 feet per second. If the wheel gives out 10,000 ft.-lbs. of energy, how much is its velocity diminished? (S. and A. Exam. 1888.) *Ans.* 1·455 feet per second.

12. The rim of a fly-wheel weighs 9 tons, and the mean linear velo of its mass is assumed to be 40 feet per second; how many foot-ton work are stored up in it? If it be required to store the additional wo 9 foot-tons, what should be the increase of velocity? *Ans.* 225 ft.-t 0·79 ft. per second.

13. Sketch a fly-press, explain its action, and state for what purpo is chiefly used. Find, in foot-pounds, the amount of work accumulat a body which weighs 80 lbs., and has a velocity of 20 feet per sec *Ans.* 500 ft.-lbs.

14. In a fly-press there are two weights, each of 60 lbs., placed a ends of an arm which drives the screw; and the velocity of each weig the instant of striking the blow is 10 feet per second. The die at th of the screw moves through $\frac{1}{16}$ inch in coming to rest; what mean st pressure does it exert on the metal subjected to the operation of stam *Ans.* 22,500 lbs.

LECTURE XXII.

CONTENTS.—Some Properties of Materials employed by Mechanics—Essential Properties—Extension—Impenetrability—Contingent Properties—Divisibility—Porosity—Density—Cohesion—Compressibility and Dilatability—Rigidity—Tenacity—Malleability—Ductility—Elasticity—Fusibility—Load, Stress, and Strain—Total Stress and Intensity of Stress—Tensile Stress and Strain—Example I.—Compressive Stress and Strain—Example II.—Limiting Stress or Ultimate Strength—Safe Loads and Elasticity—Limit of Elasticity—Hooke's Law—Factors of Safety—Modulus of Elasticity—Ratio of Stress to Strain—Example III.—Questions.

Some Properties of Materials employed by Mechanics.—The properties of matter are almost innumerable, but they may be divided into two classes: (1) *Essential* properties; (2) *Contingent* properties. The *essential* properties are those without which matter cannot possibly exist. The *contingent* properties are those which we find matter possessing, but without which we could conceive it to exist.

Essential Properties—1. Extension means that property by which every body must occupy a certain bulk or volume. When we say that one body has the same volume as another, we do not mean that it has the same quantity of matter, but only that it occupies the same space.*

2. Impenetrability means that every body occupies space to the exclusion of every other body, or that two bodies cannot exist in the same space at the same time.

Contingent Properties.—1. Divisibility means that matter may be divided into a great but not an infinite number of parts. The ultimate particles of matter are termed *atoms*, derived from a Greek word signifying indivisible.

2. Porosity signifies that every body contains throughout its mass atomic spaces or interstices to a greater or less extent. These spaces are filled with ether or gas. This has been proved to be the case with every known substance.

For example, when the steel or cast-iron cylinder of a hydraulic

* For Simple Rules of Mensuration see the Author's Elementary Manual on "Steam and the Steam Engine," Lectures I. II. III.

press is subjected to enormous pressure, water will ooze through the metal from the interior to the outside.

3. **Density** is that property by which one body differs from another in respect of the quantity of matter which it contains.*

$$\text{Let } M_1, M_2 = \text{Masses of two bodies}$$
$$\text{Let } V_1, V_2 = \text{Volumes of two bodies.}$$
$$\text{Let } D_2, D_2 = \text{Densities of two bodies.}$$

$$\text{If } V_1 = V_2, \text{ then } \frac{M_1}{M_2} = \frac{D_1}{D_2}; \text{ if } D_1 = D_2, \text{ then } \frac{M_1}{M_2} = \frac{V_1}{V_2}$$

$$\text{If both vary, then } \frac{M_1}{M_2} = \frac{V_1 \times D_1}{V_2 \times D_2}$$

4. **Cohesion** is that property by which particles of matter mutually attract each other at *insensible* or indefinitely small distances. It is therefore different from *gravitation*, since the latter acts at all distances. It is evident that without this property we could not have a solid, for if a solid body be lifted by one part, the remainder sticks to it, and the whole is kept together by cohesion.

5. **Compressibility and Dilatability** are properties common to all bodies, by which they are capable of being compressed like a sponge or extended like a piece of india-rubber in a greater or less degree.

6. **Rigidity** signifies the stiffness to resist change of shape when acted on by external forces. Unpliable materials which possess this property in a large degree are termed *hard*, whilst those which readily yield to pressure, without disconnection, are called *soft*. Substances which cannot resist a change of shape without breaking are termed *brittle*, whilst those that do resist and at the same time change their form are said to be *tough*.

7. **Tenacity** is the resistance (due to cohesion) which a body offers to being pulled asunder, and is measured by the tensile strength in lbs. per square inch of the cross section of the body. We will consider this property in the case of metals, &c., when dealing with stress and strain.

8. **Malleability** is that property by which certain solids may be pressed, rolled, or beaten out from one shape to another without giving way. It is therefore a property depending upon the softness, toughness, and tenacity of the material. Gold possesses this property in a higher degree than any other metal, and con-

* The **Density** of a substance is either the number of units of *mass* in a unit of volume, in which case it is equal to the *heaviness* (*i.e.*, weight of unit volume of substance in unit weight); or it is the ratio of the mass of a given volume of the substance to the mass of an equal volume of water, in which case it is equal to the *specific gravity*.

sequently sheets of gold are procurable of less than one-thousandth of an inch in thickness. Copper is one of the most useful of the malleable metals, and it may be beaten out into most elaborate shapes from the solid ingot. The Swedish iron of which horse-shoe nails are made is also very malleable, and is therefore highly prized by the blacksmith. Lead, although possessing softness, is not sufficiently tenacious to be considered a *very* malleable metal, but still it finds one of its most useful applications in the form of rolled lead sheathing for roofs of houses and interiors of water tanks, &c.

9. Ductility* is that property by which some metals may be drawn down through a die-plate into wire or tubes. This property depends chiefly on toughness and tenacity. For example, we find that the very fine pianoforte wire used with Lord Kelvin's deep-sea sounding machine is both hard and rigid, but possesses great toughness and tenacity. The copper wire used for electrical conductors becomes harder and harder as it gets drawn down to smaller and smaller sizes, and it has therefore to be annealed in order to comply with the many bendings and unbendings which it has afterwards to undergo in winding and unwinding it upon bobbins whilst twisting it into a stranded conductor or in covering it with a dielectric of cotton, silk, gutta-percha, or india-rubber, &c. Solid-drawn copper pipes are frequently used for conveying steam and liquids where a sound light job is required to resist great pressures. This flowing property of metals is now taken great advantage of by the engineer in a variety of ways. For example, lead and tin, when subjected to great hydraulic pressure, and properly guided through a die, can be squirted into long continuous rods or pipes, or squeezed on to insulated electric light conductors, so as to form a water-tight protecting sheathing thereto, just as if these metals were composed of so much plastic dough. In fact, all you have to do in order to cause many harder and stronger metals, such as copper, wrought-iron, and mild-steel, to *flow cold* into almost any shape of mould, is to *apply sufficient pressure and to give sufficient time* for them to retain their natural homogeneous molecular structure, or to adopt means for restoring the structure should they have departed therefrom during any part of the process. A metal or other body is said to be homogeneous when it is of the same nature or alike in every respect throughout its mass.

* Refer to the description of the Lever Testing Machine, illustrated in Lecture IV., and to Sir William Thomson's Hydrostatic Wire Testing Machine, illustrated in Lecture XVII., as examples of machines whereby the comparative ductility of certain materials may be ascertained by their percentage elongation.

10. Elasticity is that property, possessed by different solids in a greater or less degree, of regaining their original size and shape after the removal of the force which caused a change of form. We shall see later on that there are limits of elasticity beyond which the bodies will not regain their exact normal size or shape.

12. Fusibility is that property whereby metals and many other substances, such as resins, tallows, &c., become liquid on being raised to a certain temperature. The following table shows in *round numbers* the melting-points of a few of the commoner metals :—

MELTING POINTS OF METALS IN DEGREES FAHRENHEIT.

Mercury	− 38	Copper	2000
Tin	+440	German silver	2000
Bismuth	500	Gold	2000
Lead	600	Cast iron	2200
Zinc	700	Steel	2500
Antimony	800	Nickel, also Aluminium	2800
Brass	1800	Wrought iron	3300
Silver	1850	Platinum	3500

Load, Stress, and Strain.—When force is applied to a body so as to produce either elongation or compression, bending, torsion, shearing, or a tendency to any of these, the force applied is termed the *load*, the corresponding resistance or reaction in the material is termed the *stress* due to the load. Any alteration produced in the length or shape of the body is termed the *strain*.

DEFINITIONS.—*Load* is the force or forces applied to the body.

Stress is the reaction in the body due to the load.

Strain is the alteration in shape as the result of the stress.

The load is called a *dead load* when it produces a steady or a gradually increasing or diminishing stress. For example, the weight of a roof on the walls of a building is a steady or dead load. The gradually increasing pull produced on the specimen in the lever-testing machine, illustrated by the fourth figure in Lecture IV., is also a dead load.

The load is termed a *live load* when it varies from instant to instant. For example, a regiment of soldiers, or a series of vehicles, or a train passing over a bridge creates a *live load* on the bridge.

Total Stress and Intensity of Stress.—The *total stress* is the total reaction due to the total load. The *intensity of stress*, or simply the word *stress*, expresses the reaction per unit area of the cross section. Thus, if P be the total force applied in lbs., and A be the total cross section in square inches, then the

Mean *Intensity of Stress* on the section $= \frac{P}{A}$ lbs. per square inch.

Tensile Stress and Strain.—If the line of action of a load be along the axis of a bar, tie-rod, or beam, so as to tend to elongate the same, the reaction per square inch of cross section is termed the *tensile stress*, and the elongation per unit of length is called the *tensile strain*.

EXAMPLE I.—A wire $\frac{1}{10}$ square inch in cross section, and 10 feet long, is fixed at its upper end. A load of 1000 lbs. is hung from the lower end, and then the wire is found to stretch 1 inch. (1) What is the stress? (2) What is the strain?

ANSWER.—(1) Here $P = 1000$ lbs., and $A = \frac{1}{10}$ sq. in.

Let p = stress or pull per square inch in lbs.

$$\therefore \textit{ The stress, or } p = \frac{P}{A} = 1000 \Big/ \tfrac{1}{10} = \mathbf{10,000 \text{ lbs.}} \text{ per sq. inch.}$$

(2) Original length $= L = 10' = 120''$, and the increase of length $= l = 1''$.

Let e = strain or extension per unit of length, *i.e.*, per inch in this case,

$$\therefore \textit{ The Strain, or } e = \frac{\text{increase of length}}{\text{original length}} = \frac{l}{L} = \frac{1''}{120''} = \mathbf{\cdot 0083}$$

Compressive Stress and Strain.—If the line of action of a load be along the axis of a bar, shore, strut, or pillar, so as to tend to compress or shorten the same, the reaction per square inch of cross section is termed the *compressive stress*, and the diminution per unit of length is called the *compressive strain*.

EXAMPLE II.—A vertical support in the form of a hollow pillar, having 2 square inches cross section of metal, is 10 feet long. With a load of 10,000 lbs. resting on the top, it is found to be compressed $\frac{1}{10}$ of an inch in length. (1) What is the stress? (2) What is the strain?

ANSWER.—(1) Here $P = 10,000$ lbs., and $A = 2$ sq. inches. Let p = stress or compression per sq. in. of cross section in lbs.

$$\therefore \textit{ The stress, or } p = \frac{P}{A} = \frac{10,000}{2} = \mathbf{5000} \text{ per square inch.}$$

(2) Original length $= L = 10' = 120''$, and the diminution of length $= l = \frac{1}{10}''$

Let e = strain or compression per unit of length, *i.e.*, per inch in this case,

$$\therefore \textit{ The strain, or } e = \frac{\text{diminution in length}}{\text{original length}} = \frac{\cdot 1''}{120''} = \mathbf{\cdot 00083}$$

Limiting Stress or Ultimate Strength.—For every kind of material and every way in which a load is applied, there must be a value, which, if exceeded, causes rupture or fracture of the

body. The *least load* which does so is called the *limiting stress* or *ultimate strength per square inch of cross section* of the substance, for the particular way in which the load is applied.

Factors of Safety.—The ratio of the *ultimate strength* or *limiting stress* to *the safe working load* is called the *factor of safety*. This factor of necessity varies greatly with different materials, and even with the same material, according to circumstances. For materials which are subjected to oxidation or to internal changes of any kind, the factor of safety must of necessity be larger than in those which are always kept dry or are well painted and carefully handled. There is no condition in engineering structures which requires a more careful calculation, or estimate of the necessary factors of safety, than that of railway bridges, which are exposed to all sorts of weathers and to extremely variable live loads. The skill of the engineer is therefore brought out, when he designs structures so as to include all possible circumstances to which they may be subjected, and so proportions the material at his disposal, that there shall be a minimum of internal stress and strain, with a maximum resistance to dead or live loads for a minimum cost of material and workmanship.*

TABLE OF ULTIMATE STRENGTHS AND WORKING LOADS OF MATERIALS WHEN IN TENSION, COMPRESSION, AND SHEARING.

Materials.	Ultimate Strength. Tons per sq. inch.			Working Stress. Tons per sq. inch.		
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.
Cast iron . . .	7·5	45	14	1·5	9	3
Wrought-iron bars .	25	20	20	5	3·5	4
Steel bars . . .	45	70	30	9	9	5
Copper bolts . .	15	25	—	3	5	—
Brass sheet . . .	14	—	—	3	—	—

Safe Loads and Elasticity.—As a rule, however, the object of the engineer is not to put such a stress on his materials of construction as will cause rupture or destruction, but rather to

* For other tables relating to the Strength of Materials in Engineering *Constructions*, Factors of Safety, &c., refer to Rankine's "Rules and *Tables*," Molesworth's "Pocket Book of Engineering Formulæ," D. K. *Clarke's* "Rules and Tables," "The Practical Engineer's Pocket Book," and *for* Electrical Engineering Materials to Munro and Jamieson's "Pocket-book *of* Electrical Rules and Tables."

make machines and raise structures that will withstand all reasonable forces likely to be brought to bear upon them. Consequently, he is quite as much interested in what may be termed *safe loads* as in ultimate or destructive ones. He therefore requires to know what loads can be safely applied to materials under different circumstances, so as to comply with that most useful property termed *elasticity*, which we again define as *the capability of regaining their original size, shape, and even strength, after the removal of the forces which caused a change of form in them.*

Limit of Elasticity—Hooke's Law.—So long as the stress or reaction per square inch of cross section does not exceed a certain limit, called the *limit of elasticity*, then the material will return to its original shape, size, and strength, after the removal of the load. This limit has been ascertained for most materials of construction by elaborate experiments, which are to be found tabulated in the Proceedings of the Institutions of the Civil and Mechanical Engineers, and in such books as Rankine's "Rules and Tables," Molesworth's "Pocket Book of Engineering Formulæ," and D. K. Clark's "Rules and Tables." For example, with a bar of good wrought iron the elastic limit is only reached after a stress of 24,000 lbs. per square inch has been brought to bear upon it, and in a similar degree every other material has a corresponding limit, beyond which it is not safe to stress it, for fear that it should be strained, and thus lose its property of recuperation or restitution.

Within this limit there is a rule termed *Hooke's Law*, which holds good for metal bars under the action of forces tending to elongate or compress them. This law states that:

(1) The amount of extension or compression for the same bar is in direct proportion to the stress.

(2) The extension or compression is directly proportional to the length.

(3) The extension or compression is inversely proportional to the cross sectional area; consequently, if the area be doubled the extension or compression will be halved, or the resistance to the load will be doubled.

Let P = Pull, push, or load in lbs. on the bar.
,, A = Area of cross section of the bar.
,, L = Length of the bar before the load was applied.
,, l = Length by which the bar is extended or compressed.
,, p = Stress or load per square inch of cross section = P/A.

Then, so long as $\frac{P}{A}$ does not *exceed the* elastic limit, l varies ..

as P for the same bar; or $\frac{l}{L}$ varies directly as $\frac{P}{A}$, for different bars of the same material and subjected to the same conditions.

In other words, so long as the stress does not exceed the elastic limit, the strain will be proportional to the stress.

Modulus of Elasticity, or Ratio of Stress to Strain.—As we have just indicated, by HOOKE'S LAW, if a metal under test be gradually subjected to a stress, and if the load does not exceed the limits of elasticity of the material, the strain will be in proportion to the load.

Consequently, the ratio of the *stress* to the *strain* is a constant quantity for each particular substance within the limits of Hooke's Law, and is termed the *Modulus of Elasticity* of the substance.

But . . . *Stress* $\propto$ Strain

$\therefore$ . . . *Stress* $=$ E $\times$ *Strain*.*

Where E represents a constant number or modulus depending on the natural elasticity of each material—

$$\therefore \ E = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{P}{A}}{\frac{l}{L}} = \frac{PL}{Al}$$

i.e., ... $\mathbf{PL = A\mathit{l}E}$

Or, imagine—*it is pure imagination*—that a substance could be elongated to double its length or compressed to half its length by subjecting it to a certain load, we should then have an index value, or constant number, or modulus, by which we could compare it with every other substance which did likewise under similar circumstances. This imaginary value is termed the *Modulus of Elasticity*.

For example, take a bar of wrought iron of 1 square inch cross section, which is found to stretch $\frac{1}{24000000}$ part of its length under a stress of 1 lb., and consequently by Hooke's Law twice that amount under a stress of 2 lbs., and so on; then this number (24,000,000) is called the *Modulus of Elasticity* of the iron bar. For, if the elasticity of the bar were perfect, it is evident that a stress of 24,000,000 lbs. would produce a strain or elongation equal to the length of the bar, or, $\frac{24000000}{24000000} = 1$. In other words, the length of the bar would be doubled under this stress. Consequently, we have the following definition,

* *Since stress* is reckoned by so many lbs. per square inch of cross section *of a material*, and *strain* is simply an abstract number, it follows that the *Modulus of* Elasticity (E) must also be reckoned by so many lbs. per *square inch*.

DEFINITION.—*The Modulus of Elasticity of any substance is that load which would double its length on the supposition that the elongation was proportional to the stress, and that the cross section of the bar was of unit area, or one square inch, and supposing the bar to remain perfect during the operation.*

From this we again see that—

$$\textit{Modulus of Elasticity} = \frac{\textit{stress}}{\textit{strain}} = \mathrm{E} = \frac{\mathrm{P}}{\mathrm{A}} \Big/ \frac{l}{\mathrm{L}}$$

Or, ... $\mathbf{PL = A}\,l\,\mathbf{E}$

MODULI OF ELASTICITY TO STRETCHING.

(See Rankine's Rules and Tables for complete Data.)

Material.	Modulus of Elasticity in lbs. per sq. in. in round numbers.	Material.	Modulus of Elasticity in lbs. per sq. in. in round numbers.
	(Mean values.)		(Mean values.)
Wood, Elm . . .	1,000,000	Lead (sheet) . . .	700,000
„ Larch . .	1,100,000	„ (wire) . . .	1,000,000
„ Beech . .	1,300,000	Brass (cast) . . .	9,000,000
„ Birch . .	1,400,000	„ (wire) . . .	14,000,000
„ Mahogany .	1,400,000	Copper (cast) . .	15,000,000
„ Oak . . .	1,500,000	„ (wire) . .	17,000,000
„ Pine (yellow)	1,600,000	Cast Iron	18,000,000
„ Ash . . .	1,600,000	Wrought Iron . .	25,000,000
„ Teak . . .	2,000,000	Steel	35,000,000

EXAMPLE III.—A steel bar 5′ long and $2\frac{1}{4}$ sq. in. in cross section is suspended by one end; what weight hung on the other end will lengthen it by ·016 inch, if the modulus of elasticity of steel is 30,000,000 lbs. per square inch? (S. and A. Exam. 1877.)

ANSWER.—First ask what is wanted? Viz., *stress.*

Now the universal rule is $\textit{Modulus of Elasticity} = \frac{\textit{stress}}{\textit{strain}}$

Or, stress = modulus × strain.

For, *The strain is the elongation per unit of the length.*

$$\textit{Consequently, } l = \frac{\cdot 016''}{5' \times 12''} = \frac{\cdot 016}{60} = \cdot 0002\dot{6}.$$

$\therefore$ The Stress = Modulus $\times$ strain

$$,, \quad = 30{,}000{,}000 \times \cdot 0002\dot{6} = \mathbf{8000\ lbs.}\text{ per sq. in.}$$

And, The *Total Stress* = 8000 lbs. $\times$ 2·25 sq. in. = **18,000 lbs.**

Or, we might have applied the formula previously deduced—viz.,

$$PL = AlE,$$

where P is the total pull required in lbs.

$$\therefore P = \frac{AlE}{L} = \frac{2 \cdot 25 \text{ sq. in.} \times \cdot 016'' \times (30 \times 10^6)}{5 \times 12} = \mathbf{18{,}000\ lbs.}$$

EXAMPLE IV.—What do you understand by stress and strain respectively? If an iron rod, 50 feet long, is lengthened by $\frac{1}{2}$ inch under the influence of a stress, what is the strain? (S. and A. Exam. 1892.)

ANSWER.—*Stress* is the reaction per unit area of cross section due to the load. Let P = the total tension acting on area A;

$$\text{Then } stress = p = \frac{P}{A}$$

Strain is the ratio of the increase or diminution of length or volume to the original length or volume. Let L = original length of a bar of the material, l = amount by which the length is increased or diminished; then, when the bar is subjected to stress,

$$\text{The } strain = e = \frac{l}{L}$$

In the example given, L = 50′ × 12″ = 600 inches; and $l = \frac{1}{2}$ inch.

$$\therefore \text{Strain, } e = \frac{l}{L} = \frac{\frac{1}{2}}{600} = \frac{1}{1200} = \mathbf{\cdot 0008\dot{3}}$$

EXAMPLE V.—From the above question and answer determine the modulus of elasticity of the iron of which the rod is composed, if the load was 4366 lbs., and the cross section of the rod 2 square inches.

ANSWER.—(1) Stress $= \dfrac{\text{Total load}}{\text{Cross area}}$

Or, ... $p = \dfrac{P}{A} = \dfrac{4366 \text{ lbs}}{2} = 2183$ lbs.

(2) Modulus of Elasticity $= \dfrac{\text{Stress}}{\text{Strain}}$

Or, ... $E = \dfrac{p}{e} = \dfrac{2183}{\cdot 0008\dot{3}} = \mathbf{25{,}000{,}000}$

$\therefore$ *A load of* 25,000,000 lbs. *would elongate a rod of the iron to double* its length by tensile stress.

Lecture XXII.—Questions.

1. State and define the essential and contingent properties of matter, and give the names of those engineering materials with which you happen to be practically acquainted, that best exemplify each property.

2. What is the meaning of the term *ductility* as applied to wrought iron? Describe, with sketches, some apparatus for testing a piece of metal as to ductility. If a uniform bar of iron 10 inches long is found to stretch $1\frac{1}{2}$ inches at the time of fracture, what is the measure of the ductility of the material of the bar? (S. and A. Exam. 1889.) *Ans.* 15 per cent.

3. Give the approximate breaking tensile stress for a bar of cast iron of one square inch sectional area, and the same for a bar of wrought iron? What is the meaning of the term ductility as applied to wrought iron, and how is the ductility of iron measured?

4. What must be the diameter in inches of a round rod of wrought iron in order to sustain a load of 50 tons? It is given that a bar of iron 1 square inch in section will just support a load of 25 tons. *Ans.* $= \sqrt{\frac{2}{\cdot 7854}}$

5. What is the modulus of elasticity of a substance? A round bar of iron, 12 feet long and $1\frac{1}{2}$ square inches in sectional area, is held at one end and pulled by a force till it stretches $\frac{1}{8}$ inch; find the force, the modulus of elasticity being 30,000,000. (S. and A. Exam. 1891.) *Ans.* 39,063 lbs.

6. A round bar of steel 1" in diameter and 10 feet long, is fixed at its upper end, and a load is applied to the bottom end and stretches it ·05". Find the load if the modulus of elasticity is 30,000,000. *Ans.* 9817·5 lbs.

7. Find the dimensions of a transverse section of a square rod of fir to sustain a suspended load of 10 tons, the rod being held vertically. The breaking load of a rod of fir one square inch in section is 6 tons. *Ans.* 1·29 inches.

8. Find the extension produced in a bar of wrought iron 4 feet long and 2 square inches in section by a suspended weight of $4\frac{5}{8}$ tons, the modulus of elasticity of the material being 29,000,000 pounds per square inch. (S. and A. Exam. 1889.) *Ans.* ·009 inch.

9. What do you understand by the terms stress, strain, and modulus of elasticity? A tie-rod, 100' long and 2 square inches cross area, is stretched ·75" under a tension load of 32,000 lbs. What is the intensity of the stress, the strain, and the modulus of elasticity under these circumstances? (S. and A. Exam. 1888.) *Ans.* 16,000 lbs. per square inch; 0·000625; 25,600,000.

10. Define what is meant by "dead load," "live load," "limiting stress," "limit of elasticity," and "factors of safety."

11. What do you understand by stress and strain respectively? If an iron rod, 50 ft. long, is lengthened by $\frac{1}{4}$ in. under the influence of a stress, what is the strain? If the rod is 2 sq. in. in section, and the load 11,000 lbs., what is the modulus of elasticity? *Ans.* ·000417; 13,200,000.

12. Find the stress produced in a pump-rod 4" diameter, lifting a bucket 28" diameter if the pressure on the top of the bucket be 6 lbs. per square inch in addition to the atmosphere, and the vacuum below the bucket be 26" by gauge. Reckon each 2" of vacuum = 1 lb. *Ans.* 925 lbs. per sq. in.

13. *If the rod in question is* 5' long, find its extension if the modulus of *elasticity* = 9,000,000. *Ans.* ·006 inch.

LECTURE XXIII.

CONTENTS.—Stresses on Chains—Shearing Stress and Strain—Example I. —Torque or Twisting Movement—Strength of Solid Round Shafts—Example II.—Pressures on and Reactions from the Supports of Beams —Examples III. IV.—Transverse Stress or Bending Moment of Beams —(1) Load at Middle ; (2) Load Distributed—Example V.—Questions.

IN this Lecture we will continue the subject of "strength of materials," and finish the course with reasons for the shapes generally given to sections of cast iron, wrought iron, and steel girders.

Stresses on Chains.—The only stress to which the sides of the links of chains are subjected under ordinary circumstances, is that of tension. This stress tends to bring the sides of the links closer together, and consequently we find that large chain cables for mooring ships (where very sudden and severe stresses are encountered) have a cast-iron stud or wedge fitted between the inner sides of the links. These studs most effectually keep the sides of the links apart, and prevent any link jamming a neighbouring one. They add materially to the strength of the chain, for they are in compression whilst the sides of the links are in tension. Being composed of cast iron, which offers the immense resistance to compression of fully 45 tons per square inch,* there is not much fear of their giving way before the sides of the links.

The strength of a stud-link may be taken as equal to double the strength of a rod of wrought iron, of the same diameter and quality of material as that of which the chain is composed, whereas the strength of an open-link chain is only about 70 per cent. of this amount, even with perfect welding.

In Molesworth's "Pocket-Book of Engineering Formulæ," the student will find at page 54 a formula for the safe load on chains, viz.—

$$W = 7.1 d^2$$

Where W = Safe load in tons.
" d = Diameter of iron in inches.

* *See Table of the Ultimate Strengths and Safe Working Loads given in Lecture XXII.*

Now, such a formula is very easy of application, but the student should never rest content until he finds out how the constants have been arrived at, and what relation the various symbols have towards each other. If he refers back to the short table of "Ultimate Strengths and Working Loads" given in the previous Lecture, he will find opposite wrought-iron bars and under tension, the value 5 tons per square inch as the safe working load. Consequently, applying what was said above about perfect stud-link chains, he will see that—

$$W = \begin{cases} \text{twice the load of a rod of the same diameter and} \\ \quad \text{quality as that of which the chain is composed.} \end{cases}$$

$\therefore$ $W = 2 \times 5 \times$ cross area of the chain iron.

$$W = 2 \times 5 \times \frac{\pi}{4}d^2 = 2 \times 5 \times \tfrac{1}{4} \times \frac{22}{7}d^2 = \mathbf{7 \cdot 8}d^2$$

This is near enough to the constant given by the above empirical formula to enable him to see how it has been obtained.

Chains which are subjected to many sudden jerks (such as lifting chains for cranes and slings) become in time crystalline, or short in the grain, and consequently brittle and unsafe. The best precaution to adopt in order to periodically remove this enforced internal condition, is to draw them once a year very slowly through a fire, thus allowing them to become heated to a dull red, and then to cool them slowly in a heap of ashes. This method is followed at Woolwich Arsenal and some other Government works.

Shearing Stress and Strain.—The action which is produced by shearing and punching machines on iron, steel, or copper plates, &c., is to force one portion of the metal across an adjacent portion. The shearing stress is the reaction per square inch opposing the load or pressure applied to the shears or punch, and the shearing strain is the deformation per unit length or volume. Rivets holding boiler plates together, fulcra of levers, the pins of the links of the chain of a suspension bridge, the cotter keys of a pump rod, are all subjected to shearing stresses and strains. The ultimate and the working shearing stresses for a few engineering materials were given in a table in Lecture XXII.

In the case of loaded beams (which we will consider shortly in connection with bending moments) the *shearing force* at *any* point or any transverse section thereof is equal to the algebraical sum of all the forces on *either* side of the point or section.

EXAMPLE I.—A steel punch 1″ diameter is used in a large *shipyard punching* machine to make holes in steel plates 1″ thick. *What will be the total* shearing stress or least pressure required?

ANSWER.—Referring to the table in last Lecture, we see that the ultimate shearing strength or shearing stress for steel bars (which we will assume to be the same as for plates) is 30 tons per square inch. Now a hole 1″ diameter has a circumference $= \pi d = 3{\cdot}14''$, and since the plate is 1″ thick, the area of the resisting section must be the circumference of the hole × its depth, or $= 3{\cdot}14'' \times 1'' = 3{\cdot}14$ square inches.

$$\therefore \text{ The total pressure required} = 30 \text{ tons} \times 3{\cdot}14 = \mathbf{94{\cdot}2} \text{ tons.}$$

Torque, or Twisting Moment.*—In the case of a shaft having a lever, pulley, or wheel fixed to it with a force P lbs., applied at radius R feet from the centre of the shaft, then

The twisting moment is $= P \times R$ foot-lbs.

Or if R be in inches,

The torque $= P \times R$ inch-lbs.

Strength of Solid Round Shafts.—It is evident from the above, that a shaft subjected to a twisting moment must offer a sufficient *resistance* thereto, otherwise it would be twisted, or sheared, or ruptured through by the torque. It may be proved that in the case of solid round shafts their resistance to torsion is *directly proportional to the cubes of their diameters* when made of the same materal and quality.†

* The term *torque* was devised by the late Professor James Thomson, of Glasgow University, to signify twisting or tortional moment. The foot-lbs. of torque must not be confused with ft.-lbs. of work or with *resilience*, which is the work done in *straining* a body as measured by the elongation or compression in feet × the mean load causing the strain. It will therefore, perhaps, save confusion, to calculate torques in inch-lbs.—*i.e.*, to take the leverage or arm of the moment in inches, and the force applied in lbs.

† This is evident from the fact that the shaft must offer a *moment of resistance*, or *shearing moment*, equal to the *twisting moment* at the instant of rupture. Now, the area to be sheared is the cross area of the shaft $= \frac{\pi}{4}D^2$, where D is the diameter of the shaft. The mean arm or leverage at which this resistance acts is equal to *half the radius* of the shaft, for at the centre the arm is $= o$, and at the circumference it is $= r$, the radius of the shaft. The mean arm is therefore $= \frac{r}{2} = \frac{D}{4}$. And, if the shearing resistance *per square inch* of cross section of the material be $=S$, the product of these three quantities will be the *total shearing moment*, and must equal the *twisting moment*—viz. $= P \times R$, where P is the force applied at the end of the

Let D_1, D_2, D_3 = Diameters of three shafts, 1″, 2″, and 3″ diameter respectively.

T_1, T_2, T_3 = Torques which they will respectively resist when stressed to the same extent.

Then, . . . $T_1 : T_2 : T_3 :: D_1^3 : D_2^3 : D_3^3$

Or, . . . $T_1 : T_2 : T_3 :: 1^3 : 2^3 : 3^3$

$:: 1 : 8 : 27.$

In other words, the strengths of the three solid shafts will be as 1 : 8 : 27.

A good wrought-iron shaft of 1″ diameter has been found to withstand a torque of 800 lbs., or 9600 inch-lbs., which means that they will resist 800 ft.-lbs. force at 1 foot, or 12″ leverage, or 400 lbs. at 2 feet, or 24″, and so on.

Or, . $P \times R' = 800$ feet-lbs. of torque

i.e., . $P \times R'' = 9600$ inch-lbs. torque.

EXAMPLE II.—On the above basis, what force acting at the circumference of a pulley 20″ diameter will break a wrought-iron shaft 2″ diameter?

ANSWER.—By the above rule we have the proportion:

$$T_1 : T_2 :: D_1^3 : D_2^3$$

But $T_1 = P_1 \times R_1'' = 800 \text{ lbs} \times 12''$

And $T_2 = P_2 \times R_2'' = P_2 \times 10''$

$\therefore \; P_1R_1 : P_2R_2 :: D_1^3 : D_2^3$

i.e., $P_2R_2 \times D_1^3 = P_1R_1 \times D_2^3$

$$\text{Or,} \quad P_2 = \frac{P_1R_1 \times D_2^3}{D_1^3 \times R_2} = \frac{800 \times 12'' \times 8}{1 \times 10''} = \mathbf{7680\ lbs},$$

lever or circumference of the pulley, and R the length of the arm or radius of the wheel or pulley.

Consequently, $$P \times R = S\left(\frac{\pi}{4}D^2 \times \frac{D}{4}\right) = S\,\frac{\pi}{16}D^3$$

But S is a constant quantity for any particular material. Also, π and 16 are constants. $\therefore$ $P \times R$ vary as D^3.

At the instant of rupture the *strength* of the shaft just balances or is equal to the *twisting moment* $P \times R$.

$\therefore$ The strength of shaft *varies* as D^3.

This is the same as the general statement in the text above. Without some such algebraical explanation, students are sorely puzzled how the cube of the diameter crops up; or still more so when they see the following which appears in some text-books.

$$\left.\begin{array}{l}\text{The moment of resistance of}\\ \text{a round shaft to torsion}\end{array}\right\} = \frac{3\cdot1416}{16} \times \text{diameter}^3 \times \text{shearing stress.}$$

Such a statement is, however, quite evident after the above analysis. (We must leave the consideration of hollow shafts, tubes, &c., to our Advanced Course.)

POWER THAT STEEL SHAFTING WILL TRANSMIT AT VARIOUS SPEEDS.

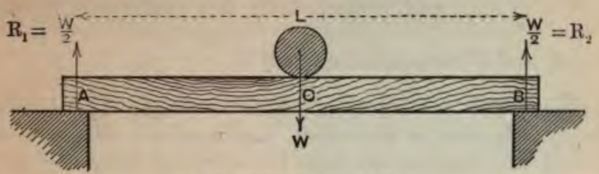
From *The Practical Engineer*, September 2, 1892. By A. G. Brown, M.E.

Revs. per Minute.	Diameters of Shafts in Inches.													
	1½	1¾	2	2¼	2½	3	3½	4	5	6	7	8	9	10
	Horse-Powers they will transmit.													
50	3·3	5·3	8·0	10·9	15·6	27	43	64	125	216	343	512	729	1000
60	4·0	6·4	9·6	13·1	18·8	32	51	77	150	259	412	614	875	1200
70	4·7	7·5	11·2	15·2	21·9	38	60	89	175	302	480	717	1021	1400
80	5·4	8·5	12·8	17·4	25·0	43	69	102	200	346	549	819	1166	1600
90	6·0	9·6	14·4	19·6	28·1	49	77	115	225	389	617	922	1312	1800
100	6·7	10·7	16·0	21·8	31·2	54	86	128	250	432	686	1024	1458	2000
110	7·4	11·8	17·6	23·9	34·4	59	94	141	275	475	755	1126	1604	2200
120	8·1	12·9	19·2	26·1	37·5	65	103	154	300	518	823	1229	1750	2400
130	8·7	13·9	20·8	28·3	40·6	70	111	166	325	562	892	1331	1895	2600
140	9·4	15·0	22·4	30·5	43·8	76	120	179	350	605	960	1434	2041	2800
150	10·1	16·1	24·0	32·6	46·9	81	129	192	375	648	1029	1536	2187	3000
160	10·8	17·1	25·6	34·8	50·0	86	137	205	400	691	1097	1638	2333	3200
170	11·5	18·2	27·2	37·0	53·1	92	146	218	425	734	1166	1741	2479	3400
180	12·2	19·3	28·8	39·2	56·3	97	154	230	450	778	1235	1843	2624	3600
190	12·8	20·4	30·4	41·3	59·4	103	163	243	475	821	1303	1945	2770	3800
200	13·5	21·4	32·0	43·5	62·5	108	172	256	500	864	1372	2048	2916	4000
225	15·2	24·1	36·6	49·0	70·3	122	193	288	563	972	1543	2304	3280	4500
250	16·9	26·8	40·0	54·4	78·1	135	214	320	625	1080	1715	2560	3645	5000
275	18·6	29·5	44·0	59·8	85·9	149	236	352	688	1188	1886	2816	4009	5500
300	20·3	32·2	48·0	65·3	93·7	162	257	384	750	1296	2058	3072	4374	6000
325	21·9	34·8	52·0	70·7	101·6	176	279	416	813	1404	2229	3328	4739	6500
350	23·6	37·5	56·0	76·2	109·4	189	300	448	875	1512	2401	3584	5103	7000
375	25·3	40·2	60·0	81·6	117·2	203	322	480	938	1620	2572	3840	5468	7500
400	27·0	42·9	64·0	87·0	125·0	216	343	512	1000	1728	2744	4096	5832	8000
425	28·7	45·6	68·0	92·5	132·8	230	364	544	1063	1836	2915	4352	6197	8500
450	30·4	48·2	72·0	97·9	140·6	243	386	576	1125	1944	3087	4608	6562	9000
475	32·1	50·9	71·0	103·4	148·4	257	407	603	1188	2052	3258	4864	6926	9500
500	33·7	53·6	80·0	108·8	156·2	270	429	640	1250	2160	3430	5120	7290	10000

... of steel shafts of the same size.

Pressures on and Reactions from the Supports of Beams.—If a beam be supported at its extremities and loaded in the middle, as shown by the following figure, then not only the weight of the beam, but also the load, produce pressures on and equal reactions from the supports A and B.

(1) Neglect the weight of the beam, and consider *only* the effect of the load W.



Reactions at A and B.

LOAD AT CENTRE AND WEIGHT OF BEAM NEGLECTED.

Let R_1 be the reaction at A, and R_2 the reaction at B. Then, by taking moments about the point B, we have—

$$R_1 \times AB = W \times CB$$

$$R_1 \times L \quad = W \times \frac{L}{2}$$

$$\therefore\ R_1 = \frac{W \times L}{2 \times L} = \frac{W}{2}$$

Also, by taking moments about the point A we have—

$$R_2 \times BA = W \times CA$$

$$R_2 \times L \quad = W \times \frac{L}{2}$$

$$\therefore\ R_2 = \frac{W \times L}{2 \times L} = \frac{W}{2}$$

We thus see that the upward reactions are each $= \frac{1}{2}W$, and since action and reaction are equal and opposite, the pressures *downwards* at A and B (due to the load W at the centre of the beam) *must also* be equal to $\frac{1}{2}W$.

(2) If we consider the beam as uniform throughout, and its weight as $= w$, then this force may be supposed to act at its centre of gravity, or at a distance $= \frac{1}{2}L$ from A and B. The load W also acts at a distance $\frac{1}{2}L$ from A and B. Consequently, taking moments about B we have—

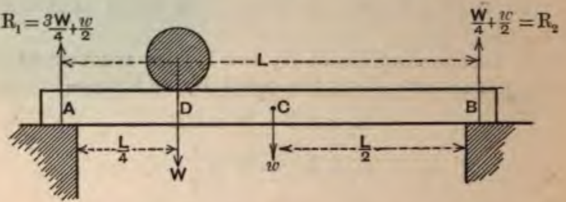
$$R_1 \times AB = W \times CB + w \times CB$$

$$R_1 \times L \;\; = W \times \frac{L}{2} + w \times \frac{L}{2} = \frac{L}{2}(W + w)$$

$$\therefore \; R_1 = \frac{L}{2L}(W + w) = \tfrac{1}{2}(W + w) = \frac{W}{2} + \frac{w}{2}$$

In the same way, by taking moments about A we should find that $R_2 = \frac{W}{2} + \frac{w}{2}$, and consequently the *downward pressures* at points A and B must also be equal to $\frac{W}{2} + \frac{w}{2}$.

EXAMPLE III.—A uniform beam of length L ft. and weight w lbs. is supported at both ends, and carries a weight W at one-fourth of the distance between the supports from one end; find the pressures and reactions at each point of support.



PRESSURES AND REACTIONS AT SUPPORTS A AND B DUE TO WEIGHT OF BEAM AND A LOAD AT D.

ANSWER.—The above figure represents the data given in the question; for, the distance between the supports A and B = L; the weight w of the uniform beam acts at its *c.g.* C, or at a distance $= \frac{L}{2}$ from each end, and the load W acts at D, or a distance $= \frac{L}{2}$ from one end A.

Then, by taking moments about the point B, we have—

$$R_1 \times AB = W \times DB + w \times CB$$

$$R_1 \times L \;\; = W \times \tfrac{3}{4}L \; + w \times \frac{L}{2}$$

(÷ both sides of the equation by L.)

$\therefore$ The *Upward Reaction* at $A = R_1 = \frac{3}{4}W + \frac{1}{2}w$.

And consequently, the *Downward Pressure* at A being equal and opposite to the *Upward Reaction* at A, must also be $= \frac{3}{4}W + \frac{1}{2}w$.

In the same way, by taking moments about the point A, we have

$$R_2 \times BA = W \times DA + w \times CA$$
$$R_2 \times L \quad = W \times \tfrac{1}{4}L \ + w \times \tfrac{1}{2}L$$

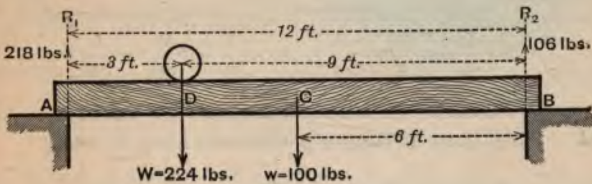
($\div$ both sides of the equation by L.)

$\therefore$ The *Upward Reaction* at $B = \tfrac{1}{4}W + \tfrac{1}{2}w$

And consequently, the *Downward Pressure* at B, being equal and opposite to the *Upward Reaction* at B, must also be equal to

$$\tfrac{1}{4}W + \tfrac{1}{2}w.$$

EXAMPLE IV.—A uniform beam, 12 feet long and weighing 100 lbs., is supported at both ends, and carries a weight of 2 cwt. at a distance of 3 feet from one end; find the pressure on each point of support.



Take moments round B, then

$$R_1 \times AB = W \times DB + w \times CB$$
$$R_1 \times 12' = 224 \times 9' + 100 \times 6'$$
$$\therefore\ R_1 = \frac{2616}{12} = \mathbf{218\ lbs.}$$

To find R_2 we get—

$$R_1 + R_2 = W + w$$
$$\therefore\ R_2 = 224 + 100 - 218$$
$$= \mathbf{106\ lbs.}$$

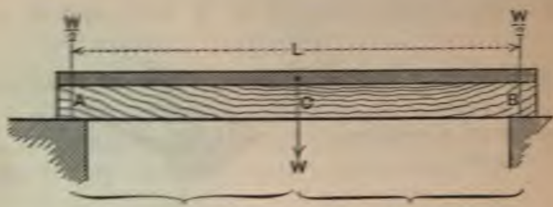
Transverse Stress or Bending Moment of Beams.—A transverse stress is produced by a force or forces acting perpendicularly to the *axis* of a bar or beam. By axis we mean a line passing through the centres of gravity of *all* the transverse or cross sections of the bar or beam.

(1) *Load at Middle.*—Consider the case of a rectangular beam (as represented by the first of the previous figures relating to beams), where the load W is placed at the centre of the beam C, about which point we desire to find the transverse stress or bending moment. Then, *neglecting the weight of the beam itself*, and

confining our attention solely to the load W, we know from the previous proof that an upward reaction $= \frac{W}{2}$ is produced at A and at B.

Then, by taking moments about the point C, we have—

$$\textit{The Bending Moment, or } \mathbf{BM}, \text{ at } C = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$$



$$\leftarrow \frac{L}{4} \rightarrow \frac{W}{2} \leftarrow \frac{L}{4} \rightarrow \leftarrow \frac{L}{4} \rightarrow \frac{W}{2} \leftarrow \frac{L}{4} \rightarrow$$

FINDING THE BENDING MOMENT AT CENTRE,
With Load W uniformly distributed along the Beam.

(2) *Load Distributed.*—Let AB represent the *same* beam, but with the load uniformly distributed along its length, and still *neglecting the weight of the beam*, we see that the loads on AC and on CB are each equal to $\frac{W}{2}$, for they together make up the whole load W. These loads may be considered as acting at points midway between A and C, and midway between B and C, or each of them at a distance $= \frac{L}{4}$ from C. We have also, as before, the reactions $\frac{W}{2}$ at A and at B.

Then, by taking moments about the point C from A or from B, we have—

$$\text{The } \mathbf{B.M.} \text{ at } C = \frac{W}{2} \times \frac{L}{2} - \frac{W}{2} \times \frac{L}{4} = \frac{W}{2}\left(\frac{L}{2} - \frac{L}{4}\right) = \frac{WL}{8}^*$$

This shows that the transverse stress, or bending moment at

* The reaction $\frac{W}{2}$ at A or at B is contrary in direction to $\frac{W}{2}$ at either middle of AC or the middle of BC; consequently, we must take the [difference] of these moments in order to get the net bending moment.

C, is only *half* the magnitude when the load is *uniformly distributed* that it was, when the *whole load* was *concentrated* at the centre C. Consequently, a uniform beam of certain dimensions will bear *double* the load, *evenly distributed*, that it can support if the load be *concentrated at its middle* or about its centre of gravity.

EXAMPLE V.—A uniform beam 12 feet long weighs 400 lbs., and is supported at its extremities. Find the bending moment tending to break the beam at a point 3 feet from one end, and the shearing force.

ANSWER.—Here we have only to consider the weight of the beam just as if we had been considering a uniformly distributed load. Consequently, the previous figure will help the student, for the point about which we have to take moments is 3 feet from one end, or $\frac{3'}{12'} = \frac{1}{4}$ of the whole length between the supports from one end.* Let that point be $\frac{L}{4}$ from A. Then the weight of this part $\frac{L}{4} = \frac{1}{4}$ of 400 lbs. = 100 lbs., and may be considered to act at a point $\frac{L}{8}$ from A, or halfway along $\frac{L}{4}$ from A and *downwards*. In the question $\frac{W}{4} = 100$ lbs. for the whole weight of the beam, or W = 400 lbs. The total weight W produces *upward* reactions at A and at B $= \frac{W}{2} = 200$ lbs.

Then, by taking moments about the point 3 feet from A, we have—

$$\textbf{B.M.} = \left(\frac{W}{2} \times \frac{L}{4} - \frac{W}{4} \times \frac{L}{8}\right) = \left(\frac{WL}{8} - \frac{WL}{32}\right) = \frac{3}{32}WL = \frac{3 \times 400 \times 12}{32} = \textbf{450 ft. lbs.}$$

N.B.—Students may always check the B.M. result as found from one end, by taking moments about the same point from the other end. The two results *must* be equal to each other, for there is equilibrium between their effects. Therefore, by taking moments about the same point 9 feet from B we have—

* If the student experiences any difficulty in understanding the above reasoning, he should at once draw down a figure to scale, marking all distances, weights, reactions, &c., at their proper places.

$$\text{B.M.} = \left(\frac{W}{2} \times \frac{3}{4}L - \frac{3}{4}W \times \frac{1}{2} \times \frac{3}{4}L\right)$$

$$= \left(\frac{3WL}{8} - \frac{9WL}{32}\right) = \left(\frac{12WL - 9WL}{32}\right) = \frac{3WL}{32} = \frac{3 \times 400 \times 12}{32} = \mathbf{50\ ft.\text{-}lbs.}$$

Or, the same result as before.

Shearing Force.—As was previously pointed out in this Lecture, and as will be still further considered in the next Lecture, the shearing force or load at *any* point or *any* transverse section of the beam is equal to the resultant or algebraical sum of all the parallel forces on *either* side of the point or section.

Consequently, in this example, the forces on the A side of the section, where the shearing force is asked for, are $\frac{W}{2}$ acting vertically upwards at A and $\frac{W}{4}$ downwards.

∴ *The Shearing Force to Left of the Section*

$$= \frac{W}{2} - \frac{W}{4} = \frac{W}{4} = \frac{400}{4} = \mathbf{100\ lbs.}\ \textit{upwards.}$$

The Shearing Force to Right of the Section

$$= \frac{3W}{4} - \frac{W}{2} = \frac{W}{4} = \frac{400}{4} = \mathbf{100\ lbs.}\ \textit{downwards.}$$

LECTURE XXIII.—QUESTIONS.

1. An open link chain is constructed of round wrought-iron rod, $\frac{5}{8}$ inch in diameter; calculate what is a probable breaking load on the chain. Wrought-iron chains are liable to deterioration by constant use; what change do they undergo, and what precaution is taken to prevent their breaking?

2. A steel punch $\frac{3}{4}$ inch in diameter is employed to punch a hole in a plate $\frac{5}{8}$ inch in thickness. What will be the least pressure necessary in order to drive the punch through the plate when the shearing strength of the material is 35 tons per square inch? (S. and A. Exam. 1890.) *Ans.* 53·7 tons.

3. Define what is meant by "shearing stress and strain," "torque or twisting moment." Show by an example that a shaft subjected to torque bears a shearing stress tending to sever it at right angles to its axis.

4. What is meant by the "twisting moment" of a shaft? If a wrought iron shaft 1 inch in diameter breaks in torsion by a force of 800 lbs. at the end of a lever 1 foot long, what force at the end of a lever 2 feet long will break a shaft of the same material, but 2 inches in diameter? Find also the diameter of a wrought-iron shaft to resist a force of 2 tons at a distance of 18 inches from its centre. *Ans.* 3200 lbs. 2 inches full.

5. If a shaft, 2 inches in diameter, is found equal to the transmission of 4 horses' power, what amount of power can be transmitted by a shaft 4 inches in diameter, all other conditions remaining the same? *Ans.* 32 horse-power.

6. If a revolving shaft, which is 2 inches diameter, is found sufficiently strong to transmit 4 horse-power, how much power may be transmitted by a shaft which is 3 inches in diameter, supposing all the other conditions to be the same, and that the iron of both shafts is subjected to the same stress? *Ans.* 13·5 H.P.

7. If 800 lbs. at the end of a 12-inch lever be a safe stress to apply to a wrought-iron bar *one* square inch in section, find the effort which a shaft 2 inches in diameter can transmit at the circumference of a pulley *one* foot in diameter, and making 300 revolutions per minute. Find also the horse-power transmitted. *Ans.* 8893 lbs.; 254 H.P.

8. If a wrought-iron shaft of 1 inch diameter is broken by the torsion of a load of 800 lbs. acting at the end of a 12-inch lever, find the weight which, when applied to the end of the same lever, would break a shaft of the same material, but 3 inches in diameter. State, in general terms, the reasoning by which you arrive at the result. (S. and A. Exam. 1891.) *Ans.* 21,600 lbs.

9. A uniform beam 10 feet long, and weighing 1000 lbs., is supported at both ends. A weight of 100 lbs. is placed at a distance of two feet from one end; find the pressure and reaction at each point of support and make a side view of the arrangement to scale, marking on your sketch the weights, distances, and reactions at each place. *Ans.* 580 lbs.; 520 lbs.

10. Define what is meant by the bending moment and the axis of a beam. A uniform beam 10 feet long weighs 500 lbs., and is supported at its extremities. Find the bending moment tending to break it at a point 4 feet from one end. (S. and A. Exam. 1890.) *Ans.* 600 ft.-lbs.

11. A beam 12 feet long is supported at its ends, and is loaded with a weight of *3 tons* at a point two feet from one end. Find the bending

moment at the centre of the beam, and also the shearing force. (S. and A. Exam. 1891. *Ans.* B.M. = 36 inch-tons, S.F. = 0·5 ton.

12. A bar of pine 44 inches long rests on props at its extremities, and just supports 10 weights of 14 lbs. each, hung at equal intervals of 4 inches along the rod. Find the amount of a single weight which, if hung at the centre of the bar, would strain it to the same extent. *Ans.* 76·36 lbs.

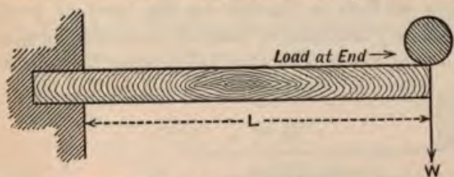
13. A batten of fir, 6 feet in length and supported at its extremities, will just sustain a load of 520 lbs. when hung at the centre. If this weight be removed, and two weights, each equal to P lbs., be hung at distances of 2 and 4 feet along the bar, what is the greatest value which may be assigned to P? *Ans.* 390 lbs.

LECTURE XXIV.

CONTENTS.—Bending Moment of Cantilevers, (1) Loaded at Centre, (2) Load Uniformly Distributed—Bending Stresses—Neutral Surface and Neutral Axis—Moment of Resistance opposed to the Bending Moment — Strength of Rectangular Beams — Relative Strengths of Rectangular Beams supported and loaded in Different Ways—Illustrations, Explanations, and Formulæ for Rectangular Beams supported in Different Ways—Comparison of the Loads and Sizes of Beams by Proportion—Example I.—Different Sections of Cast-Iron, Wrought-Iron, and Steel Beams—Questions.

Bending Moment of Cantilevers.—When a bar or beam is fixed rigidly at one end by being built into a wall or otherwise, and projects outwards for the purpose of supporting a load, it is termed a *cantilever*.

(1) *Loaded at Outer End.*—If, as shown by the following figure,



CANTILEVER.—LOAD AT OUTER END.

the load be placed at the outer end, then the maximum bending moment occurs at the point of support.

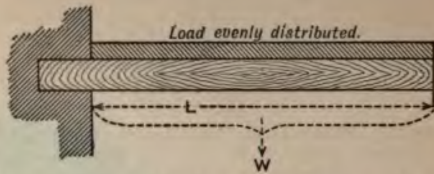
$$\because \textit{ The Maximum } \mathbf{B.M.} = W \times L$$

The beam, if of uniform section and material, would be broken *close* to the wall by a load sufficient to overcome the *moment of resistance of the beam.*

(2) *Load Distributed Uniformly.*—If the load be uniformly distributed, as shown by the following figure, the beam would also break close to the wall if the beam was of uniform section and material; but, as we shall see, it would sustain double the load of the previous case. Taking moments about the edge of the support, we have

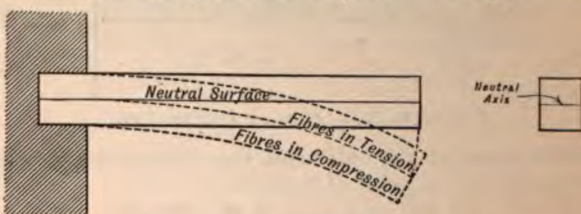
$$\textit{The Maximum } \mathbf{B.M.} = W \times \frac{1}{2}L = \frac{WL}{2}$$

Therefore, since the *bending moment is only half* what it is in the first case, it will take twice the load in the second case to break the beam close to the wall.



CANTILEVER.—LOAD UNIFORMLY DISTRIBUTED.

Bending Stresses, Neutral Surface and Neutral Axis.—If a cantilever be loaded in the manner shown by the two previous figures, then the fibres or resisting material of which the beam is composed will be subjected to a *tensile stress* tending to stretch or elongate the *upper layers;* whilst those in the *lower half* will be subjected to a *compressive stress* tending to compress or crush the fibres. These actions are graphically represented by the accompanying figure. There must therefore be one layer or horizontal longitudinal section which is neither in tension nor in compression. This surface is known as the *neutral surface*, and its intersection on any transverse section, is called the *neutral axis* of that section, as shown by the small figure on the right hand.



NEUTRAL SURFACE AND NEUTRAL AXIS IN A CANTILEVER LOADED AT OUTER END.

If a beam be supported at both ends and loaded anywhere between the bearings (as shown by any of the four last figures in the previous Lecture), then the upper set of layers are naturally in compression, whilst the under set are in tension.*

* It is therefore clear, that a wooden beam may have a saw-cut inserted into the upper set of layers without very materially affecting the strength of the beam, if the edges come together and jam up the saw-cut in the bending of the beam; or, if a wedge be inserted into the cut so as to

Moment of Resistance opposed to the Bending Moment.—It can be proved by mathematics, that the *resultant of all the tensional stresses* (on one side of the neutral axis of any transverse or cross section of a beam) *is equal to the resultant of all the compressive stresses* (on the other side of the neutral axis at the particular cross section considered). These two equal and opposite forces constitute a couple, whose moment is opposite in direction and equal in magnitude to the *bending moment* at the cross sections. It therefore constitutes the *moment of resistance* of the beam, which is opposed to the *bending moment.*

Strength of Rectangular Beams.—The *resisting moment* of a *cantilever* of rectangular cross section, loaded at the outer end, as illustrated by the first figure in this Lecture, is expressed by the formula—*

$$\mathbf{R.M.} = k\mathrm{BD}^2$$

Where RM = Resisting moment in inch lbs.
" k = Constant number found by trial depending on the *nature* of the material of which the beam is composed.
" B = Breadth of beam in *inches.*
" D = Depth of beam in *inches.*

Then if W = Weight or load tending to bend or break beam in lbs.
And if L = Length of beam in *inches.*

The *Bending Moment* = The *Resisting Moment.*

Or, . . . $\mathbf{B.M.} = \mathbf{R.M.}$

i.e., . . . $\mathrm{W} \times \mathrm{L} = k \times \mathrm{B} \times \mathrm{D}^2$

$\therefore$ . . . $\mathrm{W} = k\dfrac{\mathrm{BD}^2}{\mathrm{L}}$

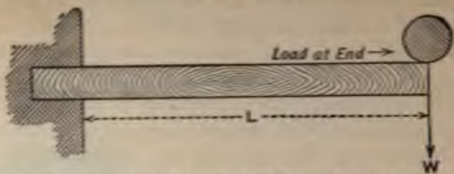
Hence the general rule, the *strength of rectangular beams to resist bending is directly proportional to the breadth, to the square of the depth, and inversely proportional to the length of the beam.*

Relative Strength of Rectangular Beams supported and loaded in Different Ways.—We have already proved the relative values of the bending moments for rectangular beams supported and loaded in the following ways, and we have also proved that the relative bending moments are inversely as the

transmit the compressive stresses. A saw-cut, however, in the lower side would very much affect the strength of the beam.

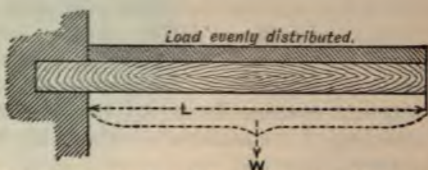
* We shall have occasion to analyse and prove this formula in our Advanced Course.

Illustrations, Explanations, and Formulæ for Rectangular Beams supported in Different Ways.



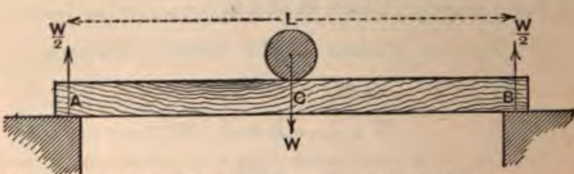
Case I.—Fixed at one end and loaded at the other. The bending moment has maximum advantage. Therefore—

$$\mathbf{W} = k\frac{\mathbf{BD}^2}{\mathbf{L}}$$



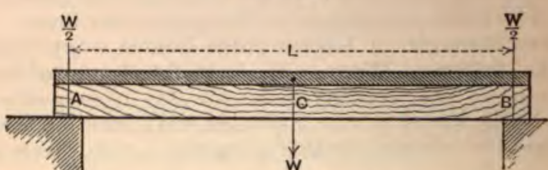
Case II.—Fixed at one end and loaded uniformly. Here the bending moment has only $\frac{1}{2}$ the advantage that it has in Case I. Therefore—

$$\mathbf{W} = 2k\frac{\mathbf{BD}^2}{\mathbf{L}}$$



Case III.—Supported at both ends and loaded at the centre. Here the bending moment has only $\frac{1}{4}$ the advantage that it has in Case I. Therefore—

$$\mathbf{W} = 4k\frac{\mathbf{BD}^2}{\mathbf{L}}$$



Case IV.—Supported at both ends and loaded uniformly. Here the bending moment has only $\frac{1}{8}$ the advantage that it has in Case I. Therefore—

$$\mathbf{W} = 8k\frac{\mathbf{BD}^2}{\mathbf{L}}$$

relative strengths or loads which they can support. Hence for beams:

WAYS IN WHICH BEAMS ARE SUPPORTED AND LOADED.	Relative B. Ms.	Relative R. Ms. or Strengths.
I. Fixed at one end and loaded at the other .	1	1
II. Fixed at one end and loaded uniformly .	$\frac{1}{2}$	2
III. Supported at both ends and loaded at centre	$\frac{1}{4}$	4
IV. Supported at both ends and loaded uniformly	$\frac{1}{8}$	8

Comparison of the Loads and Sizes of Beams by Proportion.—We have already stated that the constant numerical value represented by k (in the preceding formula for the strengths of beams having a rectangular cross section) has to be ascertained by trial. The usual way is to take a comparatively small beam of the same material, and to support as well as load it in the precise manner that the actual beam has to be supported and loaded.

Then, by carefully ascertaining *either* the *breaking load* of this elementary beam (if it should be the *ultimate strength* that is required), *or* the load which produces a certain safe *ratio* of the deflection from the horizontal to the distance of the load from its support (if it should be the *safe load* that is desired), you interpolate the numerical values of these loads in the formula, and thereby ascertain the probable ultimate strength or safe load of the full-sized beam to be used in practice.

Let w = Weight or load carried by the experimental beam.
" b = Breadth of the experimental beam in *inches*.
" d = Depth " " " "
" l = Length " " " "
" W = Weight or load *to be* carried by the full-sized beam.
" B = Breadth of full-sized beam in *inches*.
" D = Depth " " " "
" L = Length " " " "

Then, *if the two beams are supported and loaded in exactly the same way*, we have by proportion—

$$b \; : B \; :: w : W$$
$$d^2 \; : D^2 :: w : W$$
$$: l \; :: w : W$$

By combining these proportions we get—

$$bd^2L : BD^2l :: w : W$$

Or, . . . $$W \times bd^2L = w \times BD^2l$$

$$W = w\frac{BD^2l}{bd^2L}$$

Note—wl / bd^2 therefore takes the place here of the constant k in the previous formulæ.

EXAMPLE I.—A bar of teak, 1 inch square and 12 inches between the supports, breaks with a load of 820 lbs. when hung at its centre. Find the breaking load at the centre of a bar of teak, 3 inches broad and 3 inches deep and 7 feet between the supports. If the bar be 2 inches broad instead of 3 inches, what should be its depth in order to support the same weight at the centre? (S. and A. Exam. 1888.)

ANSWER.—Here, *in the first case,* $b = 1''$; $d = 1''$; $l = 12''$; $w = 820$ lbs.; $B = 3''$; $D = 3''$; $L = 7' = 7 \times 12'' = 84''$.

Consequently, by the previous formula just deduced—

$$W = w\frac{BD^2l}{bd^2L}$$

$$W = 820 \times \frac{3 \times 3 \times 3 \times \cancel{12}}{1 \times 1 \times 1 \times \underset{7}{\cancel{84}}} = \frac{820 \times 3 \times 3 \times 3}{7} = \mathbf{3162 \cdot 8 \text{ lbs.}}$$

In the *second case* we might quite easily interpolate the numerical values as we did in the *first* case, and thereby arrive at the result; but it will be seen at once by the student, that since the only variable introduced into the second part of the question is the breadth, we have only to equate the breadth and depth to the constant load W. Thus, by calling $B_1 = 3''$; $B_2 = 2''$; $D_1 = 3''$; and D_2 the depth to be found, we have at once, from the above formula, since the load W and everything else are *constants,*

$B_1 \times D_1^2$ is proportional to W;

$B_2 \times D_2^2$ is also proportional to W;

$$\therefore B_1 \times D_1^2 = B_2 \times D_2^2$$

Or, . . . $$3'' \times 9 = 2'' \times D_2^2$$

$$\therefore D_2^2 = \frac{3 \times 9}{2} = \frac{27}{2} = 13 \cdot 5$$

$$\therefore D_2 = \sqrt{13 \cdot 5} = \mathbf{3 \cdot 674} \text{ inches.}$$

Different Sections of Cast-Iron, Wrought-Iron, and Steel Beams.—Having shown how the strength of a rectangular beam varies directly as the breadth B, and as the square of the depth D, it is natural, in the case of materials such as cast-iron, wrought-iron, and steel (which vary in regard to their resistance to extension and compression), that we should endeavour to show how these materials may be most economically disposed, so as to withstand the greatest load for a minimum of weight and cost In the case of wooden beams, where it is found unprofitable to make them into any other shape, than the plain rectangular form —unless it be for purely architectural or artistic purposes—we only considered their strength when of that particular section; but it becomes quite another matter, when we have to consider the case of iron, for this material in its various modifications may be cast or rolled into any required shape, and therefore the weight and disposition of the material have a special bearing upon the cost and strength of iron beams.

Cast-Iron Beams or Girders.—The term girder is technically applied to beams of cast-iron, wrought-iron, or steel, which are used for spanning comparatively long distances, such as road or railway bridges, or large warehouses. As will have been observed from the table of ultimate strengths in Lecture XXII., the *ratio* of the ultimate strengths of cast-iron to compression and tension is as 45 to 7·5 tons; and further, since the stress on any material of which a beam may be composed is smaller and smaller the nearer the *neutral surface*, it is but natural that we should make the *upper* flange of a cast-iron beam *which is supported at both ends smaller* than the *lower* one, since the upper flange, being in compression, will obviously maintain a stress six times as great as an equal section in the lower side. On the other hand, if the cast-iron beam is of the *cantilever* type, where the upper side is subjected to tension and the lower side to compression, we should make the upper flange about six times as strong as the lower one. In practice, however, for the sake of obtaining a sound casting, (*i.e.*, having regard to the way in which the crystals of the metal naturally arrange themselves), the flange in tension is only made about four times the cross sectional area of the flange in compression as will be seen from the accompanying figure.


SECTION OF A CAST-IRON BEAM.

For vertical columns of cast iron, where the stress is purely one of compression, the H form is provided with flanges of the same dimensions at each end of the web, or, to come to the most

common form of strut—viz., that of the vertical cylindrical column—the section is that of a circle, or an **O**, since the stress is equally disposed throughout the cross section of the material.

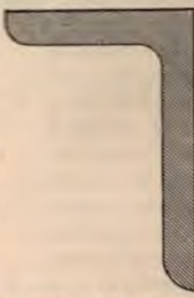
Wrought-Iron Beams and Girders.—Referring to the same Table of Ultimate Strengths and Working Loads in Lecture XXII., we see that the resistance to tension is 25 tons per square inch and to compression 20 tons, but from an average of a large

CROSS SECTION OF A WROUGHT-IRON BEAM.

CROSS SECTION OF A WROUGHT-IRON BOX GIRDER.

CROSS SECTION OF CHANNEL IRON. — STRENGTH IS LARGELY DUE TO THE DEPTH OF THE SIDES.

ORDINARY **L** IRON.



SPECIAL **L** IRON FOR HEAVY LOADS.

ORDINARY **T** IRON.

STEEL BEAM.

CROSS SECTIONS OF WROUGHT-IRON AND STEEL BEAMS.

number of specimens it is found that the resistances to those two kinds of stresses is about the same. Consequently, the material may be disposed of equally between the top and bottom flanges, and remembering that the stress at the neutral surface is zero, we have only to connect the flanges with a sufficient thickness of metal to keep them together and to transmit the vertical shearing stresses. Wrought-iron beams and girders are therefore constructed of the forms shown by the first two figures above.

In special cases, such as ship-building and bridge-building, different forms of angle iron, **T** iron, and channel iron are used, as shown by the next four figures.

Steel Beams.*—Referring once more to the Table of Ultimate Strengths and Working Loads in Lecture XXII., we see that the resistance of steel to tension is only 45 tons per square inch of cross section, whereas it withstands 70 tons per square inch against compression. Consequently, in forming beams of this material, where the beam is supported at both ends, and the load is either placed in the middle or distributed, the bottom flange which is in tension must be made about double the cross area or weight of the top one, as shown by the last figure.

We have merely touched the fringe of this important subject on metal beams. We shall therefore have to return to it again in the Advanced Course.

* The Author is indebted to Messrs. P. and W. MacLellan, the well-known manufacturers of cast-iron, wrought-iron, and steel beams, for the full-size drawings of the various sections from which these reduced figures have been made by the Publishers, with the aid of photography.

LECTURE XXIV.—QUESTIONS.

1. A wooden beam, supported at both ends and loaded in the middle, may have a saw-cut made upon the upper side without affecting the strength to an appreciable extent (if a wedge is inserted). How do you account for this result?

2. It will be observed that wooden beams are usually made rectangular in form, the depth being greater than the width. State the advantage that is derived from this form of arrangement, and the relation of strength in proportion to depth, width, and distance between the supports.

3. State the relative strengths of similar rectangular beams under the following varying conditions:—

(*a*) When fixed at one end and loaded at the other.
(*b*) " " and load distributed.
(*c*) When supported at both ends and load at centre.
(*d*) " " " " distributed.

4. Given that a rectangular rod of fir, 10 inches long, 1 inch broad, and 1 inch deep, and supported at both ends, will just sustain 540 lbs., when hung at its centre; what should be the depth of a bar of like wood 5 feet long and 2 inches broad, and supported at both ends, in order to support a load of $\frac{3}{4}$ of a ton when hung at its centre? *Ans.* 3·05 inches.

5. A wooden beam is commonly rectangular in form, the depth being greater than the breadth. State the law according to which the strength of the beam to resist a cross or transverse stress is connected with the breadth and depth. Two beams are of breadth 5 and 6 inches, and of depths 8 and 9 inches respectively; write down the numbers which represent their respective strengths in resisting transverse stresses. *Ans.* 160 : 243.

6. Two wooden beams are each loaded in the centre and supported at the ends; one is solid and measures 8 inches × 8 inches in cross section; the other is made up of two beams each 8 inches broad and 4 inches deep, and placed one over the other so as to have the same sectional area as before. Will there be any difference in the breaking load of the beams, and if so, how much will it be? State the reasoning on which you rely. (S. and A. Exam. 1889.)

7. There are two beams of the same kind of timber, each of which is supported horizontally by props at the ends. One beam is 10 feet long, 7 inches broad, and 5 inches deep, while the other is 13 feet long, 5 inches broad, and 7 inches deep, which beam will bear the greatest load on its centre? *Ans.* $W_1 : W_2 :: 13 : 14$.

8. A beam of timber, rectangular in transverse section, is 2 inches broad, 3 inches deep, and 4 feet in length, and rests upon supports at its ends. The breaking load on the centre is 2000 lbs. What would have been the breaking weight if the beam had been 4 inches deep, 2 inches broad, and 4 feet between the supports, but loaded at a distance of 1 foot from the end? (S. and A. Exam. 1887.) *Ans.* 4740·74 lbs.

9. Find the breaking weight at the centre of a beam of Memel fir, 12 inches deep, 10 inches wide, and 20 feet between the points of support. The breaking weight at the centre of a beam 1 foot long and 1 inch square is 545 lbs. *Ans.* 39,240 lbs.

10. A rectangular batten of fir, 6 feet in length, 2 inches broad, and 3 inches deep, is supported at its ends and can sustain a weight of 1100 lbs. when hung at the centre. If the load were equally distributed instead of

being hung at the centre, how much would the batten support? *Ans.* 2200 lbs.

11. A rectangular beam of timber is supported at both ends, and can just bear a weight of W lbs. evenly distributed without breaking. If the load were all brought into the centre, how much should the breadth of the beam be increased, the depth remaining unchanged? Again, if the breadth were to remain constant, by how much should the depth of the beam be increased? Explain clearly the reasons for your answer. *Ans.* Twice; $\sqrt{2}$ times.

12. A rectangular beam of timber supported at both ends, and of a given breadth and depth, just supports a load, W, at its centre. If the load be shifted to a point halfway between the centre and one end, by how much may the depth be reduced? *Ans.* 13·4 per cent.

13. Show, with sketches, the best forms and sections of flanged beams, (1) of cast iron, (2) of wrought iron, when supporting a load at the centre, and state the reasons which determine the particular forms.

14. Explain why iron girders are made with flanges at the top and bottom united by a web of metal, instead of being rectangular in section. What condition decides which of the two flanges shall contain the most metal?

15. Girders for carrying a load on their top flange, if of cast iron, have the section of metal on the bottom flange greater than on the top flange, but when made of wrought iron this rule does not hold. Why is this? Sketch a section of an ordinary cast-iron girder to carry a wall over a gateway, and of a wrought iron plate girder for the same purpose. (S. and A. Exam. 1890.)

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